Topics Donaldson–Thomas Theory

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work in progress with A. Sinkovics and R.J. Szabo

What are the Donaldson–Thomas invariants?

- In the recent years it has emerged that the so–called Donaldson–Thomas invariants provide a nice reformulation of the topological string that is both mathematically well defined and simply related to BPS states.
- The Donaldson–Thomas invariants count the number of bound states formed by a single D6 brane wrapping the Calabi–Yau X with an arbitrary number of D2 branes wrapping a curve $C \subset X$ and D0 branes
- This information is contained in the moduli space of ideal sheaves that parametrizes curves C in a homology class β with m "points" (D0 branes): $\mathcal{I}_m(X,\beta)$. It is also known as the Hilbert scheme of points and curves $\text{Hilb}^m(X,\beta)$.
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Some open problems

- Non abelian invariants on local toric CY manifolds
- We can define non abelian DT invariants by considering bound states of D0–D2 branes with an arbitrary number N of D6 branes. This would be interesting as it provides computational control on the counting of BPS states (at least in a toric CY) and one could hope to make contact with the OSV conjecture.
- Singular varieties
- Orbifold points arise in most common Calabi–Yau compactification and the topological string (and Gromov–Witten theory) is certainly well defined there. What about DT invariants? They are not even defined! Can we provide a physical picture? Can we compare this picture with Gomov–Witten invariants?

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A more physical perspective

- Perhaps it is easier to think of the DT invariants as parametrizing the classical atomic configuration in the "melting crystal" interpretation of the Topological String.
- For a toric CY they can be described as generalized instantons of a six dimensional abelian theory living on the D6 brane wrapping the CY: the problem of computing DT invariants is reduced to an instanton counting problem

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The Topological Gauge Theory Picture

- The generalized instanton configurations are classified by 3D partitions: this is how we recover the crystal picture
- Since the theory is topological the semiclassical approximation is exact

$$Z \sim \sum_{x \in \{\text{critical}\}} \left(\int_{\mathcal{M}_{\text{inst}}(\text{ch}_2,\text{ch}_3)} e(\mathcal{N}) \right) e^{\,S_{\text{inst}}(x)}$$

- Unfortunately these moduli spaces are very singular as they suffer from UV (instantons shrinking to zero size) and IR (instantons running away to infinity) divergences
- We can overcome these problems with a noncommutative deformation of the theory and working equivariantly

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- At least in the case of $X = \mathbb{C}^3_\theta$ we can provide a fairly concrete parametrization of the moduli space of these noncommutative instantons in term of generalized ADHM–like equations.
- The idea is to parametrize a generic torsion free sheaf on the resolved moduli space with the so–called Beilinson spectral sequence. This shows explicitly how the Hilbert scheme emerges from our moduli space of instantons (generalizing a 4d result by Nakajima)

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Equivariant localization

- We can do better: $\mathbb{C}^3$ and toric manifolds in general have a natural $\mathbb{T}^3$ action $z_i \longrightarrow \mathrm{e}^{\,\mathrm{i}\,\epsilon_i} z_i$
- We can work equivariantly w.r.t. this action and use equivariant localization. This can be achieved by putting the theory on the so–called Ω–background.

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- The BRST operator becomes an equivariant differential on the instanton moduli space and we should look for its fixed points.
- In our case we can show that the fixed points are isolated: the fluctuation integrals simply become sums as in the familiar Duistermaat–Heckman formula

$$\int_{\mathcal{M}} \frac{\omega^n}{n!}\,\mathrm{e}^{-\mu[\epsilon_i]} = \sum_f \frac{\mathrm{e}^{-\mu[\epsilon_i](f)}}{\prod_i w_i[\epsilon](f)}$$

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Classification of the Critical Points

- Localization techniques reduce the problem of evaluating the "volumes" of instanton moduli spaces to the classification of critical points of the equivariant BRST charge.
- Now we have a well defined problem! and these techniques can be extended to non abelian invariants (since we know what a non abelian gauge theory is) on any toric manifold (since it can be covered with $\mathbb{C}^3$ patches and has global toric $\mathbb{T}^3$ isometries)
- The critical points of the U(N) gauge theory are classified by N–tuples of 3D partitions on each $\mathbb{C}^3$ patch: $(Y_1, \ldots, Y_N)$
- Physically for what these invariants are concerned we can separate the N D6 branes and work in the Coulomb branch.

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Non Abelian DT

- We can now compute the non abelian partition function. We have developed two independent techniques:
- Work directly with the noncommutative gauge theory and compute the ratio of the determinants that constitutes the fluctuation factor around each critical point.
- Introduce an ADHM–like quantum mechanics that lives on the resolved moduli space and use localization to compute equivariant volumes of instanton moduli space.

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- The result on any toric variety is

$$\mathcal{Z}_{DT}^{U(N)} = \sum_{f} q^{\mathbb{I}} (-1)^{(N+1)\mathbb{I}} \prod_{e \in \mathrm{edge}} (-1)^{\sum_{l,l'=1}^{N} |\lambda_{l,e}||\lambda_{l',e}| m_1} \, \mathrm{e}^{-\sum_{l=1}^{N} |\lambda_{l,e}| t_e}$$

$$\mathbb{I} = \sum_{f} \sum_{l=1}^{N} |\pi_{l,f}| + \sum_{e \in \mathrm{edge}} \sum_{l=1}^{N} \sum_{(i,j) \in \lambda_{e,l}} (m_1(i-1) + m_2(j-1) + 1)$$

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Physical Interpretation

- The extra minus signs can be interpreted as a N–dependent shift of the coupling constant. The shift is exactly the one predicted by the OSV conjecture for an arbitrary number of D6 branes:
$$g_{s,OSV} = \frac{1}{G_{s,ours}} = \frac{4\pi\,\mathrm{i}}{p^0 + \mathrm{i}\,\frac{\phi_0}{\pi}}$$
This is related to our picture by S–duality.
- Unfortunately we do not find new invariants of Calabi–Yau manifolds. After the shift the non abelian partition function is given by N copies of the abelian one
- This is consistent with the computation of the IIA NS5 brane partition function in terms of topological strings by Dijkgraaf–Verlinde–Vonk. One can think of our computation as the "mirror statement"

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- We have developed two powerful approaches (noncommutative instantons and ADHM–like quantum mechanics) to compute these invariants. How general are they?
- We think they may be used to give a prescription to compute DT invariants on orbifolds
- In particular we can melt our ADHM–like parametrization with Nakajima's work on quiver varieties to define Donaldson–Thomas quiver varieties.
- We propose that the DT invariants on an orbifolds compute the equivariant volumes of these varieties. The computation of these volumes is simple with our techniques
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