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STRING DUAL OF $\mathcal{N} = 1$ SUPER-YANG-MILLS ON THE CYLINDER

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OUTLINE OF THE TALK

- $\mathcal{N} = 1$ pure super-Yang-Mills (SYM): $\mathbb{R}^{1,3}$ versus $\mathbb{R}^{1,2} \times S^1$

- String (gravity) dual description of $\mathcal{N} = 1$ pure SYM on $\mathbb{R}^{1,2} \times S^1$

- Identification of the $\mathsf{relevant\ \ degrees}$ of freedom of the gravity dual

- Generation of a $\mathsf{non-perturbative}$ $\mathsf{superpotential}$ built out of branes

- Conclusions and perspectives

$\mathcal{N}=1$ (PURE) SYM ON $\mathbb{R}^{1,3}$
THEORY WITH $SU(N)$ GAUGE GROUP

- Vector multiplet (λ, A_μ)

  $W_\alpha = \lambda_\alpha + \theta^\beta F_{\alpha\beta} + \ldots$

- Lagrangian:

  $\mathcal{L} = 2\pi i\tau \int \mathrm{d}^2\theta \, \mathrm{Tr}\, W^2 + \mathrm{h.c.}$

- Complexified coupling constant:

  $\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g_{YM}^2}$

GAUGINO CONDENSATE

- Strong Coupling Instantons computation on $\mathbb{R}^{1,3}$

$$\left\langle \frac{tr\,\lambda^2(x_1)}{16\pi^2} \cdots \frac{tr\,\lambda^2(x_N)}{16\pi^2} \right\rangle \sim \Lambda^{3N}$$

Cluster decomposition $\Rightarrow$ $\left\langle \frac{tr\,\lambda^2(x)}{16\pi^2} \right\rangle = c\,\Lambda^3$

- Weak Coupling Instanton computations

$\begin{cases} \bullet \text{ adding matter} \\ \bullet\ \mathbb{R}^{1,2} \times S^1 \end{cases} \Rightarrow \left\langle \frac{tr\,\lambda^2(x)}{16\pi^2} \right\rangle = \Lambda^3$

We miss the U.V. description of some relevant non-perturbative configurations!
Davies et al [hep-th/9905015]

SUPERGRAVITY SOLUTION

Chamseddine and Volkov [hep-th/9707176]
Maldacena and Nuñez [hep-th/0008001]

Metric:

$$ds^2_{10} = e^{\Phi}\left[dx^2_{1,3} + \frac{e^{2h}}{\lambda^2}\left(d\tilde{\theta}^2 + \sin^2\tilde{\theta}\, d\tilde{\varphi}^2\right)\right] + \frac{e^{\Phi}}{\lambda^2}\left[d\rho^2 + \sum_{a=1}^{3}(\sigma^a - A^a)^2\right] \;,$$

Dilaton:

$$e^{2\Phi} = e^{2\Phi_0}\,\frac{\sinh 2\rho}{2\,e^{h}} \;,$$

Magnetic R-R 2-form:

$$C_{(2)} = \frac{1}{4\lambda^2}\Big[\psi\left(\sin\tilde{\theta}\, d\tilde{\theta}\wedge d\tilde{\varphi} - \sin\theta\, d\theta\wedge d\varphi\right) - \cos\theta\,\cos\tilde{\theta}\, d\varphi\wedge d\tilde{\varphi}\Big] + \frac{a}{2\lambda^2}\left[d\tilde{\theta}\wedge\sigma^1 - \sin\tilde{\theta}\, d\tilde{\varphi}\wedge\sigma^2\right]$$

where

$$e^{2h} = \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4} \;,$$

$$a = \frac{2\rho}{\sinh 2\rho} \qquad \frac{1}{\lambda^2} = N g_s \alpha'$$

SYM ON THE CYLINDER ($\mathbb{R}^{1,2} \times S^1$)

$$\begin{array}{ccccc} \text{IIB} & \xrightarrow{T_1} & \text{IIA} & \xrightarrow{\tilde{UP}} & \text{11D} \\ \text{D5} & & \text{D4} & & \text{M5} \\ S \updownarrow & & & & ||| \quad \mathbb{R}^{1,2} \times S^1 \times \tilde{S}^1 \times CY_3 \\ \text{IIB} & \xrightarrow{\tilde{T}_1} & \text{IIA} & \xrightarrow{\text{UP}} & \text{11D} \\ \text{NS5} & & \text{NS5} & & \text{M5} \end{array}$$

Metric:

$$ds_{11}^2 = e^{\frac{2}{3}\Phi} \left[dx_{1,2}^2 + \frac{e^{2h}}{\lambda^2} \left(d\tilde{\theta}^2 + \sin^2 \tilde{\theta}\, d\tilde{\varphi}^2 \right) \right] + \frac{e^{\frac{2}{3}\Phi}}{\lambda^2} \left[d\rho^2 + \sum_{a=1}^{3} (\sigma^a - \lambda A^a)^2 \right] + e^{\frac{2}{3}\Phi}\, dy^2 + e^{-\frac{4}{3}\Phi}\, dz^2$$

Magnetic 3-form:

$$C_{(3)} = C_{(2)} \wedge dz$$

M5-BRANE WORLDVOLUME THEORY

The worldvolume “PST” action:

$$S = T_{M5}\int d^6\xi\Bigg(-\sqrt{-det(g+\tilde{H})}+$$

$$\frac{\sqrt{-det\, g}}{4\partial a\cdot\partial a}\partial_i a(\star H)^{ijk}H_{jkl}\partial_a^l\Bigg)+\frac{T_{M5}}{2}\int F\wedge C_{(3)}$$

where

$$H = F - C_{(3)} \quad , \quad F = dA_{(2)}$$

$$\tilde{H}^{ij} = \frac{1}{3!\sqrt{-det\, g}}\frac{1}{\sqrt{-(\partial a)^2}}\epsilon^{ijklmn}\partial_k a H_{lmn}$$

Worldvolume geometry: $\mathbb{R}^{1,2}\times\tilde{S}^1_y\times(S^2\subset CY_3)$

Gauge choice: $a = y$

Two-form worldvolume potential:

$$\frac{A_{(2)}}{(2\pi)^2 g_s} = \alpha'\frac{y}{4\pi g_s}F_{ab}dx^a\wedge dx^b + (\alpha')^{3/2}\Sigma\, d\Omega_2$$

We get:

$$S = -\frac{\pi\sqrt{\alpha'}g_s}{V_2}\int d^3x\,\partial_c\Sigma\partial^c\Sigma - \frac{1}{2}\int d^3x\,\epsilon^{abc}F_{ab}\partial_c\Sigma$$

Σ is the scalar dual to the three dimensional vector field $F_{ab} = \partial_a A_b - \partial_b A_a$!

In our case we have $SU(N) \to U(1)^{(N-1)}$

with $b_i \sim z_i \sim S^1_z$ (fluctuations in z direction)

$$S_i = -\frac{1}{2}\int d^3x \left[\frac{2\pi}{g^2_{YM}R}\partial_a b_i\,\partial^a b_i + \frac{g^2_{YM}}{2\pi R}\left(\partial\Sigma_i - \theta_{YM}\partial b_i\right)^2\right]$$

Seiberg and Witten [hep-th/9607163]

We can define $\gamma_i = \frac{g^2_{YM}}{2\pi}\Sigma_i$

and get an holomorphic action in terms of

$$\Phi_i = b_i + i\,\gamma_i$$

GENERATION OF A (NON-PERT.) SUPERPOTENTIAL

Euclidean M2-brane configurations preserving two zero-modes generate:

$$W \sim M^3 \sum_i \mathrm{e}^{i\, S_{M2_i}}$$

Consider (open) M2-branes extended between two adjacent $M5$-branes and wrapping $S^2 \subset CY_3$:

$$iS_{M2_i} = -T_{M2} \int d^3\xi \sqrt{det\, g} + i\, T_{M2} \int \left(C_{(3)} - F \right)$$

$C_{(3)}$ and $det\, g$ are functions of $\partial b \sim \partial z$

F is function of $\partial \Sigma$ and is necessary for gauge invariance

The generated superpotential is:

$$W = M^3 \left(\sum_{i=1}^{N-1} \mathrm{e}^{-\frac{8\pi^2}{g_{YM}^2}\Phi_i} + \mathrm{e}^{-\frac{8\pi^2}{g_{YM}^2}\frac{R}{\sqrt{\alpha'}} + \sum_{i=1}^{N-1}\frac{8\pi^2}{g_{YM}^2}\Phi_i} \right)$$

Perfect agreement with field theory on the cylinder! Davies et al [hep-th/9905015]

Extremizing it we get:

$$\langle \Phi_i \rangle = \frac{R}{N\sqrt{\alpha'}} + i\,\frac{g_{YM}^2}{4\pi}\,\frac{k}{N} \quad , k = 0, \ldots, N-1$$

$$\langle W \rangle = N\Lambda^3$$

. . . mass gap, confinement, domain walls, gaugino condensation!

CONCLUSIONS and PERSPECTIVES:

- Proper M-theory description of three dimensional gauge theory via wrapped M-branes
- Applications to other cases (*e.g.* $\mathcal{N} = 4$ Atiyah-Hitchin space)
- Four dimensional $\mathcal{N} = 1$ “smooth” description on the cylinder
- Explicit derivation of a $\mathsf{non-perturbative}$ $\mathsf{superpotential}$
- It can be used in cosmological KKLT scenarios
- Tension of $\mathsf{confining\ (p,q)\ strings}$
- Other (problematic) computations in this set-up ($\mathsf{adding\ flavors}$, $\mathsf{domain\ \ walls}$,. . .)