Backreacting flavors in the Klebanov-Strassler theory: a new duality cascade

Stefano Cremonesi

SISSA/ISAS and INFN, Trieste

RTN Workshop, Valencia, October 5, 2007

based on:

F. Benini, F. Canoura, SC, C. Núñez and A. V. Ramallo,
JHEP **0709**, 109 (2007) [arXiv:0706.1238].

Outline of the talk

- **Motivation and framework**:
  AdS/CFT and large N expansions with flavors

- **Review of KT/KS**:
  Field theory and duality cascade

- **Backreacting flavors in the KS background**:
  - Supergravity + branes solution
  - Field theory and IIA brane configuration
  - Seiberg duality cascade from supergravity solution: methods and results

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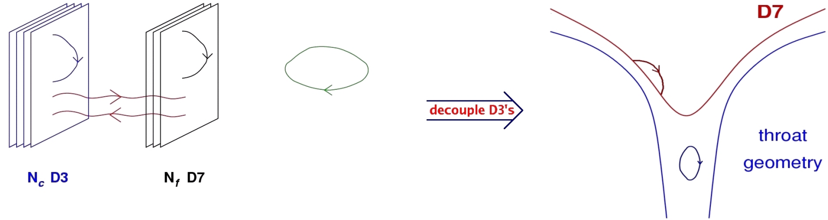
Addition of flavors to AdS/CFT

Karch-Katz '02



$g^2_{YM\,(D3)} \propto g_s \qquad g^2_{YM\,(D7)} \propto g_s\,\alpha'^2 \qquad l^8_{Pl} \propto g_s^2 \alpha'^4$

$\mathcal{N} = 4$ $SU(N_c)$ SYM + N_f hyper $\equiv$ D7 open strings and IIB closed strings on $AdS^5 \times S^5$

Probe vs. Backreacting D7

Probe approximation ($N_f/N_c \ll 1$) Karch-Katz '02

- Background from D3-branes (after decoupling limit)
- Stable D7 embeddings in that background

$\longrightarrow$ Quenched approx in lattice gauge theories.

Backreacting D7's (any N_f/N_c) Graña-Polchinski, Bertolini et al. '00
Casero-Núñez-Paredes '06 (D5)

- Solve simult. EoM for IIB sugra + DBI-WZ action of flavor branes.

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Flavors in Large N expansions

't Hooft limit: $N_c \to \infty$, $\lambda = N_c g_{YM}^2$ and N_f fixed

- Leading: planar diagrams w/o quark internal loops
- Topological expansion of a theory of closed and open strings

Probe approx in sugra: leading order in 't Hooft limit.

Veneziano limit: $N_c \to \infty$, $\lambda = N_c g_{YM}^2$ and $x = N_f / N_c$ fixed

- Leading: planar diagrams w/ and w/o quark internal loops

Backreacting D7 in sugra: leading order in Veneziano limit.

NEW PHYSICS at leading order in Veneziano limit:

- New hadronic physics, e.g. finite mass for η' (Veneziano-Witten)
- Screening of color charges, breaking of chromoelectric flux tubes
- RG flow

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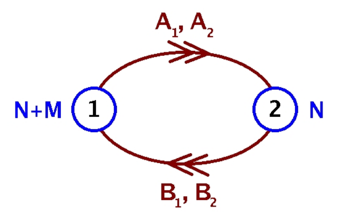
KT/KS: field theory and backgrounds

N regular and M fractional D3 on the conifold

($z_1 z_2 - z_3 z_4 = 0$ in $\mathbb{C}^4$):

$$W = h\, \epsilon^{ij}\, \epsilon^{kl}\, \mathrm{Tr}\,(A_i B_k A_j B_l)$$



- F_5, F_3, H_3 constant ϕ $\quad \int_{S^3} F_3 \propto M, \quad \int_{T^{1,1}} F_5 \propto N_{eff}(r)$ Klebanov-Tseytlin '00
- RG flow: cascade of Seiberg dualities. Klebanov-Strassler '00
  $N = kM$: IR $SU(2M) \times SU(M)$ on a $\mathbb{Z}_2$-symm pt of the baryonic branch.
  $\hookrightarrow \mathcal{N} = 1$ $SU(M)$ pure SYM + Goldstone of $U(1)_B$.

Complex deformation $z_1 z_2 - z_3 z_4 = \epsilon^2$	$\longleftrightarrow$	Quantum deformation of the moduli space
Size of S^3 at the tip	$\longleftrightarrow$	Gluino condensate

$\Longrightarrow$ Smooth supergravity solution: Warped deformed conifold.

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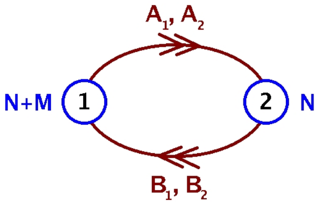

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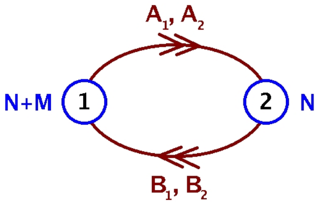

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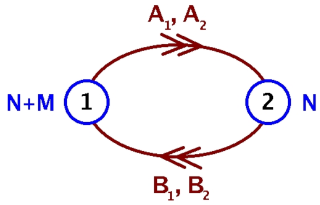

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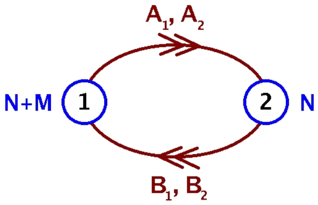

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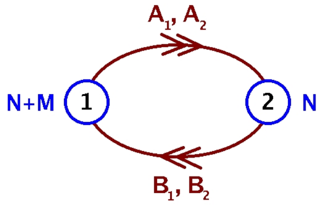

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KT/KS: a cascade of Seiberg dualities

Field theory side

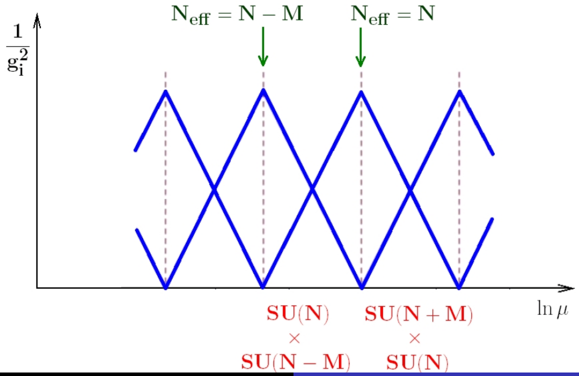
$N \gg M$: $\quad \gamma = -\frac{1}{2} + \mathcal{O}\left(\left(\frac{M}{N}\right)^2\right)$

$\Longrightarrow \quad \beta_{\frac{8\pi^2}{g_1^2}} \simeq 3M, \quad \beta_{\frac{8\pi^2}{g_2^2}} \simeq -3M$

Gravity side

$$\frac{8\pi^2}{g_1^2} + \frac{8\pi^2}{g_1^2} = 2\pi e^{-\phi}$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_1^2} = 4\pi e^{-\phi}\left[\frac{1}{4\pi^2\alpha'}\int_{S^2} B_2 - \frac{1}{2}\right]$$



Supergravity ansatz for flavored KT/KS

- Look for susy solutions of EoM of $S = S_{IIB} + S_{D7}$ with g_{MN}, ϕ, F_3, H_3, F_5, F_1 and D7-embedding.
- D7: $dF_1 = \star j_{D7} \neq 0$, source in Einstein and dilaton eqns.
  κ-symmetric embeddings $z_1 + z_2 = 0$ and $SU(2)_D$-related: smear!
  $\Longrightarrow \quad S_{D7}^{WZ} = T_7 \int \Omega \wedge C_8, \qquad \Omega = -\frac{N_f}{4\pi} \sum_i \sin\theta_i \, d\theta_i \wedge d\varphi_i .$
- Solve Bianchi identities by ansatz, DBI eqns. by κ-symmetry, reduce dilaton and Einstein eqns. to BPS system by requiring supersymmetry of the solution.
- Analytic solutions for backreacting flavors in KT and KS: supergravity duals of the same kind of gauge theories, with the same duality cascade but different IR dynamics (due to different initial ranks).

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Supergravity ansatz for flavored KT/KS

$$
\begin{aligned}
ds_{10}^2 &= h(\tau)^{-1/2} dx_{1,3}^2 + h(\tau)^{1/2} ds_6^2 \\
\phi &= \phi(\tau) \\
F_1 &= \frac{N_f}{4\pi}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i \, d\varphi_i\right) \\
F_3 &= \frac{M}{2}\Big\{ g^5 \wedge \Big[\Big(F(\tau) + \frac{N_f}{4\pi} f(\tau)\Big) g^1 \wedge g^2 + \Big(1 - F(\tau) + \frac{N_f}{4\pi} k(\tau)\Big) g^3 \wedge g^4 \Big] + \\
&\quad + F'(\tau) d\tau \wedge \left(g^1 \wedge g^3 + g^2 \wedge g^4\right) \Big\} \\
F_5 &= (1 + \star)\, dh^{-1}(\tau) \wedge d^4x_{1,3} \\
B_2 &= \frac{M}{2}\left[f(\tau)\, g^1 \wedge g^2 + k(\tau)\, g^3 \wedge g^4 \right]
\end{aligned}
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with suitable ansatz for 6-dimensional vielbein which includes both KT and KS cases.

Supergravity ansatz for flavored KT/KS

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Flavored KS solution

$$ds_6^2 = \frac{1}{2}\mu^{\frac{4}{3}}\Lambda(\tau)\left\{\frac{4(\tau_0-\tau)}{3\Lambda^3(\tau)}\left[d\tau^2+(g^5)^2\right]+\cosh^2\frac{\tau}{2}\left[(g^3)^2+(g^4)^2\right]+\sinh^2\frac{\tau}{2}\left[(g^1)^2+(g^2)^2\right]\right\}$$

$$\Lambda(\tau) = \frac{\left[2(\tau-\tau_0)\left(\tau-\sinh(2\tau)\right)+\cosh(2\tau)-2\tau\tau_0-1\right]^{1/3}}{\sinh\tau}$$

$$h(\tau) = -\frac{2\pi M^2}{\mu^{8/3}N_f}\int^{\tau} dx\,\frac{x\coth x-1}{(x-\tau_0^2)\sinh^2 x}\cdot\frac{-\cosh(2x)+4x^2-4x\tau_0+1-(x-2\tau_0)\sinh(2x)}{\left[\cosh(2x)+2x^2-4x\tau_0-1-2(x-\tau_0)\sinh(2x)\right]^{2/3}}$$

$$e^{\phi(\tau)} = \frac{4\pi}{N_f}\frac{1}{\tau_0-\tau} \qquad (0\le\tau<\tau_0)$$

$$e^{-\phi(\tau)}f(\tau) = \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau-1)$$

$$e^{-\phi(\tau)}k(\tau) = \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau+1)$$

$$F(\tau) = \frac{\sinh\tau-\tau}{2\sinh\tau}$$

- $N_f \to 0$, $N_f\tau_0$ fixed: recover KS.

Flavored KS solution

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Curvature singularities and interpretation

UV $(\tau \to \tau_0^-)$

- Usual D7 singularity.
- Interpretation: Landau pole nature of the duality wall. (More later)

IR $(\tau \to 0^+)$

- Limits of metric components as in KS, but with $\mathcal{O}(\tau)$ corrections instead of $\mathcal{O}(\tau^2)$. $\Longrightarrow$ Curvature singularity.
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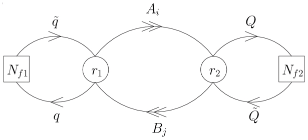
The flavored KS field theory



$$W = \lambda(A_1B_1A_2B_2 - A_1B_2A_2B_1) + h_1\tilde{q}(A_1B_1 + A_2B_2)q + h_2\tilde{Q}(B_1A_1 + B_2A_2)Q + \alpha q\tilde{q}q\tilde{q} + \beta Q\tilde{Q}Q\tilde{Q}$$

IIB brane engineering

(Convention: $r_l > r_s$, $l, s = 1, 2$)

- r_l $D3_f$ of 1st kind (wrapped $D5$, no wv flux)
- r_s $D3_f$ of 2nd kind (wrapped $\overline{D5}$, wv flux)
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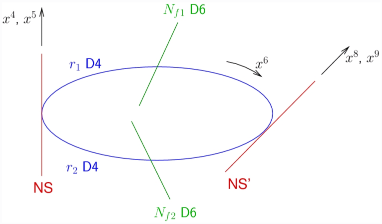

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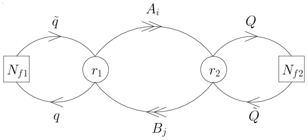

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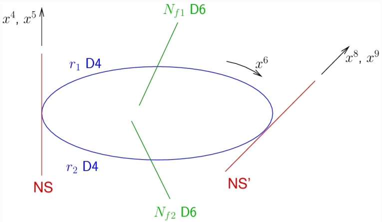

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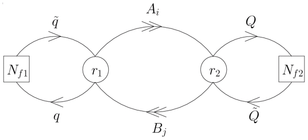

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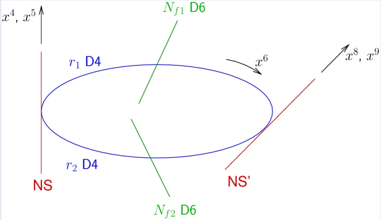

Seiberg duality cascade and self-similarity

Field theory analysis

- Assume that g_l runs to strong coupling, g_s to weak coupling as in KS.
- At some energy scale flowing towards the IR, $\frac{1}{g_l^2} = 0$: Seiberg-dualize.
- The theory with quartic flavor couplings is self-similar: same W before and after duality, only ranks change.
- If the RG flow proceeds like this: cascade of Seiberg dualities.

Analysis from supergravity solutions

- Flavored KS solution: dual gauge theory with duality cascade until the IR, where nonperturbative gauge dynamics occurs.
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Group ranks and fractional branes: Maxwell charges

BRANE CHARGES at a given τ $\longleftrightarrow$ GROUP RANKS at a given E

Maxwell charges

$$dF_{8-p} = \star j_{Dp}^{Maxwell}$$

F_{8-p}: gauge invariant (improved) field strength
j_{Dp}: magnetic current

$$Q_{Dp}^{Maxwell} \propto \int_{V_{9-p}} \star j_{Dp}^{Maxwell} = \int_{\partial V_{9-p}} F_{8-p}$$

Gauge invariant, conserved, not localized (carried by fluxes), not quantized.

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Group ranks and fractional branes: Maxwell charges

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Object	frac D3 (1)	frac D3 (2)	frac D7 (1)	frac D7 (2)
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Page charges and group ranks (I)

Page charges

Write Bianchi id's in absence of (magnetic) sources as: exterior derivatives of some differential forms (in general not gauge invariant) =0. Then substitute RHS with $\star j^{Page}$.

$$d(F_3 - B_2 \wedge F_1) = \star j^{Page}_{D5}$$
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- Exact Seiberg duality along the RG flow; infinitely many Seiberg dual FT descriptions, but at any given energy scale only one is 'good'.
- Seiberg dualities in the FT are large gauge transf's on the sugra solutions.

Large gauge transformation

$$\Upsilon_2 \equiv \frac{1}{2}(\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2)$$

$$B_2 \to B_2 - \pi n \Upsilon_2 \quad \Rightarrow \quad b_0 \to b_0 - n \qquad (n \in \mathbb{Z})$$

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$$Q_{D7}^{Page} = N_f = N_{fl} + N_{fs}$$
$$Q_{D5}^{Page} = r_l - r_s - \frac{N_{fl}}{2} \qquad \hookrightarrow \qquad \text{Comparison with ranks in FT.}$$
$$Q_{D3}^{Page} = r_s + \frac{3N_{fl} - N_{fs}}{16}$$

- After a step of the duality cascade, Page charges of our solution change in the way predicted by the changes of the ranks in the field theory.

Page charges and group ranks (II)

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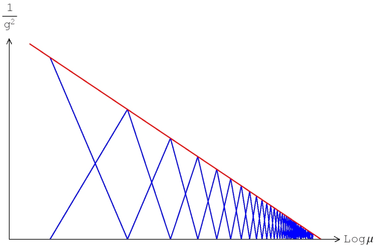
Cascade: R-anomalies and β-functions

Dictionary

$$\frac{4\pi^2}{g_l^2} + \frac{4\pi^2}{g_s^2} = \pi e^{-\phi} \qquad \theta_l^{YM} + \theta_s^{YM} = 2\pi C_0$$

$$\frac{4\pi^2}{g_l^2} - \frac{4\pi^2}{g_s^2} = 2\pi e^{-\phi}\left[\frac{1}{4\pi^2\alpha'}\int_{S^2} B_2 - \frac{1}{2}\right] \qquad \theta_l^{YM} - \theta_s^{YM} = \frac{1}{\pi}\int_{S^2} C_2$$

- R-anomalies match FT expectations.
- Running gauge couplings: duality wall.
  (Divergence of b_0 at finite proper distance from the bulk: infintely many cascade steps to reach a finite energy scale E_{UV}.)



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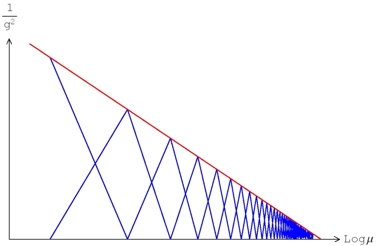

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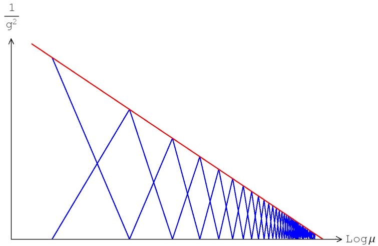

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