ATTRACTORS IN DIVERSE DIMENSIONS

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3rd RTN Workshop

Constituents, Fundamental Forces and Symmetries of the Universe

Valencia, Spain, 3rd October 07

" Attractor mechanism, was first considered in the framework of $N=2$ supergravity in $D=4$ dimensions

S.F., Kallosh, Strominger
S.F., Kallosh; Strominger
S.F. Gibbons, Kallosh

Extremal Black-Holes $(T=0)$

$$\phi^i(r) \underset{r\to\infty}{\longrightarrow} \phi^i_\infty \in \mathcal{M}$$

$$\phi^i(r) \underset{r\to r_H}{\longrightarrow} \phi^i_H(e_\Lambda, m^\Lambda)$$

The flow is non singular provided

$$\left.\frac{\partial V_{BH}}{\partial \phi^i}\right|_{\phi^i=\phi^i_H} = 0$$

The Bekenstein-Hawking entropy formula

$$S = \frac{A_H}{4} = \pi \left. V_{BH}\right|_{\phi^i=\phi^i_H}$$

Calabi-Yau B-H (Attractor varieties) G. Moore...

Recent advances:

Extremal non-BPS black-holes, attractors and critical points

$$M^2_{ADM}\Big|_{\text{Horizon}} > |Z|^2_H \quad \text{Central charge}$$

Kallosh, S.F. hep-th/0603247

Gimon, Kallosh, S.F. hep-th/0606211

Bellucci, Gunaydin, Marrani, S.F. hep-th/0606209

Marrani, S.F. arXiv:0705.3866
arXiv:0706.1674

Cerchiai, Marrani, S.F. arXiv:0707.0964

Tripathy-Trivedi

Goldstein, Iizuka, Jena, Trivedi

Kallosh, De Wit et al (Higher derivative corrections..)

Sen, Kallosh et al.

Dabholkar, Sen, Trivedi Kraus, Larsen (black rings, beyond Einstein)

Saraikin, Vafa

Cardoso, Dall'Agata

- - - - - Kallosh, Sivanandam, Soroush

O.S.V. (Ooguri, Strominger, Vafa) (top. part. fun.)

Sen (Entropy function formalism)

Entropy of charged black-holes in N - extended supergravities and BH charge configurations studied and classified before

Gibbons, Hull ; Breitenlohner, Maison, Gibbons
Kallosh, Linde, Ortin, Peet, van Proeyen
Kallosh, Ortin, Peet ; Kallosh, Kol
Khuri, Ortin
Duff, Khuri, Rahmfeld
Cvetic, Hull; Cvetic, Youm; Cvetic, Tseytlin

Electric-Magnetic Duality:

Gaillard. Zumino
S.F., Scherk, Zumino

Reviews on Black Holes

Duff. Khuri ; Maldacena; Peet ;
Pioline; Larsen; Dabholkar

Recent work on "attractors" of $N=8$ supergravities (32 local supersymmetries) in $D=4, 5, 6$ dimensions (non-degenerate orbits).

(S.F., R. Kallosh; S.F., A. Marrani; Andrianopoli, Marrani, Trigiante, S.F.; Ceresole, Marrani, S.F.)

previous related work

(S.F., Maldacena; S.F. Gunaydin; Andrianopoli, D'Auria, Lledó, S.F.)

Moduli space of BPS and nonBPS orbits of maximally extended sugras (32 charges)

Moduli space of nonBPS orbits of minimally extended sugras (8 charges)

Moduli space of BPS and nonBPS orbits of $N=4$ (16 charges) extended sugras

Relations between $D=4, 5, 6$ attractor equations and BPS nature

Black-holes coupled to scalar fields:

(4d N-extended supergravity)

$$\mathcal{L} = \sqrt{-g}\Big(-\frac{1}{2}R - g_{ab}\,\partial_\mu\phi^a\partial_\nu\phi^b g^{\mu\nu} +$$

$$\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^{\mu\nu\Sigma} + \frac{1}{2}\frac{1}{\sqrt{-g}}\,\mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}\,\epsilon^{\mu\nu\rho\lambda} F^\Lambda_{\mu\nu} F^\Sigma_{\rho\lambda} \dots\Big)$$

In $N=2$ special geometry

(similar formulae exist in all $N>2$ extended supergravities)

there is a relation among g_{ab}, $\mathcal{N}_{\Lambda\Sigma}$

$$f_I^\Lambda = e^{K/2}(X^\Lambda, \bar{\mathcal{D}}_{\bar a}\bar{X}^\Lambda)$$

$$h_{I\Lambda} = e^{K/2}(F_\Lambda, \bar{\mathcal{D}}_{\bar a}\bar{F}_\Lambda) \qquad \left(F_\Lambda = \frac{\partial}{\partial X^\Lambda}F(X)\right)$$

$$\mathcal{N}_{\Lambda\Sigma} = h_{I\Lambda}(f^{-1})^I_\Sigma$$

$$g_{a\bar b} = -i\left(\bar{\mathcal{D}}_{\bar b}\bar{X}^\Lambda \mathcal{D}_a F_\Lambda - \bar{\mathcal{D}}_{\bar b}\bar{F}^\Lambda \mathcal{D}_a X_\Lambda\right)e^K$$

For asymptotically flat extremal black-holes the black-hole potential is given in terms of the complex symmetric matrix

$$\mathcal{N}_{\Lambda\Sigma} = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} + i\,\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}\,, \quad \mathrm{Im}\,\mathcal{N} < 0$$

$(\Lambda, \Sigma = 1 \ldots n_V$ vector fields of the theory$)$

$\mathcal{N}(\phi^i)$ (over the moduli space)

$$\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\mu\nu\Sigma} + \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} \tilde{F}^{\mu\nu\Sigma}$$

and of the background charges

$$\frac{1}{4\pi}\int_{S_2} F^{\Lambda} = m^{\Lambda}\,, \quad \frac{1}{4\pi}\int_{S_2} \frac{\delta \mathcal{L}}{\delta F^{\Lambda}} = G_{\Lambda} = e_{\Lambda}$$

(S.F., Gibbons, Kallosh)

$$V_{BH}(\phi^i, e, m) = -\frac{1}{2}\left(e_{\Lambda} - \mathcal{N}_{\Lambda\Sigma} m^{\Sigma}\right)\left(\mathrm{Im}\,\mathcal{N}\right)^{-1\,\Lambda\Delta}\left(\bar{e}_{\Delta} - \bar{\mathcal{N}}_{\Delta\Gamma} m^{\Gamma}\right)$$

This formula is valid for any theory coupling Einstein gravity to scalars and Maxwell vector fields

Equivalent (but manifestly duality covariant form of the potential)

$$V = -\frac{1}{2} Q^T M Q$$

Q^T is the $2n_v$-dim vector (m^Λ, e_Λ) and M the $Sp(2n_v, R)$ $2n_v \times 2n_v$ symmetric matrix

$$M = R^T M_D R$$

$$R = \begin{pmatrix} 1 & 0 \\ -\mathrm{Re}\,N & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} \mathrm{Im}\,N & 0 \\ 0 & \mathrm{Im}\,N^{-1} \end{pmatrix}$$

with the property

$$M \Omega M = \Omega \qquad \left(\Omega = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\right)$$

$$M = \begin{pmatrix} \mathrm{Im}\,N + \mathrm{Re}\,N\,\mathrm{Im}\,N^{-1}\,\mathrm{Re}\,N & -\mathrm{Re}\,N\,\mathrm{Im}\,N^{-1} \\ -\mathrm{Im}\,N^{-1}\,\mathrm{Re}\,N & \mathrm{Im}\,N^{-1} \end{pmatrix}$$

For supergravity theories we can express V_{BH} in terms of dressed-charges which appear in the "fermion" transformation rules in a B-H background

$$\delta_\epsilon \lambda^I\Big|_{B\text{-}H} = \ldots Z^I(\phi, e, m)\,\epsilon$$

then $V_{BH} = \ldots |Z^I|^2$

Hence in N-extended supergravity

$$V_{BH} = \frac{1}{2}|Z_{AB}|^2 + |Z^I|^2$$

$Z_{AB} = -Z_{BA}$ central charge matrix

Z^I matter charges

$$\delta_\epsilon \psi_{\mu A} = \ldots Z_{AB}\gamma_\mu \epsilon^B$$

$$\delta_\epsilon \lambda^I_A = \ldots Z^I \epsilon_A$$

$D=4 \quad N=8$

BPS orbit : $I_4 > 0 \quad E_{7(7)}/E_{6(2)}$

Moduli space! $\longrightarrow E_{6(2)}/SU(2)\times SU(6)$

non BPS orbit : $I_4 < 0 \quad E_{7(7)}/E_{6(6)}$

Moduli space $\quad E_{6(6)}/USp(8)$

$D=5 \quad N=8$

BPS orbit $\quad I_3 \neq 0 \quad E_{6(6)}/F_{4(4)}$

Moduli space $\longrightarrow F_{4(4)}/USp(2)\times USp(6)$

$D=6 \quad N=8$

BPS orbit $\quad I_2 \neq 0 \quad SO(5,5)/SO(5,4)$

Moduli space $\longrightarrow SO(5,4)/SO(5)\times SO(4)$

(BPS) moduli spaces are always quaternionic!

Relation between $D=4, 5, 6$ $(N=8)$

(S.F., Maldacena)

$$E_{7(7)} \to E_{6(6)} \times SO(1,1)$$

$$E_{6(6)} \to SO(5,5) \times SO(1,1)$$

$$56 \to 27_1 + 1_{-3} + 27'_{-1} + 1_3$$

$$27 \to 16_1 + 10_{-2} + 1_4$$

$$I_4 = (56)^4 = (27_1)^3 1_{-3} + (27'_{-1})^3 1_3$$
$$+ 1_3 1_3 1_{-3} 1_{-3} + 27_1 27_1 27'_{-1} 27'_{-1} +$$
$$+ 27_1 27'_{-1} 1_3 1_{-3}$$

$$I_3 = (27)^3 = 10_{-2} 10_{-2} 1_4 + 16_1 16_1 10_{-2}$$

Singlets: K.K modes

$27, 27'$ (B.H − BS) $d=5$

10 (BS) $d=6$

16 (BH) $d=6$

Same for $N=2$ $d=\cancel{4}, 5, 6$

(Ceresole, Mancini, S.F.) (Andrianopoli, Mancini, Trigiante S.F.)

Symmetric spaces (Gunaydin, Sierra, Townsend)

(generic series + magic supergravities)

Particular cases of homogeneous spaces

(de Wit, van Proeyen, vanderseypen)

$L(q, P)$ spaces

$(L(q,0) = L(0,P)$ generic series

$SO(1,1) \times SO(1, q+1)/SO(q+1))$

$L(1,1)$ $L(2,1)$ $L(4,1)$ $L(8,1)$

$\Downarrow$

J_3^{R} J_3^{C} J_3^{H} J_3^{O}

at $d=6$:

$n_T = q+1$, $n_V =$ spinor of $SO(1, q+1)$

$N=2$ orbits of attractors classified
(Bellucci, Gunaydin, Marrani S.F.)
and their moduli space computed.
(S.F., Marrani; S.F., Gunaydin)

($N=4$ orbits and their moduli
spaces computed) (AFMT
ADFL)

$N=4$, $D=5$

BPS orbit $\quad SO(5,n)/SO(4,n)$

Moduli space $\quad SO(4,n)/SO(4)\times SO(n)$

non BPS orbit $\quad SO(5,n)/SO(5,n-1)$

Moduli space $\quad SO(5,n-1)/SO(5)\times SO(n-1)$

Two general classes of non-BPS attractor solutions $(\mathcal{D}_i Z \neq 0)$

$$2\bar{Z}\mathcal{D}_i Z + i C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \mathcal{D}_{\bar{j}} \bar{Z} \mathcal{D}_{\bar{k}} \bar{Z} = 0$$

1 $\bar{Z} \neq 0$, $J_4 < 0$

2 $\bar{Z} = 0$ $J_4 > 0$

Violation of the BPS bound:

$$M^2_{ADM}\big|_H = 4|Z|^2_H > |Z|^2_H \qquad 1)$$

$$M^2_{ADM}\big|_H = |\mathcal{D}_i Z|^2 > 0 \qquad 2)$$

$$\left(C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \mathcal{D}_{\bar{j}} \bar{Z} \mathcal{D}_{\bar{k}} \bar{Z} = 0 \right)$$

For symmetric spaces one has

$$\mathcal{D}_i C_{jkp} = 0$$

$$g^{k\bar{k}} g^{r\bar{j}} C_{r(pq} C_{i)jk} \bar{C}_{\bar{k}\bar{r}\bar{j}} = \frac{4}{3} g_{(q|\bar{r}} C_{|ijp)}$$

$$\left(E_{\bar{r}qijp} = 0 \right)$$

Special Geometry

$$R_{i\bar{j}\ell\bar{k}} = -g_{i\bar{j}}\,g_{\ell\bar{k}} - g_{i\bar{k}}\,g_{\ell\bar{j}} + C_{i\ell p}\,C_{\bar{j}\bar{k}\bar{p}}\,g^{p\bar{p}}$$

$$\left(\bar{D}_{\bar{i}}\,C_{i\ell p} = 0 \qquad D_{[k}\,C_{i]\ell p} = 0\right)$$

Symmetric spaces

$$D_k\,C_{i\ell p} = 0$$

$$C_{i\ell p} = e^{K}\,\partial_i\partial_\ell\partial_p\,f(t^i)$$

$$f(t^i) = \frac{1}{3!}\,d_{ijk}\,t^i t^j t^k \qquad ijk \to ABC$$

$$d_{ABC}\,d^{B(PQ}\,d^{LM)C} = \frac{4}{3}\,\delta_A^{(P}\,d^{QLM)}$$

(Cremmer, van Proeyen ; Gunaydin, Sierra, Townsend)

The associated moduli spaces for non BPS attractors are given in the tables.

Hessian of V (2n×2n sym. matrix)
Eigenvalues

1) $Z \neq 0$ n−1 massless (Tripathy, Trivedi)

2) $Z = 0$ massless according to tables

J_3^A ($A = 1, 2, 4, 8$ for $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$)

flat directions $Z \neq 0$ $3A + 2$
$Z = 0$ $2A_c$

($D = 5$) $\cancel{Z \neq 0}$ $2A$

For the 4D potential of cubic geometries (at vanishing axions) (CFM)

(p^0, q_0)

1) $V_{d=4} = \frac{1}{2}\left[(p^0)^2 \mathcal{V} + (q_0)^2 \mathcal{V}^{-1}\right]$

(p^0, q_i)

2) $V_{d=4} = \frac{1}{2}\left[(p^0)^2 \mathcal{V} + \mathcal{V}^{-1/3} V_{d=5}\right]$

$V_{d=5} = a^{ij} q_i q_j$ $\qquad a^{ij} = (a_{ij})^{-1}$

$a_{ij} = -\frac{\partial^2}{\partial\lambda^i \partial\lambda^j} \log \mathcal{V} \Big|_{\mathcal{V}=1}$ $\qquad \left(\mathcal{V} = \frac{1}{3!} d_{ijk} \lambda^i \lambda^j \lambda^k\right)$

Attractors

1) $\mathcal{V}_h = |q_0/p^0|$ $\qquad$ non BPS

$S = \pi |p^0 q_0|$

2) $\mathcal{V}_h^{4/3} = (p^0)^{-2}\left(\frac{1}{3!}|d^{ijk} q_i q_j q_k|\right)^{2/3}$

$S = \pi\, 2 |p^0 d^{ijk} q_i q_j q_k|^{1/2}$

For the 5d potential ($Y^\alpha = 0$) (ADFM)

(e_z, e_Λ, $e_\alpha = 0$)

$$V_{d=5} = z^2 e_z^2 + z^{-1} V_{d=6}(\hat{X}, e_\Lambda)$$

$$\mathcal{V} = \frac{1}{2} z X^\Lambda X^\Sigma \eta_{\Lambda\Sigma} + \frac{1}{2} X_\Lambda \underset{\overset{\shortparallel}{0}}{Y^\alpha} Y^\beta \Gamma^\Lambda_{\alpha\beta}$$

Attractor equations

$$z = \left(\frac{V^h_{d=6}}{2 e_z^2} \right)^{1/3}$$

$$S^{4/3} = 3 \left| \frac{1}{2} e_z e^\Lambda e_\Lambda \right|^{2/3}$$

	$\frac{\widehat{H}}{\widehat{h}}$	r	$dim_{\mathbb{R}}$
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$SO(1,1) \otimes \frac{SO(1,n-1)}{SO(n-1)}$	$1\ (n=1)$ $2\ (n \geqslant 2)$	n
$J_3^{\mathbb{O}}$	$\frac{E_{6(-26)}}{F_{4(-52)}}$	2	26
$J_3^{\mathbb{H}}$	$\frac{SU^*(6)}{USp(6)}$	2	14
$J_3^{\mathbb{C}}$	$\frac{SL(3,\mathbb{C})}{SU(3)}$	2	8
$J_3^{\mathbb{R}}$	$\frac{SL(3,\mathbb{R})}{SO(3)}$	2	5

Table 1: **Moduli spaces of non-BPS $Z \neq 0$ critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}=2$, $d=4$ homogeneous symmetric supergravities. They are the $\mathcal{N}=2$, $d=5$ homogeneous symmetric real special manifolds.**

	$\frac{\tilde{H}}{h}=\frac{\tilde{H}}{h'\otimes U(1)}$	r	$dim_{\mathbb{C}}$
quadratic sequence $n\in\mathbb{N}$	$\frac{SU(1,n-1)}{U(1)\otimes SU(n-1)}$	1	$n-1$
$\mathbb{R}\oplus\mathbf{\Gamma}_n,\ n\in\mathbb{N}$	$\frac{SO(2,n-2)}{SO(2)\otimes SO(n-2)\otimes U(1)},\ n\geqslant 3$	$1\ (n=3)$ $2\ (n\geqslant 4)$	$n-2$
$J_3^{\mathbb{O}}$	$\frac{E_{6(-14)}}{SO(10)\otimes U(1)}$	2	16
$J_3^{\mathbb{H}}$	$\frac{SU(4,2)}{SU(4)\otimes SU(2)\otimes U(1)}$	2	8
$J_3^{\mathbb{C}}$	$\frac{SU(2,1)}{SU(2)\otimes U(1)}\otimes\frac{SU(1,2)}{SU(2)\otimes U(1)}$	2	4
$J_3^{\mathbb{R}}$	$\frac{SU(2,1)}{SU(2)\otimes U(1)}$	1	2

Table 1: **Moduli spaces of non-BPS $Z=0$ critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}=2$, $d=4$ homogeneous symmetric supergravities. They are (non-special) homogeneous symmetric Kähler manifolds.**

	$\frac{\tilde{H}_5}{\tilde{K}_5}$	r	$dim_{\mathbb{R}}$
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$\frac{SO(1,n-2)}{SO(n-2)},\ n \geqslant 3$	$1\ (n \geqslant 3)$	$n-2$
$J_3^{\mathbb{O}}$	$\frac{F_{4(-20)}}{SO(9)}$	1	16
$J_3^{\mathbb{H}}$	$\frac{USp(4,2)}{USp(4) \otimes USp(2)}$	1	8
$J_3^{\mathbb{C}}$	$\frac{SU(2,1)}{SU(2) \otimes U(1)}$	1	4
$J_3^{\mathbb{R}}$	$\frac{SL(2,\mathbb{R})}{SO(2)}$	1	2

Table 1: **Moduli spaces of non-BPS critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}=2$, $d=5$ homogeneous symmetric supergravities.**