HIGH ENERGY SCATTERING IN ADS/CFT

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Basic Question

- In flat space high energy interactions are dominated by gravitational interaction

- Resum in G using eikonal methods

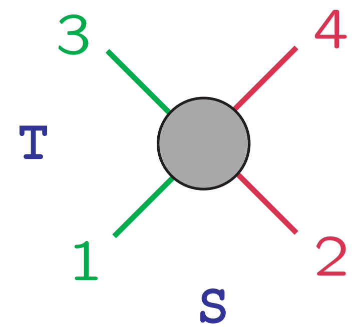
- Can we extend to AdS and to dual CFT's ?

Preliminaries

- CFT$_4$ amplitude

$$A = \langle \mathcal{O}_1(\mathbf{x}_1)\,\mathcal{O}_1(\mathbf{x}_3)\,\mathcal{O}_2(\mathbf{x}_2)\,\mathcal{O}_2(\mathbf{x}_4)\rangle$$

$$= \frac{1}{\mathbf{x}_{13}^{2\Delta_1}\mathbf{x}_{24}^{2\Delta_2}}\mathcal{A}(z,\bar{z})$$



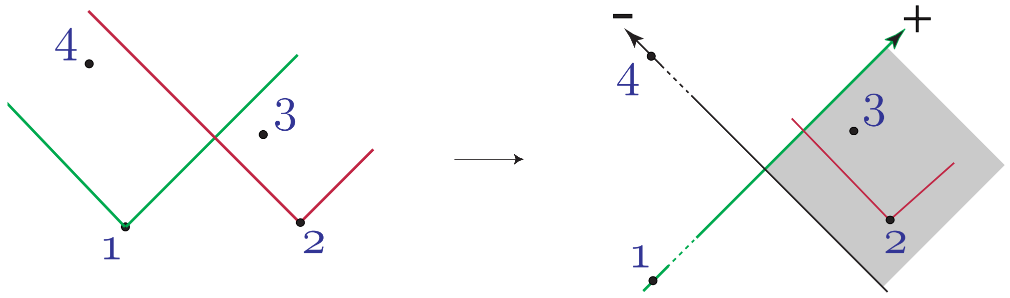
with cross–ratios

$$z\bar{z} = \frac{\mathbf{x}_{13}^2\mathbf{x}_{24}^2}{\mathbf{x}_{12}^2\mathbf{x}_{34}^2} \qquad (1-z)(1-\bar{z}) = \frac{\mathbf{x}_{14}^2\mathbf{x}_{23}^2}{\mathbf{x}_{12}^2\mathbf{x}_{34}^2}$$

- T– and S–channel $z,\bar{z} \to 0$ and $z,\bar{z} \to \infty$

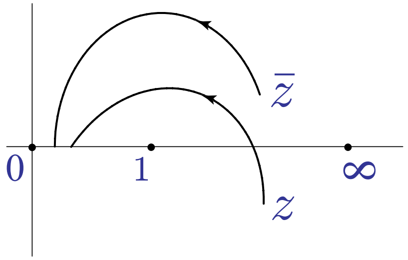
Lorentzian Kinematics

- Causal relations

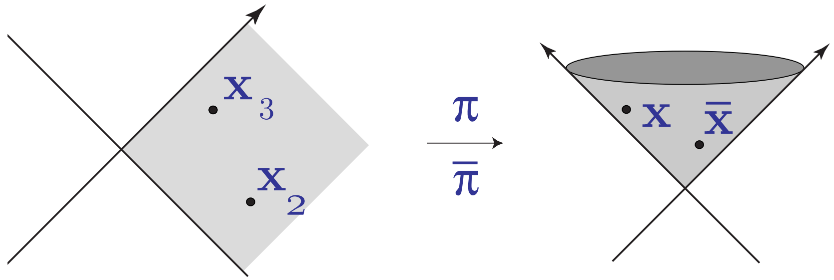


with coordinates $\mathbf{x} = (x^+, x^-, x)$

- Lorentzian reduced amplitude $\hat{\mathcal{A}}(z, \bar{z})$



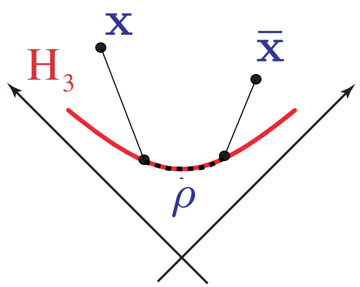
- Parameterize A by $\mathbf{x}, \bar{\mathbf{x}}$ in the Milne cone M



as

$$A = \frac{1}{\mathbf{x}^{2\Delta_1}\bar{\mathbf{x}}^{2\Delta_2}}\mathcal{A}(z, \bar{z})$$

- Cross–ratios



$$z \sim \sigma\, e^{\rho}$$

$$\bar{z} \sim \sigma\, e^{-\rho}$$

$$\sigma \sim |\mathbf{x}||\bar{\mathbf{x}}|$$

with $\mathrm{H}_3 \subset \mathrm{M} \subset \mathbb{M}^4$ the transverse hyperbolic space

Regge Theory

- T−channel exchange $\mathcal{T}_{E,J}\ (z,\bar{z})$ of primary of

- Euclidean OPE for $\sigma \to 0$

$$\mathcal{T}_{E,J}\ \sim \sigma^{2+E}$$

- Lorentzian regime

$$\hat{\mathcal{T}}_{E,J} \sim \sigma^{1-J} \ \Pi_E(\rho)$$

$\Pi_E(\rho)$ propagator of energy $1+E$ in H_3

General spin J contribution to $\hat{\mathcal{A}}$

$$\sigma^{1-J} \ \alpha(\rho)$$

- Maximal spin dominates. If spin J unbounded resum contributions. Leading Regge pole at $J = j(\nu)$ gives

$$\int d\nu \, \sigma^{1-j(\nu)} \, \alpha(\nu) \ \Omega_{i\nu}(\rho) \qquad \left(-\Box_{H_3} = 1+\nu^2\right)$$

Impact Parameter Representation

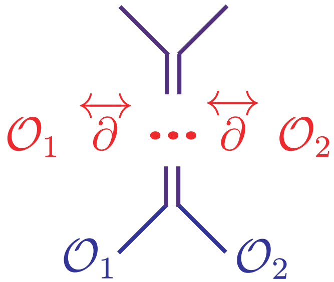
- Amplitude

$$A(\mathbf{x},\bar{\mathbf{x}}) \sim \int_{\mathsf{M}} \frac{d\mathbf{y}\,d\bar{\mathbf{y}}}{\mathbf{y}^{d-2\Delta_1}\bar{\mathbf{y}}^{d-2\Delta_2}}\, e^{-2i\mathbf{x}\cdot\mathbf{y}-2i\bar{\mathbf{x}}\cdot\bar{\mathbf{y}}}\; e^{-2\pi i\; \Gamma(\mathbf{y},\bar{\mathbf{y}})}$$

- No interaction

$$\mathcal{A} = 1 \qquad\qquad \Gamma = 0$$

- Lattice sum



$$1 = \sum_{E,J} \mathcal{S}_{E,J}$$

$$E = \Delta_1 + \Delta_2 + J + 2n$$

- For large E, J replace $\sum \to \int d\mathbf{y}\, d\bar{\mathbf{y}}$ with

$$
\begin{aligned}
4\,|\mathbf{y}|\,|\bar{\mathbf{y}}| = E^2 - J^2 = s \\
-4\mathbf{y}\cdot\bar{\mathbf{y}} = E^2 + J^2 = s \cosh r
\end{aligned}
$$

with r impact parameter on H_3

- Interactions in the two–particle approximation

$$
\sum_{E,J} \mathcal{S}_{E+2\Gamma\;,\;J}
$$

Phase Shift $\quad\leftrightarrow\quad$ Anom. Dim. of Double Trace Op.

- Regge pole

$$
\Gamma \sim \int d\nu\;\, s^{j(\nu)-1}\;\beta\,(\nu)\;\,\Omega_{i\nu}\,(r) \qquad\qquad (\beta\,(\nu) \sim \alpha\,(\nu))
$$

Eikonal Scattering in AdS

- Dual theory in AdS_5 in the gravity limit

- AdS Poincarè coordinates

$$w^+, w^-, \underbrace{w, r}_{H_3}$$

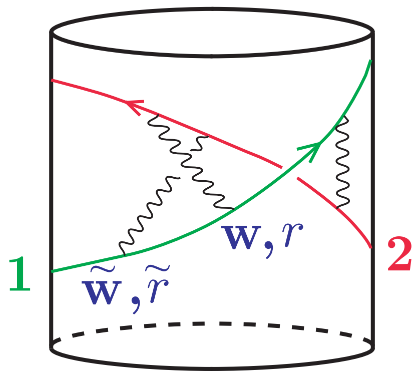
- Wave–functions dual to $\mathcal{O}_1$ for $E \to \infty$

$$e^{-\frac{i}{2}E\ w^-}\ f\,(w, r)$$

- Along $\mathcal{O}_1$ trajectory $2i\partial_- \sim E$ and propagator is

$$\frac{1}{E}\ \Theta(w^+ - \tilde{w}^+) \cdot \delta(w^- - \tilde{w}^-)\ \delta_{\mathsf{H}_3}(w, r\ |\tilde{w}, \tilde{r})$$

with



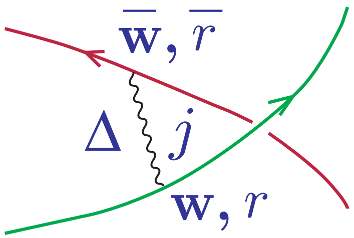
$w^-, \overbrace{w, r}^{\mathsf{H}_3}$ parameterize geodesics

w^+ affine

- Amplitude

$$E^2 \int dw^- d\epsilon_{\mathsf{H}_3}(w,r) \int d\bar{w}^+ \; d\epsilon_{\mathsf{H}_3}(\bar{w},\bar{r})$$
$$\times\, f_1(w,r)\, f_3(w,r) \quad f_2(\bar{w},\bar{r})\, f_4(\bar{w},\bar{r}) \times e^{-2\pi i\;\Gamma}$$

- Phase



Spin j interaction of dimension $2+\Delta$

$$\Gamma = -2G\;(E^2 r\bar{r})^{j-1}\;\Pi_\Delta(w,r|\bar{w},\bar{r})$$

- Boundary wave–functions of $\mathcal{O}_i$ can be localized in global coordinates so that Γ is identified with phase–shift

$$\Gamma\left(s,r\right) \sim -2G\ s^{j-1}\Pi_{\Delta}\left(r\right)$$

For gravity $j=2$, $\Delta = 2$ and

$$\Gamma \sim -\frac{G}{4\pi}\,\frac{(E-J)^4}{EJ}$$

(Exact in $G = \pi/2N^2$)

Including String Effects

- Regge trajectory of the graviton $j(\nu, \lambda)$ with

$$\lambda = \frac{\ell^4}{\alpha'^2} = g_{YM}^2 N$$

- Flat space limit

$$\lim_{\ell \to \infty} j\left(\ell q, \ell^4/\alpha'^2\right) = 2 - \frac{\alpha'}{2} q^2$$

- Energy–momentum tensor $j(\pm 2i, \lambda) = 2$

- Decreasing intercept [Brower et al., Lipatov]

$$j(\nu, \lambda) = 2 - \frac{4 + \nu^2}{2\sqrt{\lambda}} - \cdots$$

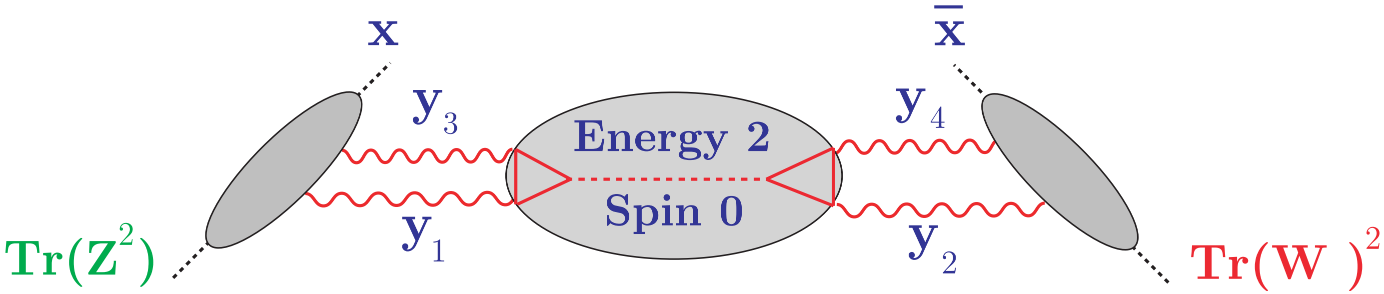
Small λ and Pomeron Exchange

- High energy scattering in QCD for $s \gg |t| \gg \Lambda_{\text{QCD}}$ dominated by hard Pomeron exchange

    - Quantum numbers of the vacuum

    - Spin $j\,(\nu, \lambda) = 1$ for $\lambda \to 0$

- BFKL picture for $\mathcal{N} = 4$ SYM [Lipatov]

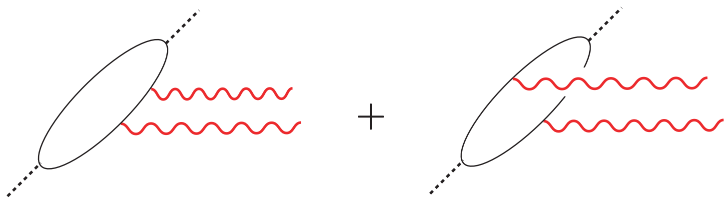


- $\mathbf{x}, \bar{\mathbf{x}} \in$ M and $\mathbf{y}_i \in \partial$M points in transverse conformal 2d space

- Amplitude

$$\hat{\mathcal{A}} \sim \int_{\partial \mathrm{H}_3} \frac{d\mathbf{y}_1 d\mathbf{y}_3 d\mathbf{y}_2 d\mathbf{y}_4}{\mathbf{y}_{13}^2 \mathbf{y}_{24}^2} \quad V(\mathbf{x}, \mathbf{y}_1, \mathbf{y}_3) \quad F(\mathbf{y}_i) \quad \bar{V}(\bar{\mathbf{x}}, \mathbf{y}_2, \mathbf{y}_4)$$

- Impact factor $V(\mathbf{x}, \mathbf{y}_1, \mathbf{y}_3)$ function of single cross–ratio and computable in perturbation theory



- We obtain

$$\hat{\mathcal{A}}(\sigma, \rho) \sim \frac{\rho^2}{\sinh^2(\rho)}$$

matching exact 4–point function to order λ^2

[Bianchi et al., Arutyunov et al.]

Open Problem

- Extend the relation Phase Shift $\sim$ Anomalous Dimension to the weak coupling regime
  - Anomalous Dimension $\sim$ Dilatation Operator on double–trace states [Minahan, Zarembo, Beisert, ...]
  - Phase Shift promoted to Operator acting on two–string states in flat space [Amati, Ciafaloni, Veneziano]
- Is there a separation of orbital and internal excitations at high energy when string effects are dominant ?