First-Order Flow Equations for Extremal Non-BPS Black Holes

Gabriel Lopes Cardoso

with Ceresole, Dall'Agata, Oberreuter, Perz, arXiv 0706.3373

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ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Extremal Non-BPS Black Hole Solutions

Goal: construct extremal, non-BPS black hole solutions in $N = 2, D = 4$ sugra theories at 2-derivative level, by

- solving first-order flow equations;
- use $5D/4D$-connection for extremal BHS to achieve this
  `Gaiotto+Strominger+Yin, hep-th/0503217`
  - single-center, static solutions to $N = 2, D = 4$
  - dyonic $(p^0, p^A; q_0, q_A)$, $\quad A = 1, \ldots, n$
  - supported by complex scalar fields z^A
  - spherical symmetry: metric factor $U = U(r)$
    $z^A = z^A(r)$

Supersymmetric Case

BPS case: solutions obtained by solving first-order differential flow equations associated with $\delta\,\mathrm{Fermion} = 0$:

`Ferrara+Gibbons+Kallosh+Strominger 1996`

$$\partial_\tau \mathrm{e}^{-U} = |Z_4| \, ,$$
$$\partial_\tau z^A = -2\,\mathrm{e}^{U}\, g^{A\bar{B}}\, \partial_{\bar{B}} |Z_4| \, ,$$

where $\tau = 1/r$, and $Z_4 = Z_4(p, q, z^A)$ central charge.

Solutions expressed in terms of harmonic functions $(H^0, H^A; H_0, H_A)$.

Flow: spatial infinity $r = \infty$, $z^A(\infty)$; horizon $r = 0$, $z^A(0)$.

Under good conditions, flow in moduli space from $z^A(\infty)$ to $z^A(0)$ evolves smoothly to a fixed-point at the horizon with $Z_4 \neq 0$:

attractor point $\partial_{\bar{B}} |Z_4| = 0$, entropy $S_{\mathrm{macro}} = \pi |Z_4|^2_{\mathrm{hor}}$

Non-Supersymmetric Case

There exist non-BPS black hole solutions that can be constructed by solving first-order flow equations.

`Ceresole+Dall'Agata, hep-th/0702088`

Associated flow equations:

$$\partial_\tau \mathrm{e}^{-U} = W_4 \ ,$$
$$\partial_\tau z^A = -2\, \mathrm{e}^U\, g^{A\bar{B}}\, \partial_{\bar{B}} W_4 \ ,$$

with W_4 real, $W_4 \neq |Z_4|$.

Under good conditions, flow has again a fixed-point at horizon with $W_4 \neq 0$:

attractor point $\partial_{\bar{B}} W_4 = 0$, entropy $S_{\rm macro} = \pi W^2_{4\,\rm hor}$

Effective Black Hole Potential

When do such first-order flow equations exist?

Consider the effective black hole potential

$$F^2 \longrightarrow V_{\rm BH} = Q^T \, \mathcal{M}(z,\bar{z})\, Q \quad , \quad Q^T = (p,q) \, .$$

Whenever

$$V_{\rm BH} = W_4^2 + 4\, g^{A\bar{B}}\, \partial_A W_4 \, \partial_{\bar{B}} W_4 \, ,$$

can rewrite 2-derivative Lagrangian (dim. reduced on spher. symm. background) as

$$\int L_4 = \sum \int (\text{first} - \text{order flow equations})^2 + \int \text{T.D.}$$

Solution to first-order eqs satisfies EOM.

See also `Andrianopoli+D'Auria+Orazi,Trigiante 0706.0712`

Effective Black Hole Potential

Rewriting of V_{BH} is not unique:

- if in terms of $Z_4 \longrightarrow$ BPS
- if in terms of $W_4 \neq |Z_4| \longrightarrow$ non-BPS

Here: construct W_4 by using the $5D/4D$-connection for extremal black holes. Gaiotto+Strominger+Yin, hep-th/0503217

Rotating extremal BHS in $D = 5$ in Taub-NUT space $\leftrightarrow$ static extremal BHS in $D = 4$.

In the vicinity of NUT charge, spacetime looks five-dimensional, whereas far away from it spacetime looks four-dimensional.

VSG ($N = 2, D = 5$) simpler to deal with.

Related work by Ceresole+Ferrara+Marrani, 0707.0964

Static Extremal BHS in Taub-NUT

First: static extremal 5D BHS in Taub-NUT space.

Electric charges q_A , Taub-NUT charge $p^0 > 0$

Effective black hole potential in $D = 5$:

$$G_{AB}\, F^A\, F^B \longrightarrow V_{\rm BH} = q_A\, G^{AB}\, q_B$$

Suppose G^{-1} possesses an invariance group with elements $R^A{}_B$, i.e.

$$R^T\, G^{-1}\, R = G^{-1} \quad , \quad R = {\rm const}\,,\ {\rm real}$$

Then

$$V_{\rm BH} = \frac{2}{3}\, W_5^2 + G^{AB}\, \partial_A W_5\, \partial_B W_5$$

with

$$W_5 = Q_A\, X^A \quad , \quad Q_A = q_B\, R^B{}_A$$

- if $R^A{}_B = \delta^A{}_B \longrightarrow W_5 = Z_5$, BPS black hole in 5D
- if $R^A{}_B \neq \delta^A{}_B \longrightarrow W_5 \neq Z_5$, non-BPS black hole in 5D

Static Extremal BHS in Taub-NUT

Ex: G_{AB} diagonal, then $R = \mathrm{diag}(\pm 1, \mp 1, \ldots, \pm 1)$

Can now rewrite

$$\int L_5 = \sum \int (\mathrm{first-order\,flow\,equations})^2 + \int \mathrm{T.D.}$$

$\longrightarrow$ solved in terms of harmonic functions ($\tau = 1/r$)

$$H_A = h_A + |Q_A|\,\tau \quad , \quad h_A > 0$$

Reducing to $4D$ over Taub-NUT circle yields first-order flow eqs. for a static extremal dyonic BH with charges (p^0, q_A), based on

$$W_4 = \frac{1}{8} \mathrm{e}^{K/2} \left(\mathrm{e}^{-K} p^0 + 4 |Q_A \, (z^A - \bar{z}^A)| \right)$$

When $R^A{}_B \neq \delta^A{}_B \quad , \quad W_4 \neq |Z_4| \longrightarrow$ non-BPS BH in $4D$

$4D$ BH solution is constructed out of harmonic functions ($\tau = 1/r$)

$$H^0 = h^0 + p^0\,\tau \quad , \quad H_A = h_A + |Q_A|\,\tau$$

Rotating Extremal BHS in Taub-NUT

Now: consider rotating extremal 5D BHS in Taub-NUT space.

Stationary part $\omega = \omega_5\, d\psi$ of line element along Taub-NUT circle ψ.

Chern-Simons term $C_{ABC}F^A \wedge F^B \wedge F^C$ in L_5 now contributes. Its first-order rewriting exists if $R^A{}_B$ satisfies, in addition, the relation

$$2G_{AB}\, R^B{}_C\, X^C = C_{ABC}\, R^B{}_D\, X^D\, R^C{}_E\, X^E \qquad (*)$$

Then, denoting $s = sgn(W_5) \quad , \quad W_5 = Q_A\, X^A$,

- $s = 1 \quad , \quad \omega_5 = H_0 = h_0 + q_0\, \tau \quad , \quad$ harmonic
- $s = -1 \quad , \quad \omega_5 = c/H^0 \quad , \quad c = \mathrm{const}$

Ex: take $R^A{}_B = \delta^A{}_B$ (satisfies $(*)$) and $s = -1$, and reduce over Taub-NUT direction ψ. This yields a 4D flow based on

$$W_4 = \mathrm{e}^{K/2} | \frac{1}{6} p^0 C_{ABC} z^A z^B \bar{z}^C - q_A z^A | \neq |Z_4|$$

Since $\omega_5 \neq 0$ along flow, have non-trivial axions $\mathrm{Re} z^A$ along the flow.

Conclusions

1. Using $5D/4D$-connection, constructed extremal non-BPS BHS with charges (p^0, q_A). Apply duality transformations to obtain other non-BPS BHS, for instance (q_0, p^A).
2. An example of an extremal non-BPS BH not captured by the above construction:

   $4D \;\; (p^0, q_0) \longleftrightarrow 5D \quad (p^0, J)$ Kerr BH
3. Multiple-center solutions? Split attractor flow for non-BPS black holes?