Deconstructing the Little Hagedorn Holography

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based on arXiv:0707.1158
in collaboration with

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Valencia

Objective & Motivation

- Objective:

Phase diagram of Little String Theory

-- through its holographic dual --

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  Phase diagram of Little String Theory

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- Motivation:
  - LST presents semiclassically a Hagedorn behavior. Detail study of Hagedorn regime.
  - LST has a holographic dual asymptotic to flat space except for a linear dilaton. Toy model for holography in flat spaces.

Little String Theory

- Definition: worldvolume theory of a stack of *N* type II NS5-branes in the decoupling limit

$$g_s \to 0 \qquad\qquad \ell_s = \text{fixed}$$

[Seiberg]

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We will focus on the type IIA case

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$$g_s \to 0 \qquad\qquad \ell_s = \text{fixed}$$ [Seiberg]

We will focus on the type IIA case

Characteristics:

- Lives in 6 dimensions
- It has $\mathcal{N} = (2,0)$ supersymmetry.
- It is non-gravitational: no graviton in the spectrum
- It is non-local; e. g. LST exhibits T-duality

Holographic dual of LST

Preceding definition not useful for treating the LST. However one can construct a holographic dual along the lines of the AdS/CFT correspondence.

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NS5-brane

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$$ds^2 = dx_6^2 + \left(1 + \frac{R^2}{r^2}\right)(dr^2 + r^2 d\Omega_3^2)$$

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LST dual:

$g_s \to 0$ ↓ $r \to 0$

[Aharony,Berkooz Kutasov,Seiberg]

$$ds^2 = dx_6^2 + dz^2 + R^2 d\Omega_3^2$$

$$\Phi = -\frac{z}{R} \qquad R = \sqrt{N\alpha'}$$

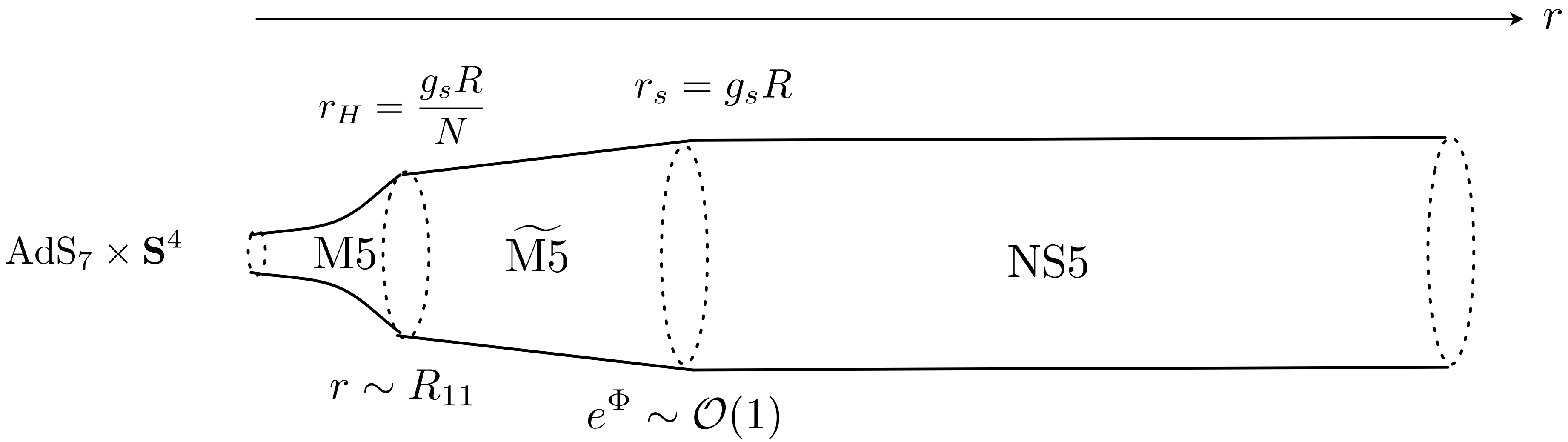
Type IIA on

$$\mathbb{R}^{5,1} \times \mathbb{R}_z \times S^3$$

Exact CFT

$$r = g_s R e^{\frac{z}{R}}$$

Holographic dual of LST



IR

UV

$\mathrm{AdS}_7 \times \mathbf{S}^4$

$\updownarrow$

$\mathcal{N} = (2,0)$ 6d SCFT

$\longleftrightarrow$

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Thermodynamics of LST

Finite temperature = near-extremal NS5-branes with a horizon at r_0

$$ds^2 = dt^2\left(1-\frac{r_0^2}{r^2}\right)+dy_5^2+\left(1+\frac{R^2}{r^2}\right)\left(\frac{dr^2}{1-r_0^2/r^2}+r^2\,d\Omega_3^2\right)$$

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Energy with respect to extremal NS5-brane

$$E(r_0) = \frac{V_5 m_s^8 r_0^2}{(2\pi)^5 g_s^2}$$

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Finite T dual:

$r = r_0 \cosh\sigma$

$g_s \to 0$ ↓ $r_0 \to 0$

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Semi-infinite cigar

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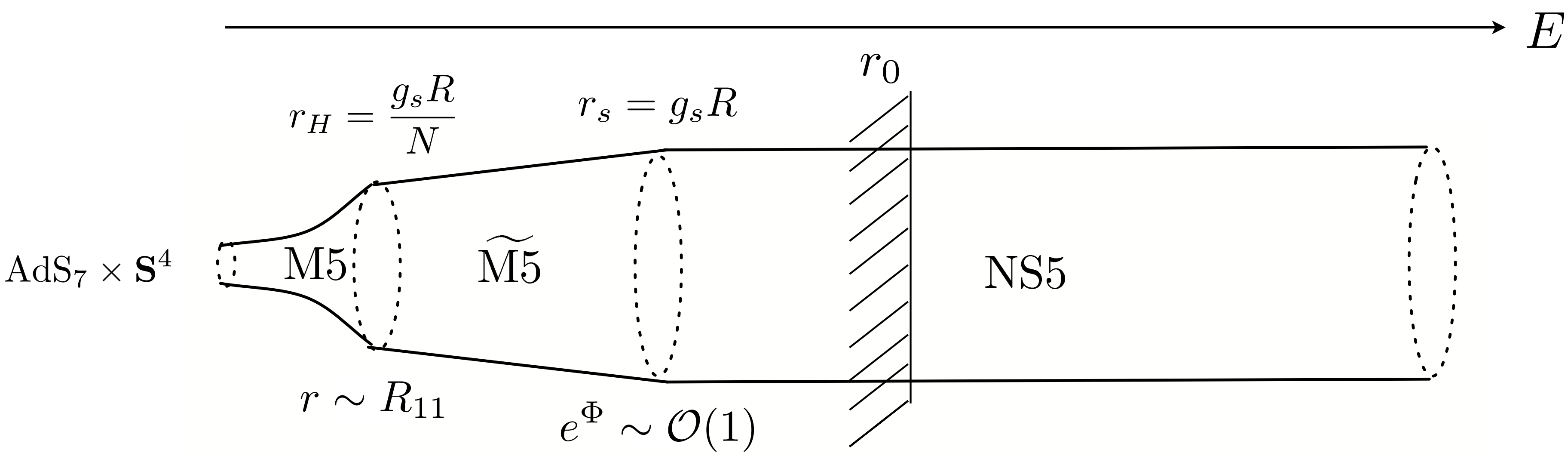
Semi-infinite cigar

$$\beta_H = 2\pi R$$

$$S = \beta_H E$$

Thermodynamics of LST

The position of the horizon gives the energy we are probing



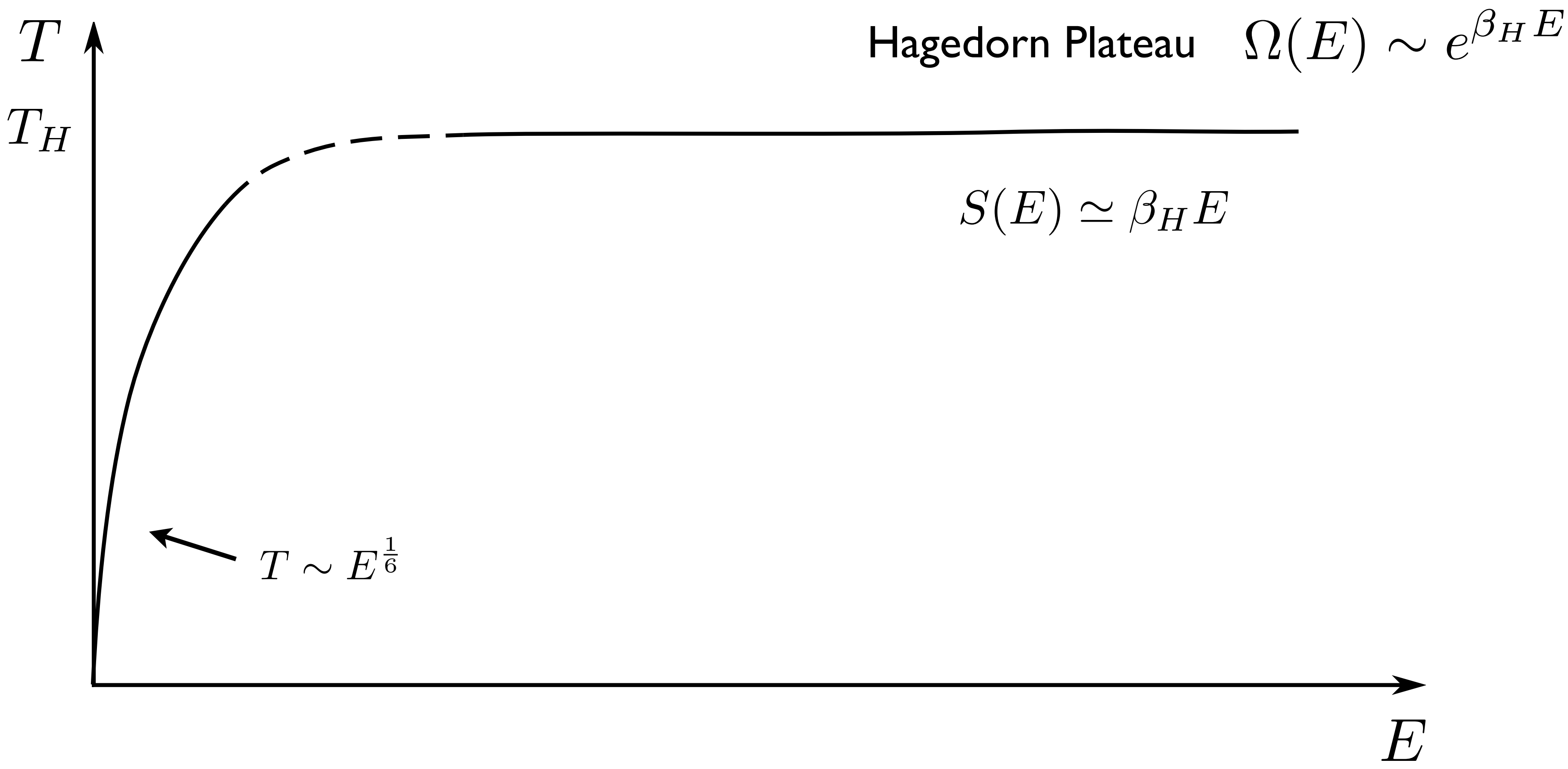
$$S_{\rm CFT} \sim N^3 V_5 T^5$$

$$E_{\rm CFT} \sim N^3 V_5 T^6$$

$$S_H \sim \beta_H E_H \qquad T_H = \frac{1}{2\pi R}$$

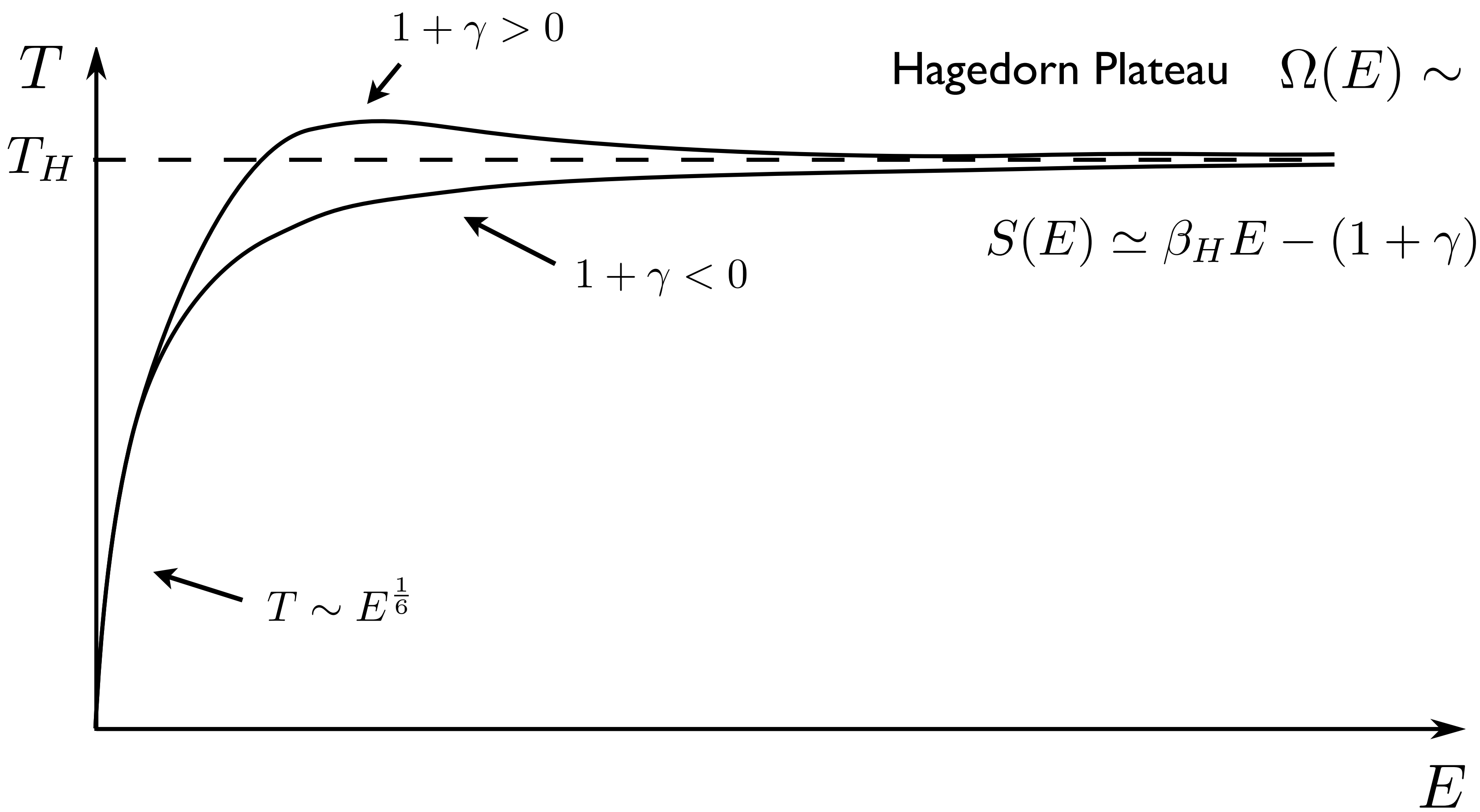
Thermodynamics of LST

Microcanonical View



Thermodynamics of LST

Microcanonical View



$$\Omega(E) \sim \frac{e^{\beta_H E}}{E^{1+\gamma}}$$

$$S(E) \simeq \beta_H E - (1+\gamma)\log\frac{E}{E_\Lambda}$$

Thermodynamics of LST

One loop correction to free energy has two contributions

$$-\beta F \simeq (1+\gamma)\log E$$

- Horizon region

- Tube region

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$$e^{2\Phi(r_0)} = \frac{g_s^2 R^2}{r_0^2} = N\,\frac{E_H}{E} \qquad \left.\frac{\delta T(E)}{T_H}\right|_{\mathrm{horizon}} \approx C\,\frac{N E_H}{E}$$

[Harmark, Obers]
[Berkooz,Rozali]

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[Harmark, Obers]
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- Tube region

Strings (radiation gas of the black brane) propagating in the tube

$$ds^2 = dx_6^2 + dz^2 + R^2 d\Omega_3^2 \qquad \Phi = -\frac{z}{R}$$

Corrections are extensive in the tube length!
They completely dominate

$$F_{\text{tube}} \propto \Delta z$$

[Kutasov, Sahakyan]

Radiation in the Tube

Free energy can be written as a sum over field excitations (flat space)

$$F^{(1)}_{\rm flat} = V_5\,\Delta z \sum_{f,j} (-1)^{F_f} \int \frac{d\vec{p}\,dp_z}{(2\pi)^6} \log\left(1-(-1)^{F_f} e^{-\beta\omega_{f,j}}\right)$$

$$\omega_{f,j} = \sqrt{\vec{p}^{\,2}+p_z^2+M_{f,j}^2}$$

$$= -V_5\,\Delta z \int_0^\infty \frac{d\tau_2}{2\tau_2}(4\pi^2\alpha'\tau_2)^{-\frac{7}{2}} \sum_{f,j}\sum_{\ell\in\mathbb{Z}} (-1)^{(\ell+1)F_f}\,\exp\left(-\pi\alpha'\tau_2 M_{f,j}^2 - \frac{\ell^2\beta^2}{4\pi\alpha'\tau_2}\right)$$

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Sugra

$$\begin{aligned} S_\Psi &= -\frac{1}{2}\int d^D x\,\sqrt{-g}\,e^{-2\Phi}\ (g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi) \\ &= -\frac{1}{2}\int dt dz \prod_i dV_i \quad \overline{\Psi}\ \left(\partial_t^2 - \partial_z^2 + V_{\text{eff}}(z)\right)\ \overline{\Psi} \end{aligned}$$

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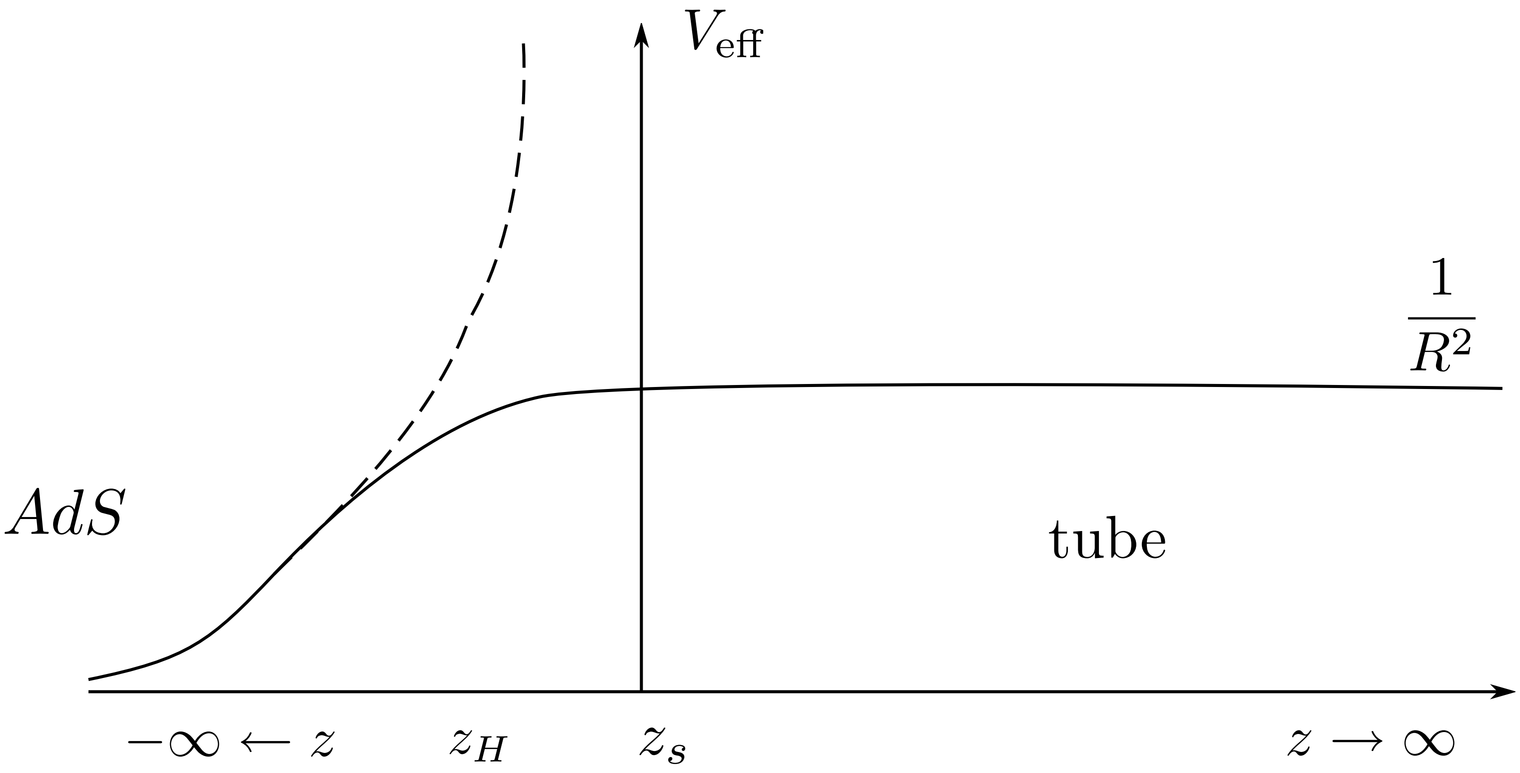
WKB approximation

$$F^{(1)}_{\mathrm{tube}} = -V_5 \int_0^\infty \frac{d\tau_2}{2\tau_2}(4\pi^2\alpha'\tau_2)^{-\frac{7}{2}} \sum_{f,j}\sum_{\ell\in\mathbb{Z}}(-1)^{(\ell+1)F_f} \int dz\ \exp\left(-\pi\alpha'\tau_2 V_{\mathrm{eff}}(z) - \frac{\ell^2\beta^2}{4\pi\alpha'\tau_2}\right)$$

Radiation in the Tube

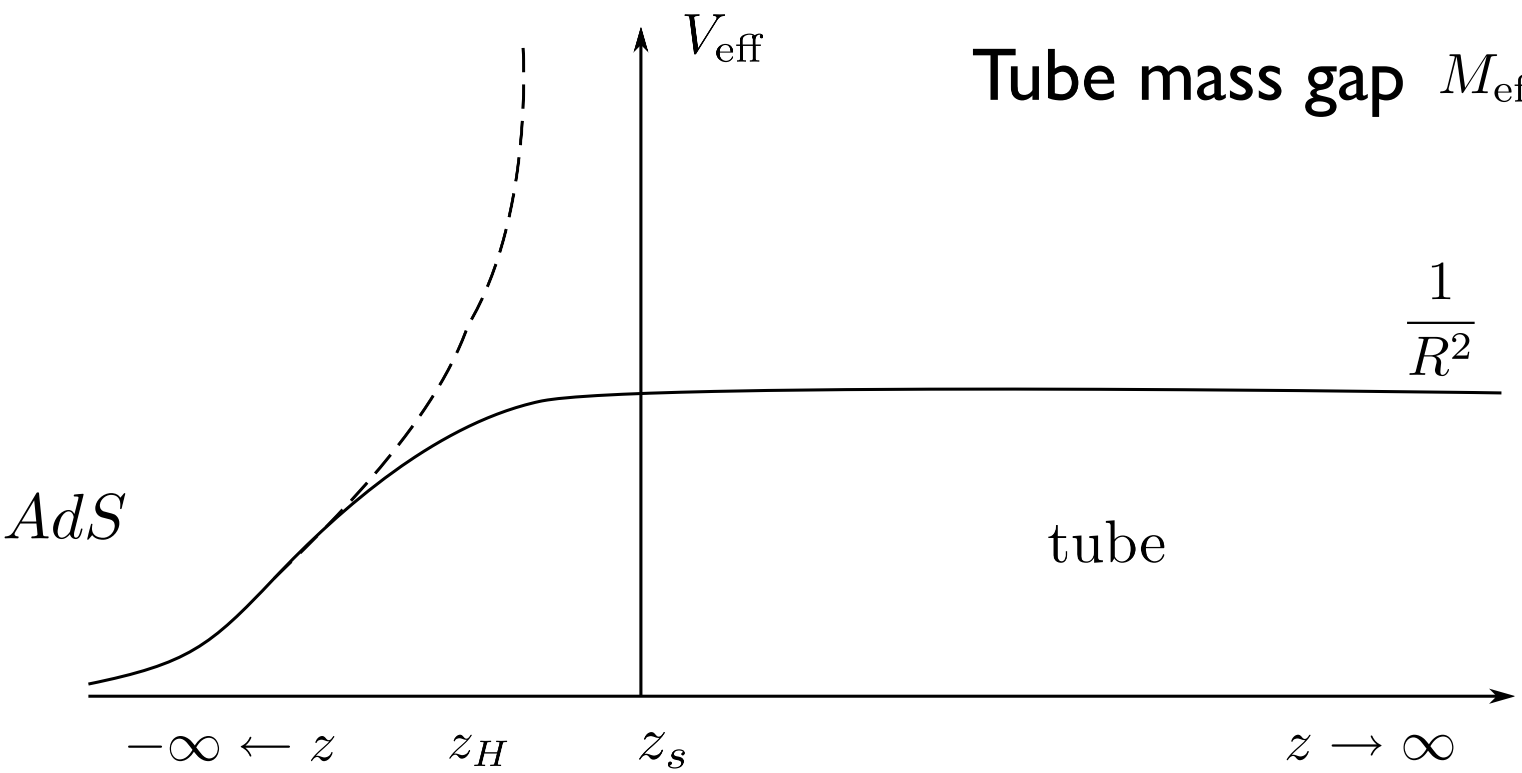


$$F^{(1)}_{\rm tube} = -V_5 \int_0^\infty \frac{d\tau_2}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-\frac{7}{2}} \sum_{f,j} \sum_{\ell \in \mathbb{Z}} (-1)^{(\ell+1)F_f} \int dz \, \exp\left(-\pi\alpha'\tau_2 V_{\rm eff}(z) - \frac{\ell^2\beta^2}{4\pi\alpha'\tau_2}\right)$$

For flat $V_{\rm eff}$ one has extensive behaviour in z: one has to regularize the length of the tube $\Delta z = z_\Lambda - z_b = \frac{1}{2} R \log(E_\Lambda / E_b)$

Radiation in the Tube

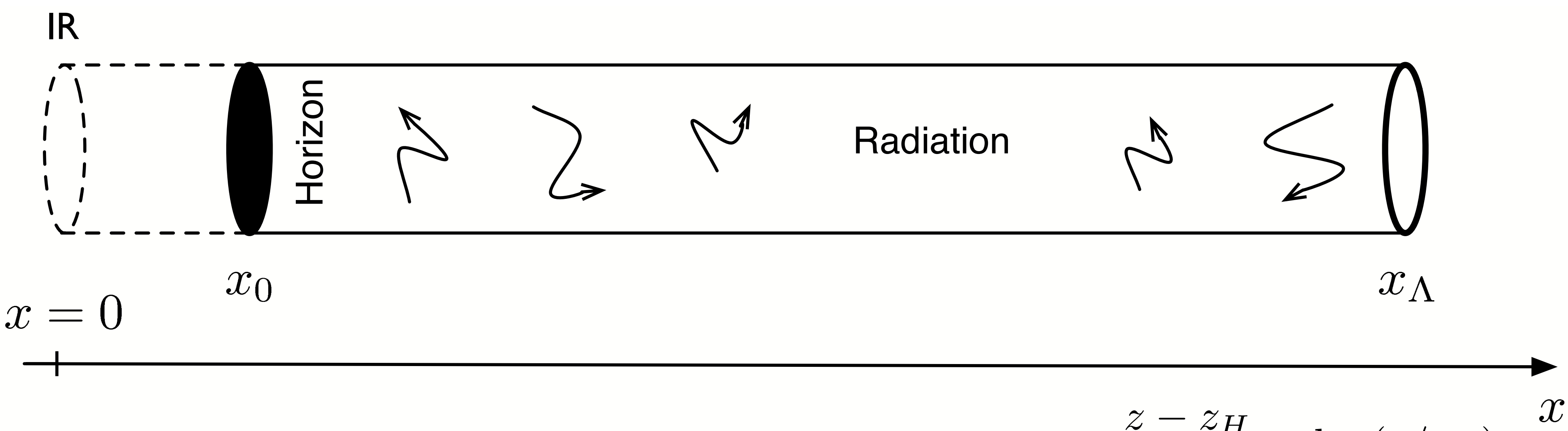
Tube mass gap $M_{\rm eff} \geq \frac{1}{R} \sim T_H$



States:

IR	behave 6-d relativistic	$S_{\rm IR} \sim N^{\frac{1}{2}} V_5^{\frac{1}{6}} E^{\frac{5}{6}}$
$E < T_H$	behave 7-d non-relativistic	$S_{nr} \sim \beta_H E (\log(V_6 \frac{T_H^7}{E}) + 1)$
$E > T_H$	behave 10-d relativistic	$S_r \sim V_9^{\frac{1}{10}} E^{\frac{9}{10}}$

Thermodynamics cutoff LST



$$x \equiv \frac{z - z_H}{R} = \log(r/r_H)$$

$$S_{\rm total}(E_T) = S_{\rm Black\,Brane}(E_{BB}) + S_{\rm Radiation}(E_R)$$

$$E_T = E_{BB} + E_R$$

$$S_{\rm IR} \sim N^{\frac{1}{2}} V_5^{\frac{1}{6}} E^{\frac{5}{6}}$$

$$S_{nr} \sim \beta_H E (\log(V_6 \frac{T_H^7}{E}) + 1)$$

$$S_r \sim V_9^{\frac{1}{10}} E^{\frac{9}{10}}$$

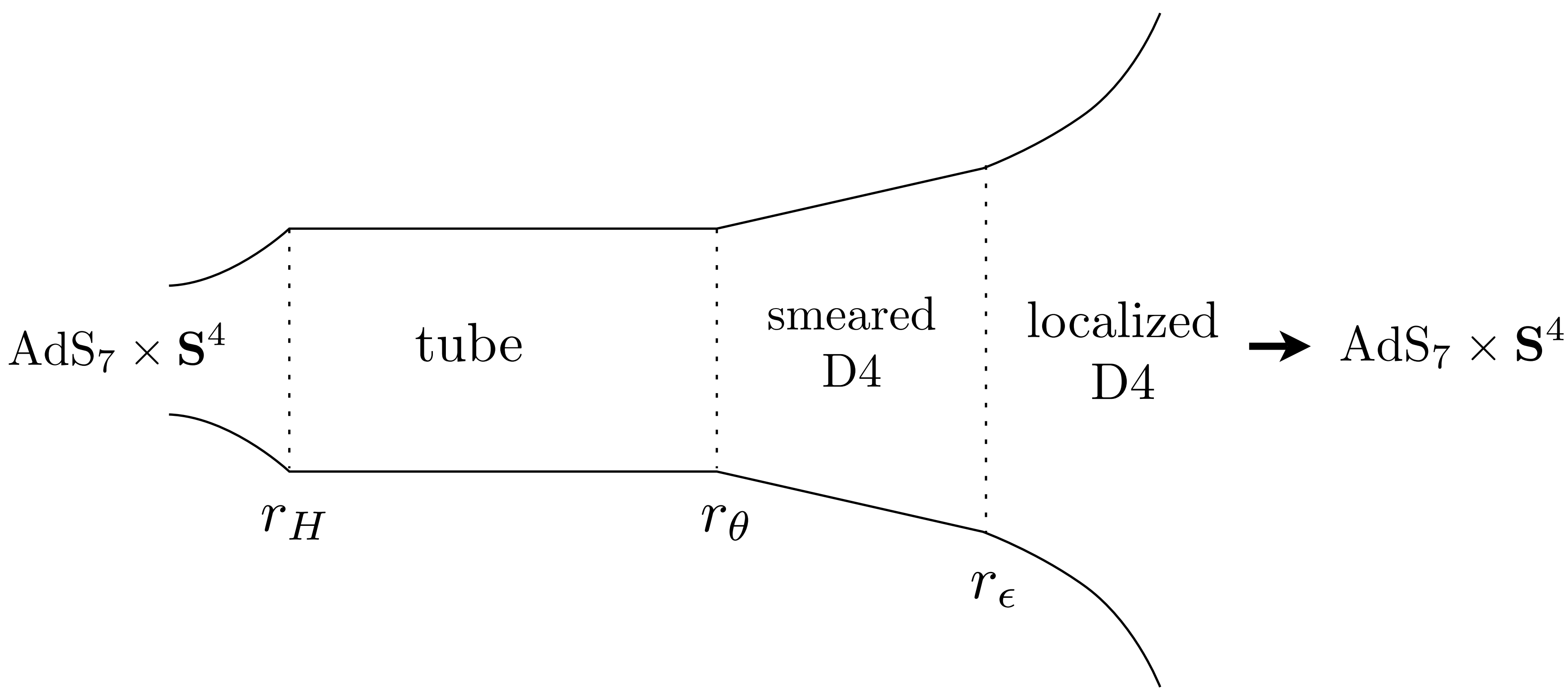
$$S_{BB} \sim \beta_H E$$

UV completion LST

Deconstruction:

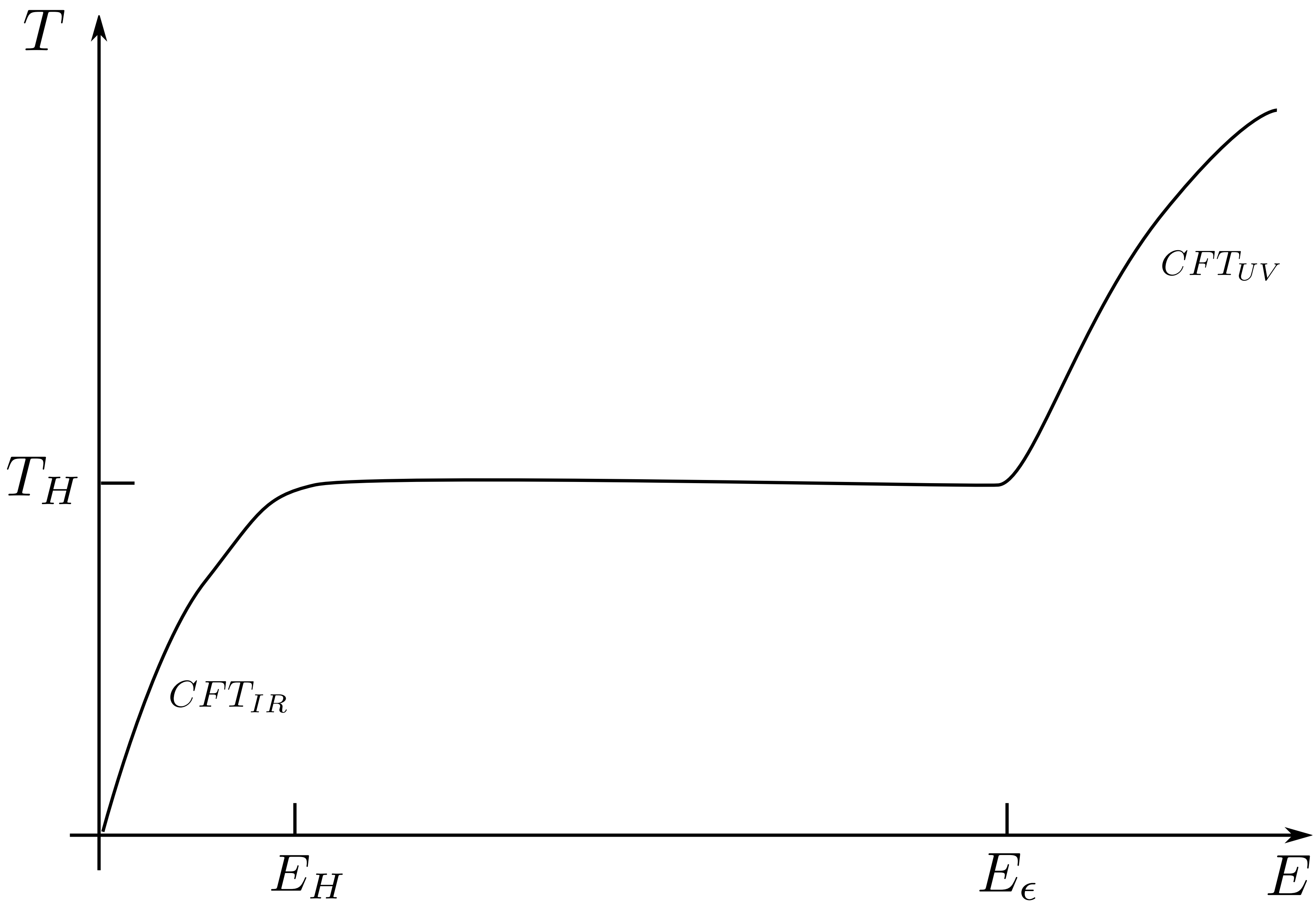
[Arkani-Hamed, Cohen, Kaplan, Karch, Molt]
[Dorey]

Physical realization of cutoff (AdS)
Stable degrees of freedom in the UV (CFT)



Smeared D4-branes on a torus with wrapped NS5-branes

Phase Diagram of LST



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