

Deconstructing the Little Hagedorn Holography

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based on [arXiv:0707.1158](#)

in collaboration with

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Valencia

Objective & Motivation

- Objective:

Phase diagram of Little String Theory
-- through its holographic dual --

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Phase diagram of Little String Theory
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- Motivation:

- ▶ LST presents semiclassically a Hagedorn behavior. Detail study of Hagedorn regime.
- ▶ LST has a holographic dual asymptotic to flat space except for a linear dilaton. Toy model for holography in flat spaces.

Little String Theory

- Definition: worldvolume theory of a stack of N type II NS5-branes in the decoupling limit

$$g_s \rightarrow 0$$

$$\ell_s = \text{fixed}$$

[Seiberg]

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We will focus on the type IIA case

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Characteristics:

- ▶ Lives in 6 dimensions
- ▶ It has $\mathcal{N} = (2, 0)$ supersymmetry.
- ▶ It is non-gravitational: no graviton in the spectrum
- ▶ It is non-local; e. g. LST exhibits T-duality

Holographic dual of LST

Preceding definition not useful for treating the LST. However one can construct a holographic dual along the lines of the AdS/CFT correspondence.

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NS5-brane

$$e^{2\Phi} = g_s^2 \left(1 + \frac{R^2}{r^2}\right) \quad \int_{S^3} dB = N \quad R = \sqrt{N\alpha'}$$

$$ds^2 = dx_6^2 + \left(1 + \frac{R^2}{r^2}\right) (dr^2 + r^2 d\Omega_3^2)$$

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LST dual:

$$g_s \rightarrow 0 \quad \downarrow \quad r \rightarrow 0$$

[Aharony, Berkooz
Kutasov, Seiberg]

$$r = g_s R e^{\frac{z}{R}}$$

$$ds^2 = dx_6^2 + dz^2 + R^2 d\Omega_3^2$$

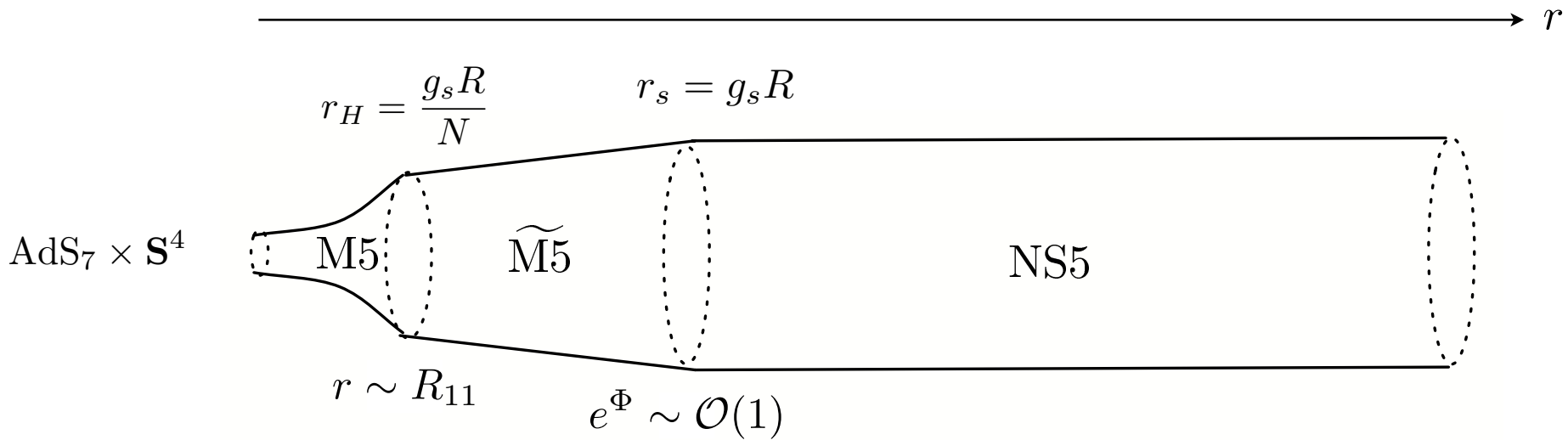
$$\Phi = -\frac{z}{R} \quad R = \sqrt{N\alpha'}$$

Type IIA on

$$\mathbb{R}^{5,1} \times \mathbb{R}_z \times S^3$$

Exact CFT

Holographic dual of LST



IR

UV

$$\text{AdS}_7 \times \mathbf{S}^4$$

$$ds^2 = dx_6^2 + dz^2 + R^2 d\Omega_3^2$$

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$$\mathcal{N} = (2, 0) \text{ 6d SCFT}$$

$$\mathbb{R}^{5,1} \times \mathbb{R}_z \times S^3$$

Thermodynamics of LST

Finite temperature = near-extremal NS5-branes with a horizon
at r_0

$$ds^2 = dt^2 \left(1 - \frac{r_0^2}{r^2}\right) + dy_5^2 + \left(1 + \frac{R^2}{r^2}\right) \left(\frac{dr^2}{1 - r_0^2/r^2} + r^2 d\Omega_3^2\right)$$

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$$E(r_0) = \frac{V_5 m_s^8 r_0^2}{(2\pi)^5 g_s^2}$$

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Finite T dual:

$$r = r_0 \cosh \sigma$$

$$g_s \rightarrow 0$$

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Semi-infinite cigar

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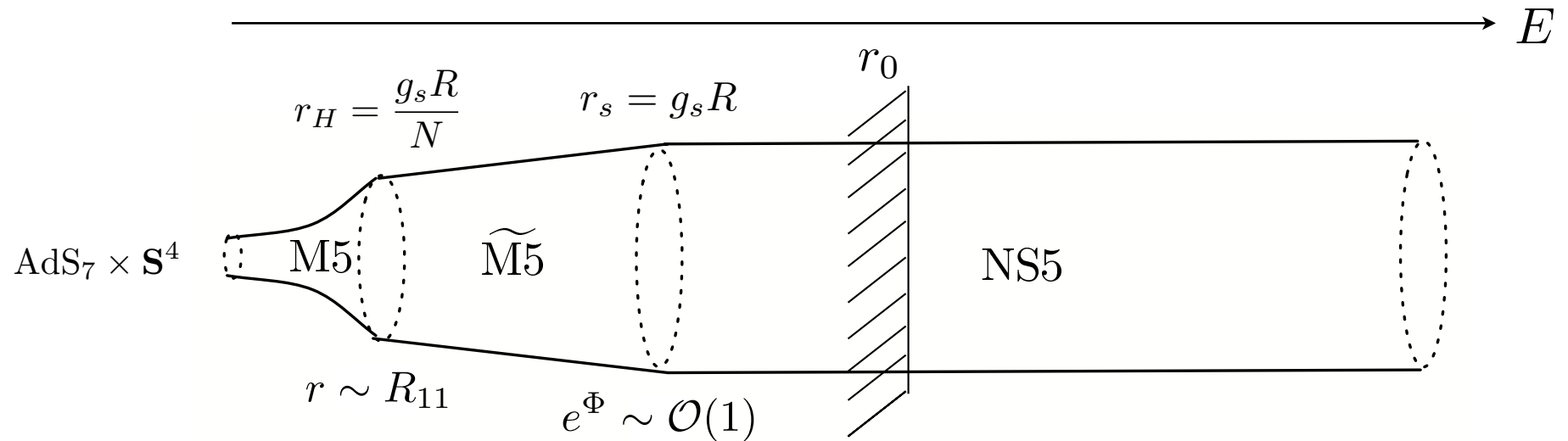
Semi-infinite cigar

$$\beta_H = 2\pi R$$

$$S = \beta_H E$$

Thermodynamics of LST

The position of the horizon gives the energy we are probing



$$S_{\text{CFT}} \sim N^3 V_5 T^5$$

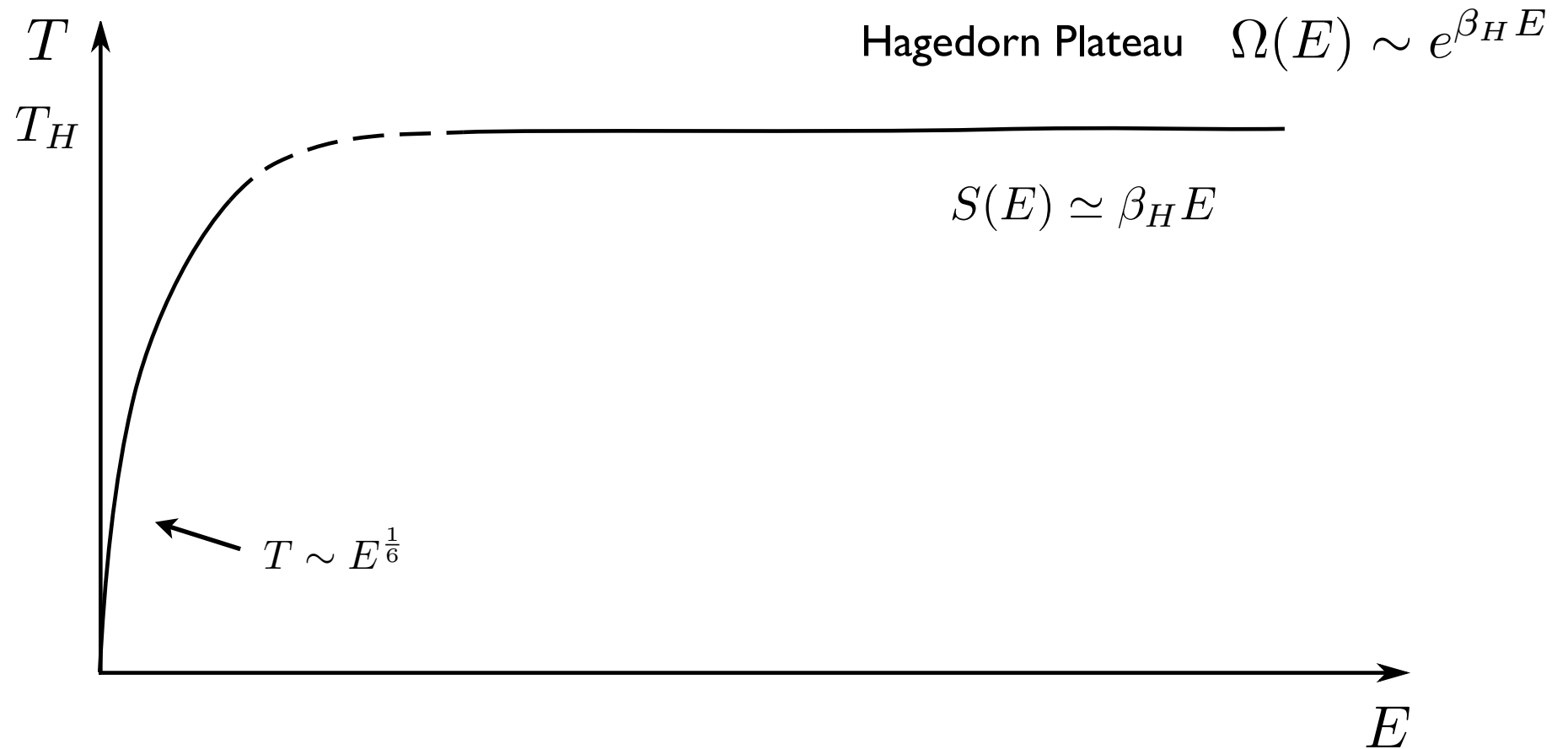
$$E_{\text{CFT}} \sim N^3 V_5 T^6$$

$$S_H \sim \beta_H E_H$$

$$T_H = \frac{1}{2\pi R}$$

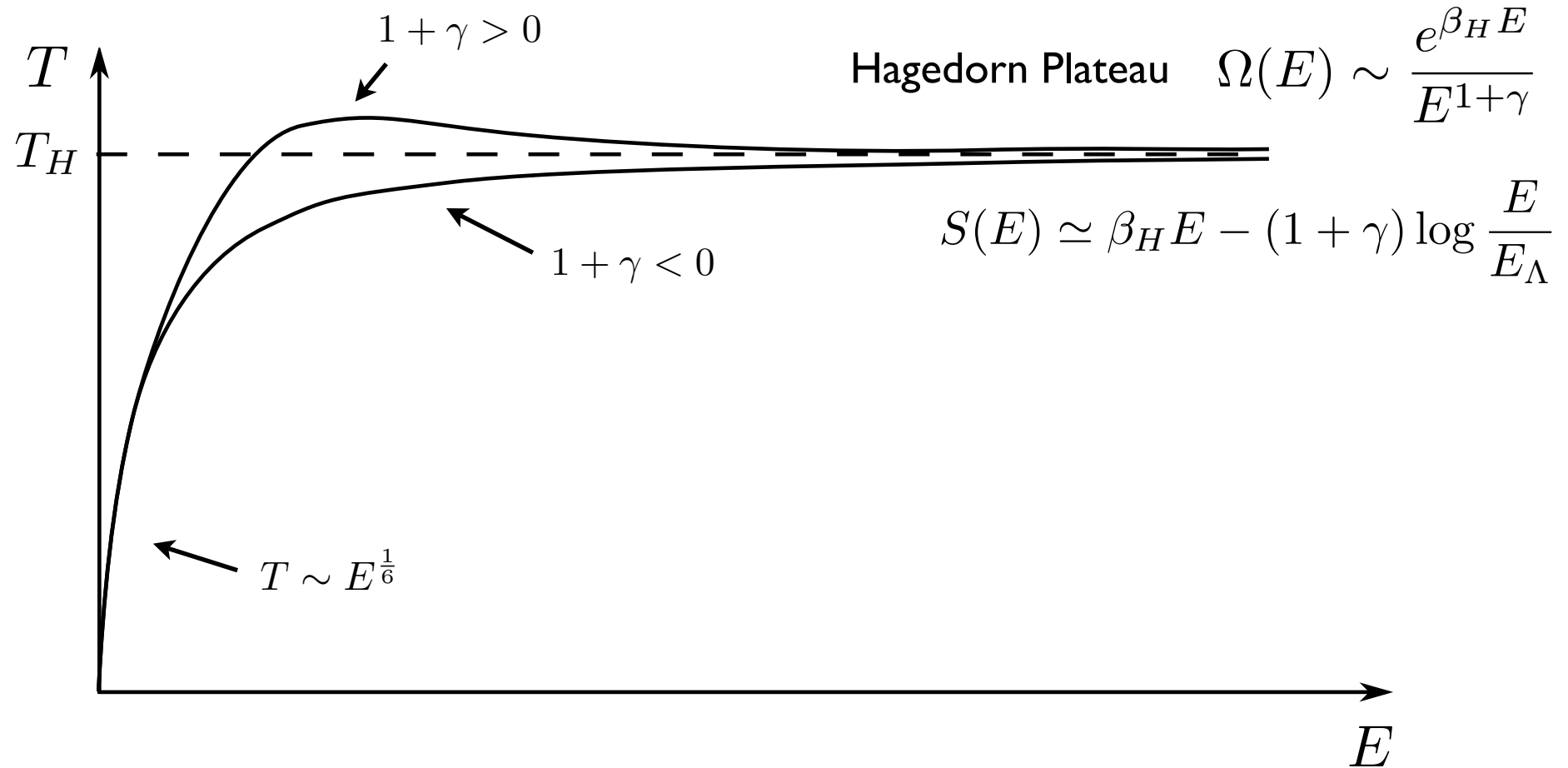
Thermodynamics of LST

Microcanonical View



Thermodynamics of LST

Microcanonical View



Thermodynamics of LST

One loop correction to free energy has two contributions

$$-\beta F \simeq (1 + \gamma) \log E$$

- Horizon region

$$e^{2\Phi(r_0)} = \frac{g_s^2 R^2}{r_0^2} = N \frac{E_H}{E} \quad \frac{\delta T(E)}{T_H} \Big|_{\text{horizon}} \approx C \frac{N E_H}{E}$$

[Harmark, Obers]
[Berkooz, Rozali]

- Tube region

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- Tube region

Strings (radiation gas of the black brane) propagating in the tube

$$ds^2 = dx_6^2 + dz^2 + R^2 d\Omega_3^2 \quad \Phi = -\frac{z}{R}$$

Corrections are extensive in the tube length!
They completely dominate

$$F_{\text{tube}} \propto \Delta z$$

[Kutasov, Sahakyan]

Radiation in the Tube

Free energy can be written as a sum over field excitations (flat space)

$$\begin{aligned} F_{\text{flat}}^{(1)} &= V_5 \Delta z \sum_{f,j} (-1)^{F_f} \int \frac{d\vec{p} dp_z}{(2\pi)^6} \log \left(1 - (-1)^{F_f} e^{-\beta \omega_{f,j}} \right) & \omega_{f,j} = \sqrt{\vec{p}^2 + p_z^2 + M_{f,j}^2} \\ &= -V_5 \Delta z \int_0^\infty \frac{d\tau_2}{2\tau_2} (4\pi^2 \alpha' \tau_2)^{-\frac{7}{2}} \sum_{f,j} \sum_{\ell \in \mathbb{Z}} (-1)^{(\ell+1)F_f} \exp \left(-\pi \alpha' \tau_2 M_{f,j}^2 - \frac{\ell^2 \beta^2}{4\pi \alpha' \tau_2} \right) \end{aligned}$$

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Sugra

$$\begin{aligned} S_\Psi &= -\frac{1}{2} \int d^D x \sqrt{-g} e^{-2\Phi} (g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi) \\ &= -\frac{1}{2} \int dt dz \prod_i dV_i \bar{\Psi} (\partial_t^2 - \partial_z^2 + V_{\text{eff}}(z)) \Psi \end{aligned}$$

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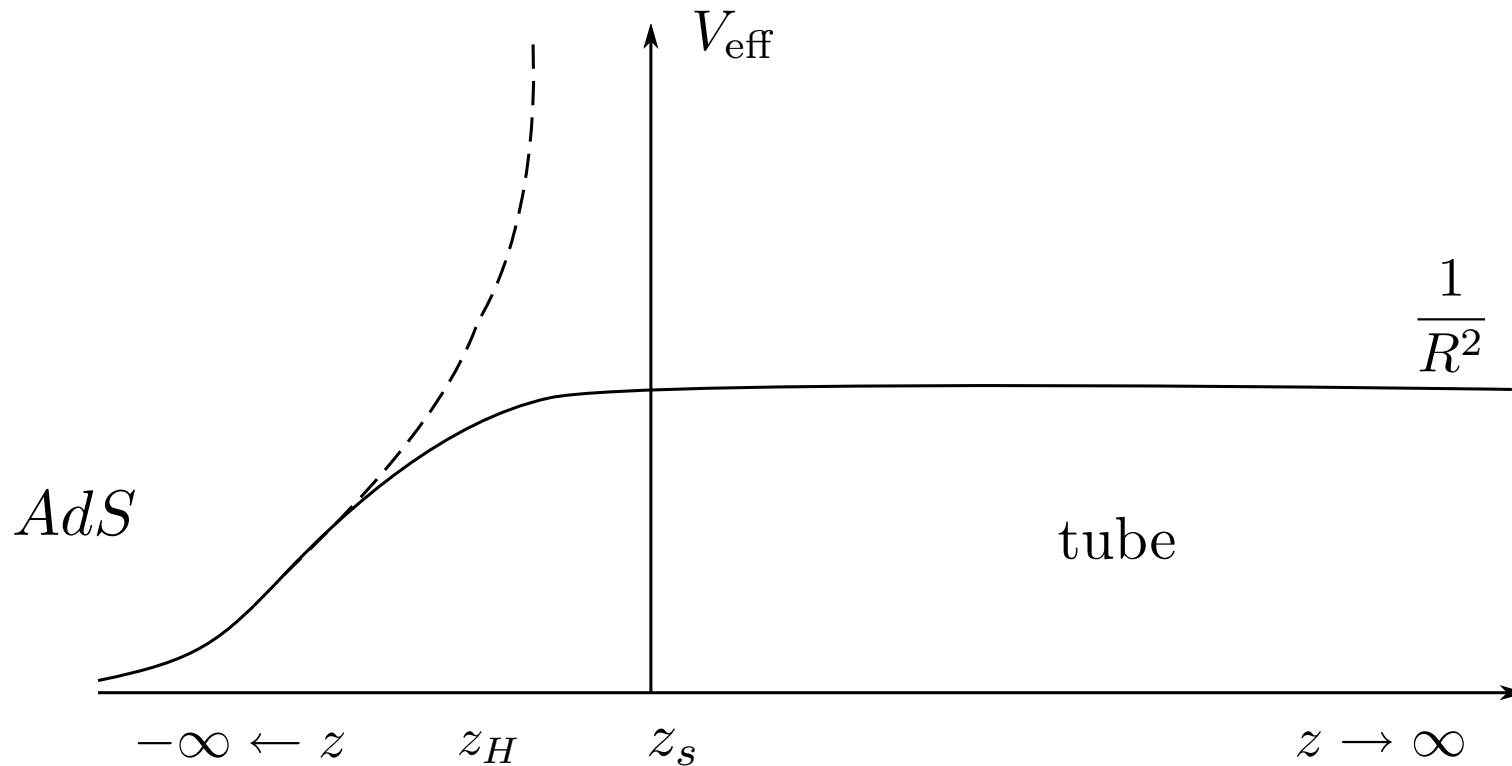
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WKB approximation

$$F_{\text{tube}}^{(1)} = -V_5 \int_0^\infty \frac{d\tau_2}{2\tau_2} (4\pi^2 \alpha' \tau_2)^{-\frac{7}{2}} \sum_{f,j} \sum_{\ell \in \mathbb{Z}} (-1)^{(\ell+1)F_f} \int dz \exp \left(-\pi \alpha' \tau_2 V_{\text{eff}}(z) - \frac{\ell^2 \beta^2}{4\pi \alpha' \tau_2} \right)$$

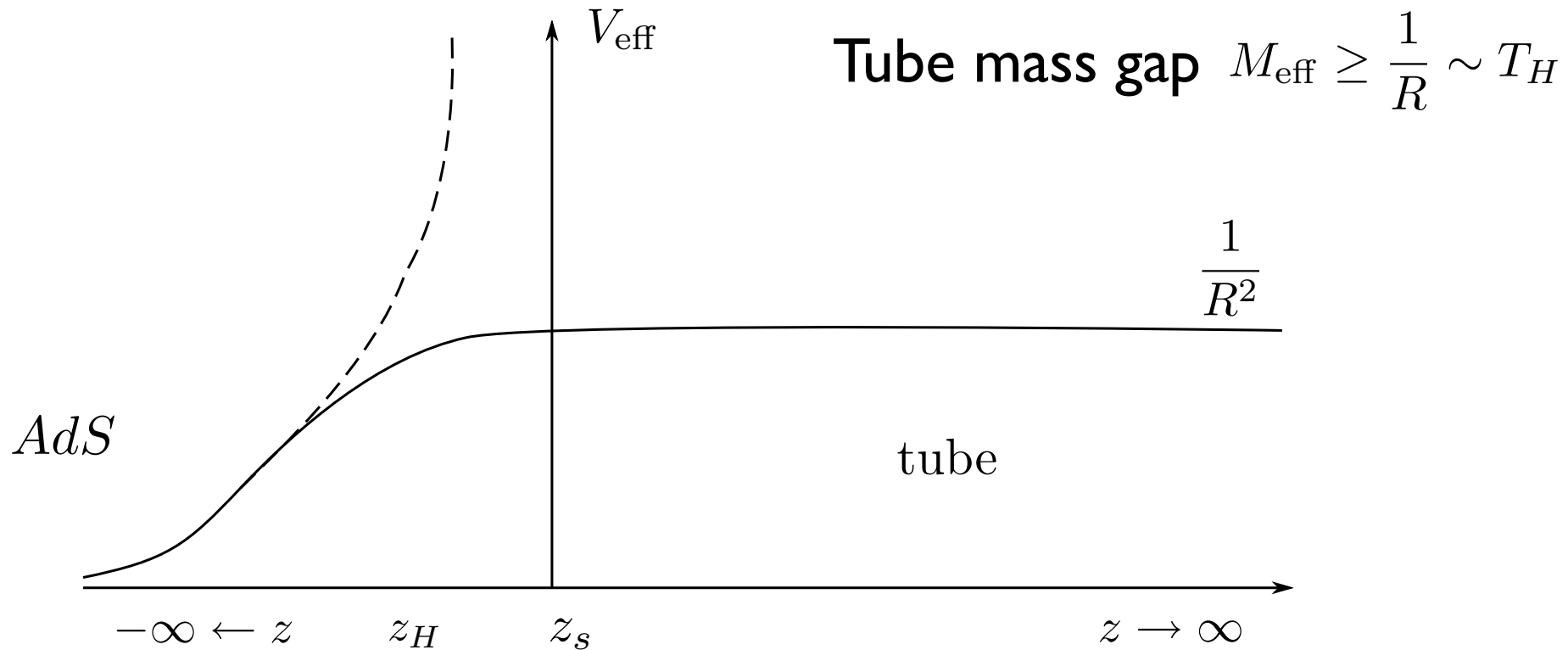
Radiation in the Tube



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For flat V_{eff} one has extensive behaviour in z : one has to regularize the length of the tube $\Delta z = z_\Lambda - z_b = \frac{1}{2} R \log(E_\Lambda/E_b)$

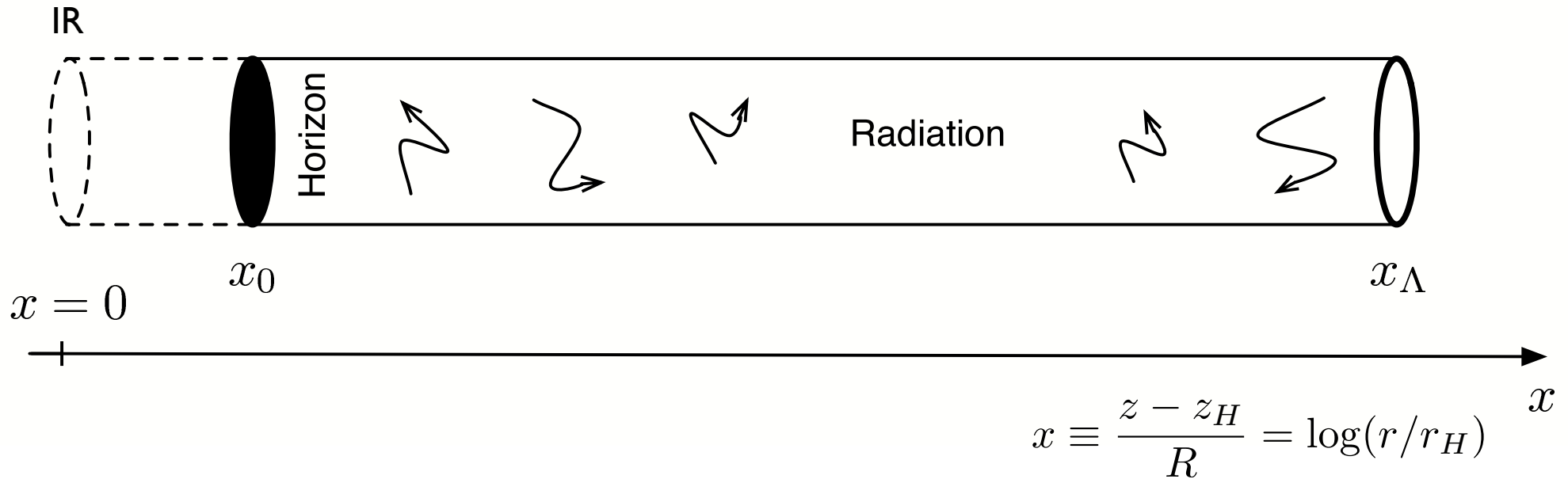
Radiation in the Tube



States:

IR	behave 6-d relativistic	$S_{\text{IR}} \sim N^{\frac{1}{2}} V_5^{\frac{1}{6}} E^{\frac{5}{6}}$
$E < T_H$	behave 7-d non-relativistic	$S_{nr} \sim \beta_H E (\log(V_6 \frac{T_H^7}{E}) + 1)$
$E > T_H$	behave 10-d relativistic	$S_r \sim V_9^{\frac{1}{10}} E^{\frac{9}{10}}$

Thermodynamics cutoff LST



$$S_{\text{total}}(E_T) = S_{\text{Black Brane}}(E_{BB}) + S_{\text{Radiation}}(E_R)$$

$$E_T = E_{BB} + E_R$$

$$S_{\text{IR}} \sim N^{\frac{1}{2}} V_5^{\frac{1}{6}} E^{\frac{5}{6}}$$

$$S_{nr} \sim \beta_H E (\log(V_6 \frac{T_H^7}{E}) + 1)$$

$$S_r \sim V_9^{\frac{1}{10}} E^{\frac{9}{10}}$$

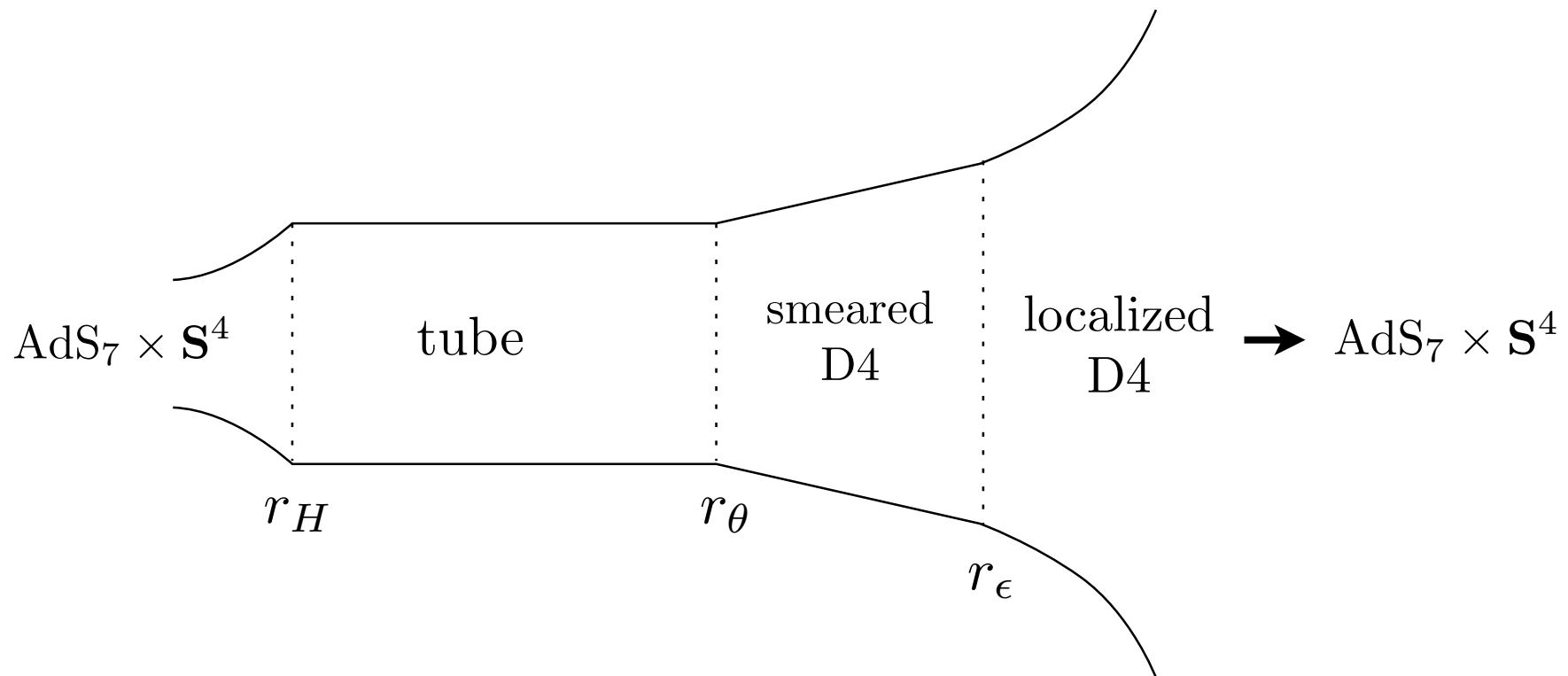
$$S_{BB} \sim \beta_H E$$

UV completion LST

Deconstruction:

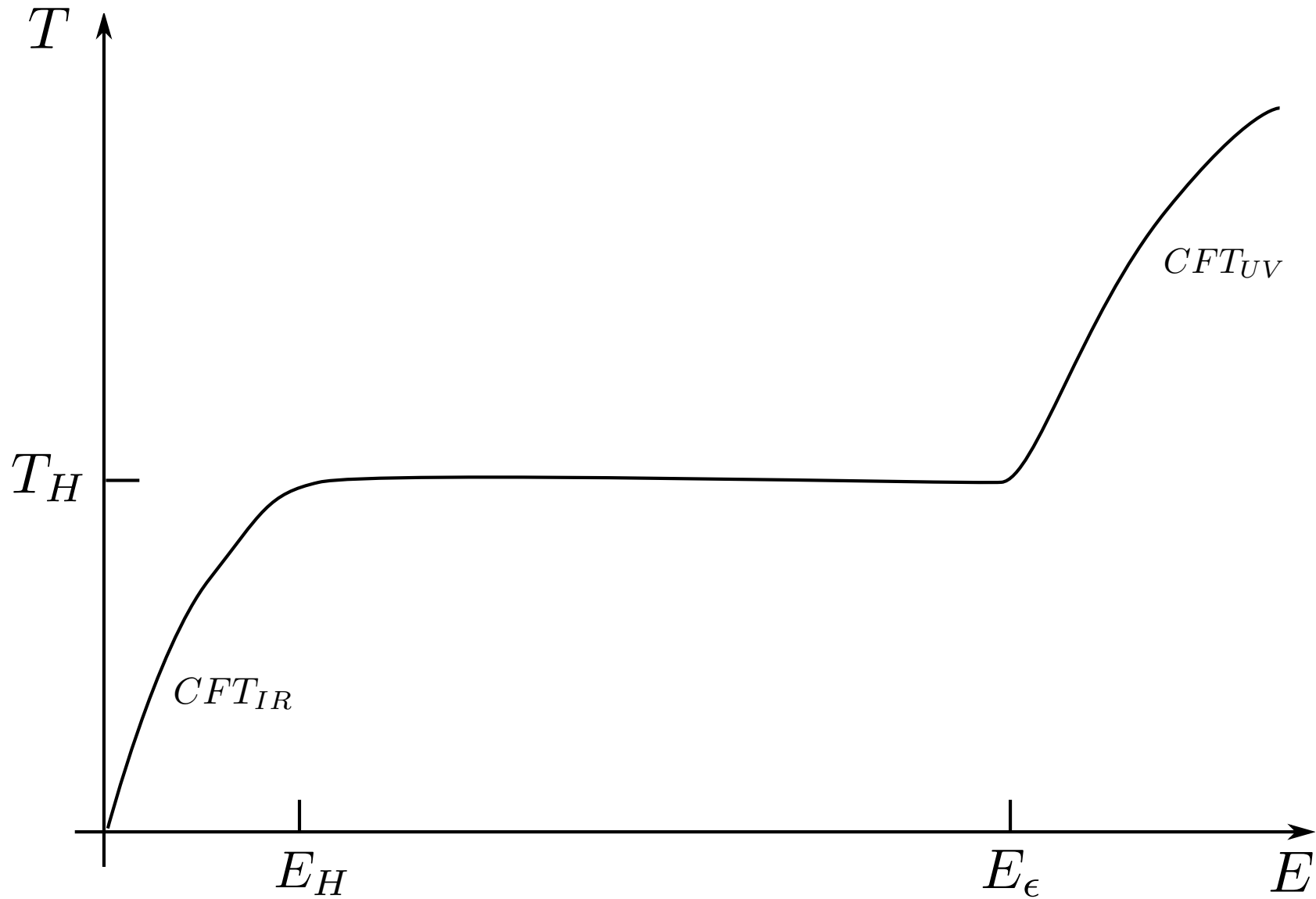
[Arkani-Hamed, Cohen,
Kaplan, Karch, Molt]
[Dorey]

Physical realization of cutoff (AdS)
Stable degrees of freedom in the UV (CFT)



Smeared D4-branes on a torus with wrapped NS5-branes

Phase Diagram of LST



Phase Diagram of LST

