Tadpole conditions in F theory and the Euler characteristic of singular varieties

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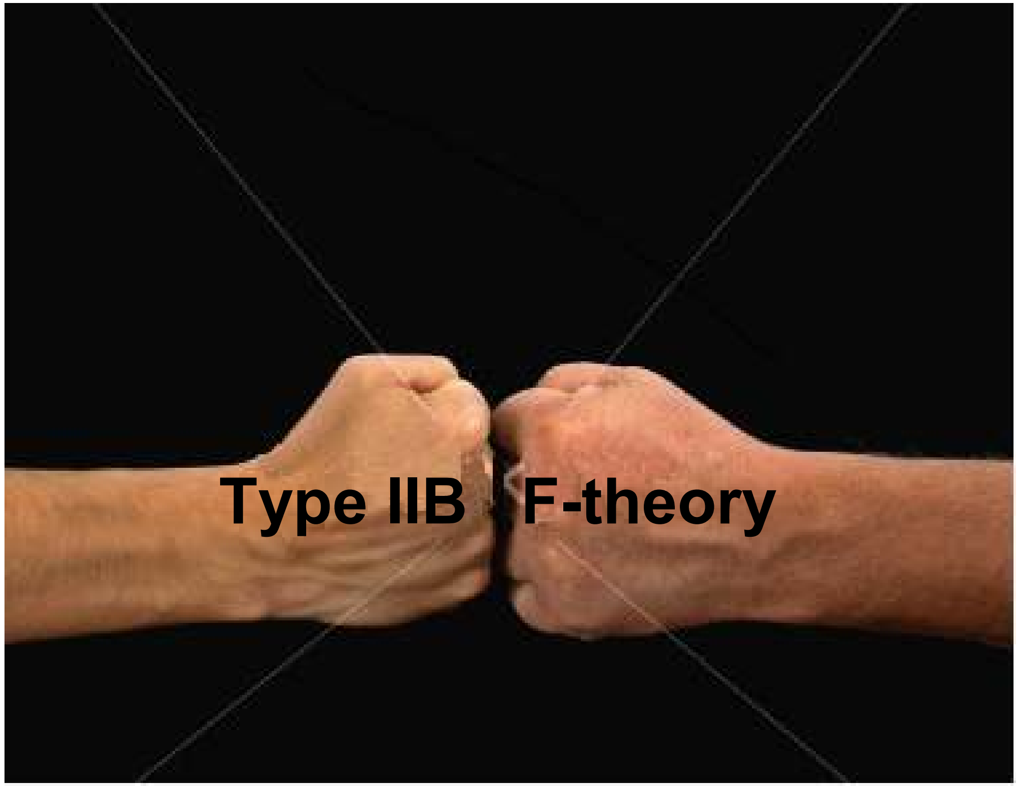
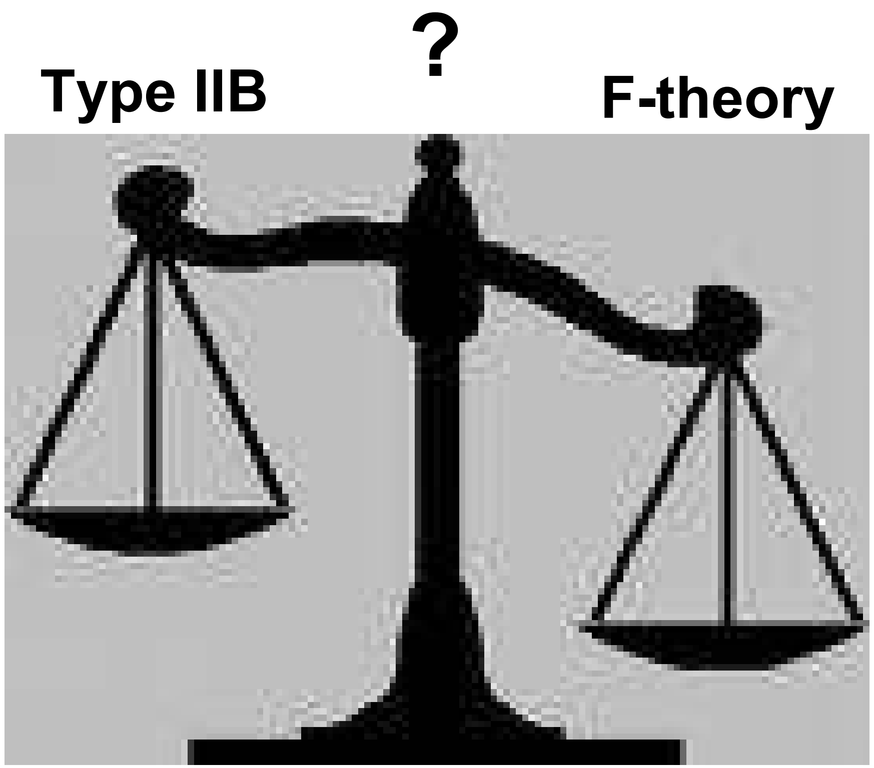
P. Aluffi and M.E., Chern Class identities from tadpole matching in type IIB and F-theory, to appear

P. Aluffi, A. Collinucci, F. Denef, and M.E,D-brane Deconstruction in type IIB orientifolds, to appear

Type IIB vs F-theory



D3 branes as a probe…



…tadpole conditions as the test

Tadpole conditions

In a non-compact space, fluxes can escape to infinity.

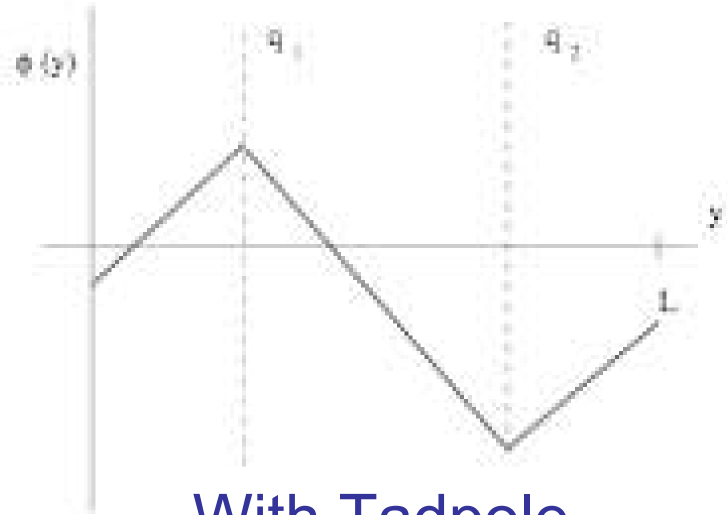
In a compact space fluxes have no where to go so the total flux has to vanish

Homework

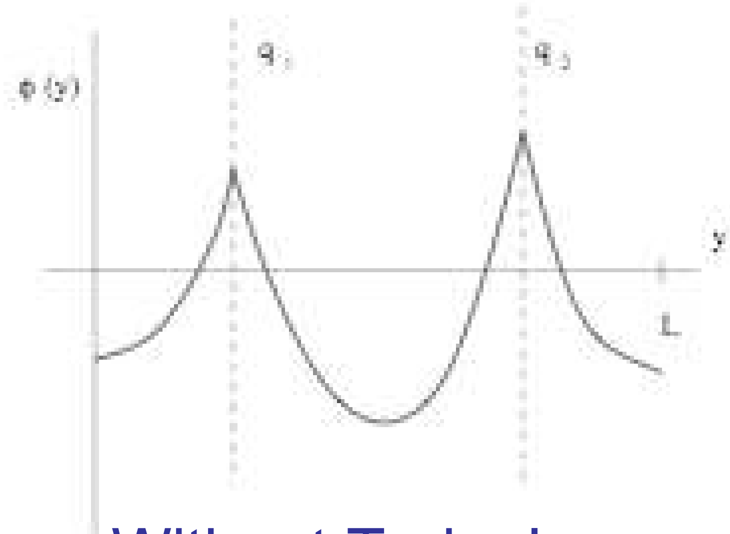
Study the electrostatic potential for two point-like charges on a cir
of radius L.

Example: Electrostatic potential of two charges on a circle

When we ignore tadpole conditions,
we describe a very different physics…



With Tadpole cancellation:
Linear potential



Without Tadpole cancellation:
Parabolic potential!

Tadpole conditions with D-branes

We start from a Lagrangian of the type

$$\mathcal{L} = -\frac{1}{4}\int |F_{p+2}|^2 - \mu_p \sum_a C_{p+1} \wedge \pi^a_{9-p}$$

The equation of motions are

$$d * dC_{p+1} = 2\sum_a \pi^a_{9-p}$$

Tadpole condition
for a D-brane

$$\sum_a \Pi^a = 0$$

Chern-Simons terms and induced charges

Anomaly cancellation required the presence of CS terms that contributes to the tadpole: a brane has lower brane charge !

$$S_W = \int_W C \wedge e^{-B} \mathrm{ch}(E) \wedge \sqrt{\frac{\hat{A}(T_W)}{\hat{A}(N_W)}}$$

$$S_{WZ}^{Oplane} = -2^{p-4} \int_W C \wedge \sqrt{\frac{L(R_T/4)}{L(R_N/4)}}$$

$$C = C_{(0)} + C_{(2)} + C_{(4)} + C_{(6)} + C_{(8)} + C_{(10)}$$

F-theory

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Vafa has proposed an elegant geometric formulation of Type IIB with (p,q)-branes:

- A theory in 12 dimensions: 10 dimensions + an elliptic curve (a two torus)

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

- Complex structure of the elliptic curve= type IIB axion-dilaton

- F-theory on an elliptically fibered CY_4 = Type IIB on the base

ACT I: The set up

- Type IIB orientifold with spacefilling D3 and D7 branes and O7 planes.
- The D7 branes are wrapped around complex surfaces D_i, while the O7 planes are wrapped around complex surfaces O_j.
- The corresponding F-theory is given by a Calabi-Yau four-fold CY_4

Act II:The tadpole conditions

D7 tadpole:
In IIB

$$\sum_i [D_i] = 4 \sum_j [O_j]$$

Type IIB :
D3 tadpole

$$\frac{1}{2} N_{D3} = \sum_i \frac{\chi(D_i)}{24} + \sum_j \frac{4\chi(O_j)}{24}$$

F-theory:
D3 tadpole

$$N_{D3} = \sum_i \frac{\chi(CY_4)}{24}$$

Act III:Type IIB and F-theory tadpoles matching condition

$$2\chi(CY_4) = \sum_i \chi(D_i) + 4\sum_j \chi(O_j)$$

An embarrassing mismatch

The type IIB/F-theory tadpole matching condition is not satisfied

One can prove that this is not an exception… It is the norm!

Is it the end of the honeymoon between string theory and self-consistency?

Not so fast…

Seven branes in F-theory

Elliptic fibration: $Y \rightarrow B$

Weierestrass form: $y^2 = z^3 + fz + g$

Discriminant: $\Delta = 4f^3 + 27g^2$

The seven branes are located at the singularities of the elliptic fibration is singular:

$$\Delta = 0$$

Sen's weak coupling limit of F-theory

$$y^2 = z^3 + fz + g$$

Sen's ansatz:

$$\begin{cases} f = & -3h^2 + C\eta \\ g = & -2h^3 + Ch\eta + C^2\chi \end{cases}$$

$$\Delta \sim C^2 h^2 (\eta^2 + 12h\chi)$$

Calabi-Yau three-fold: $x_o^2 = h$

Orientifold O7-plane: $x_o = h = 0$

D7 branes: $\eta^2 + 12x_o^2\chi = 0$

I see …

Singularities!!!

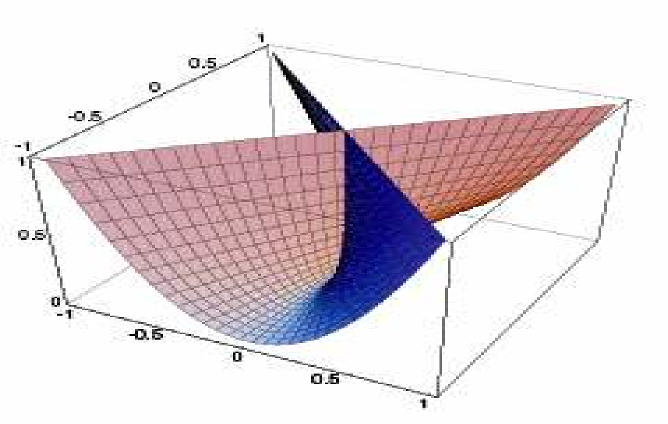
Anything special in Sen’s weak coupling limit?

$$\eta^2 + 12h\chi = 0$$

Singularities of the D7 brane

The divisor $\eta^2 + 12x_0^2\chi = 0$ can be mapped to

the Withney umbrella: $x^2 = y^2 z$



QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

•A double curve along the z axis (x=y=0)

•A pinch point at the origin (x=y=z=0)

Topological invariant and singularities

- Singularities are an invitation to diversity:

There are many ways to extend the usual topological invariants to singular varieties.

- There will be casualties :

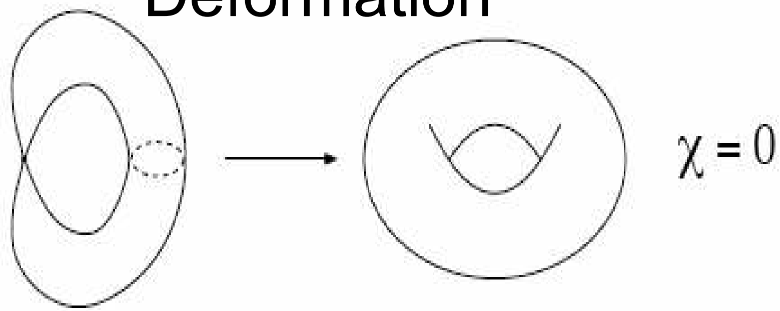
Not all the properties are preserved in the process

- Hard choices:

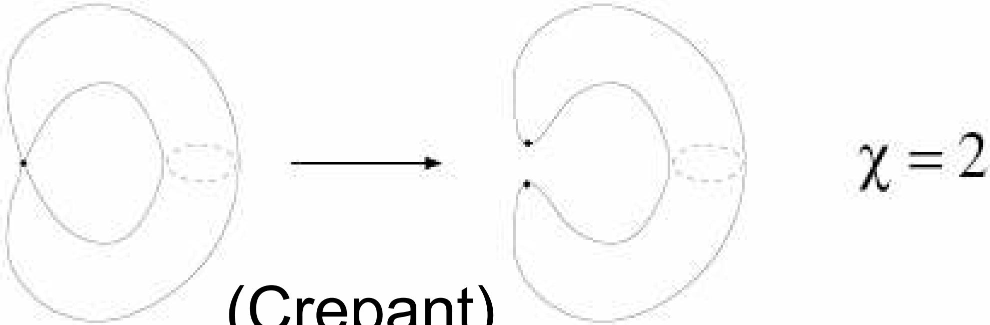
what are the properties that really matter?

Example: Euler characteristic of a singular elliptic curve

Fulton Euler characteristic



Stringy Euler characteristic



Defining topological invariant for singular spaces

- We work in homology
- We defined generalized Chern classes
- The topological invariants follow

For example, the Euler characteristic is defined from the “top Chern class”, which in homology is the term of order zero.

The pinch points story

Smooth prejudice

$$2\chi(CY_4) = \chi(D7) + 4\chi(O7)$$

Pinch points discrepancy

$$2\chi(CY_4) = 4\chi(O7) + \chi(D7) - \chi(\text{Pinch Points})$$

Generalization

We can upgrade the previous relation to a more general Theorem :

- in any dimension
- Without the CY condition
- At the level of the total Chern classes

$$2c(Y) = 4c(O) + c(D) - c(S)$$

Recovering the tadpole matching conditition

It is possible to define an Euler characteristic χ^{∞} such that we recover the relation expected from the smooth case:

$$2\chi(Y) = \chi^{\infty}(D) + 4\chi(O)$$

χ^{∞} is defined as the limit of χ^{m} as m goes to infinity.

χ^{m} is defined through the Hopf-Poincaré theorem from a generalized Chern class $c^{(m)}$

m has to do with the definition of the "relative canonical divisor".

Conclusions

Singularities are there and they can be very useful.

• In F-theory, we have to take them seriously into account since might show up in the weak coupling limit

•This represents a challenge to define the topological invariant

•Can we get the same result from a direct string calculation or different physical arguments? The answer seems to be yes, but this is a different story…

D-brane Deconstruction in Type IIB Orientifolds

P. Aluffi, A. Colinnuci, F. Denef and M. E.

The Euler characteristic formula for singular varieties that we have presented here is confronted to

- K-theory
- Sen-Witten deconstuction of D-brane configuration,
- Dirac quantization
- moduli counting
- and more

It passes successfully several physical tests!