N=1 4d Supermembrane from 11D

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OUTLINE OF THE TALK

- MOTIVATION
- 11D SUPERMEMBRANE
- SUPERMEMBRANE WITH CENTRAL CHARGES
- 4D ACTION
- SPECTRUM ANALYSIS
- N=1 SUPERSYMMETRY
- MODULI STABILIZATION

1. MOTIVATION

•**GOAL**: Perform the quantization of M2 in 4d with N=1

No Supergravity approach. Top- Down Approach.

• **OPEN PROBLEM**: NONPERTURBATIVE QUANTIZATION OF STRING TH.

⇩

M−theory Quantization: M2, M5

Bergshoeff,Sezgin,Townsend 87

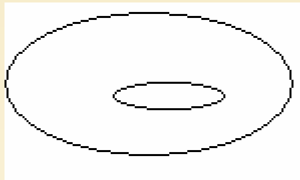
•**TOOLS:** Semiclassical Analysis: *Duff, Inami, Pope, Sezgin, Stelle 88; Restucia et al. 96*
Matrix Formulation: *de Witt, Hoppe, Nicolai 88' ;…,GM, Restuccia 00'* +…
Exact Formulation *Boulton, G.M. Restuccia 06'*

2. 11D SUPERMEMBRANE IN THE L.C.G

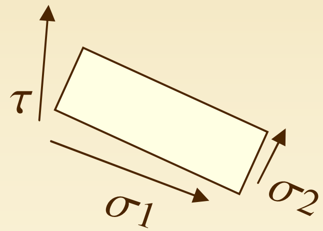
Bergshoeff, Sezgin, Townsend; De Witt, Hoppe, Nicolai; De Witt,Luscher, Nicolai, De Witt, Marquard, Nicolai, De Witt, Peeters, Plefka. 87-96

2+1D CLOSED RIEMANN SURFACE IN M_{11}

Σ_g



$(\sigma_1, \sigma_2, \tau)$



$$H = \int_\Sigma \sqrt{W} \left(\frac{1}{2} \left(\frac{P_M}{\sqrt{W}} \right)^2 + \frac{1}{4} \{X^M, X^N\}^2 + \text{ Fermionic terms} \right)$$

with $\{X^M, X^N\} = \frac{\varepsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a X^M \partial_b X^N$

Subject to the following constraints:

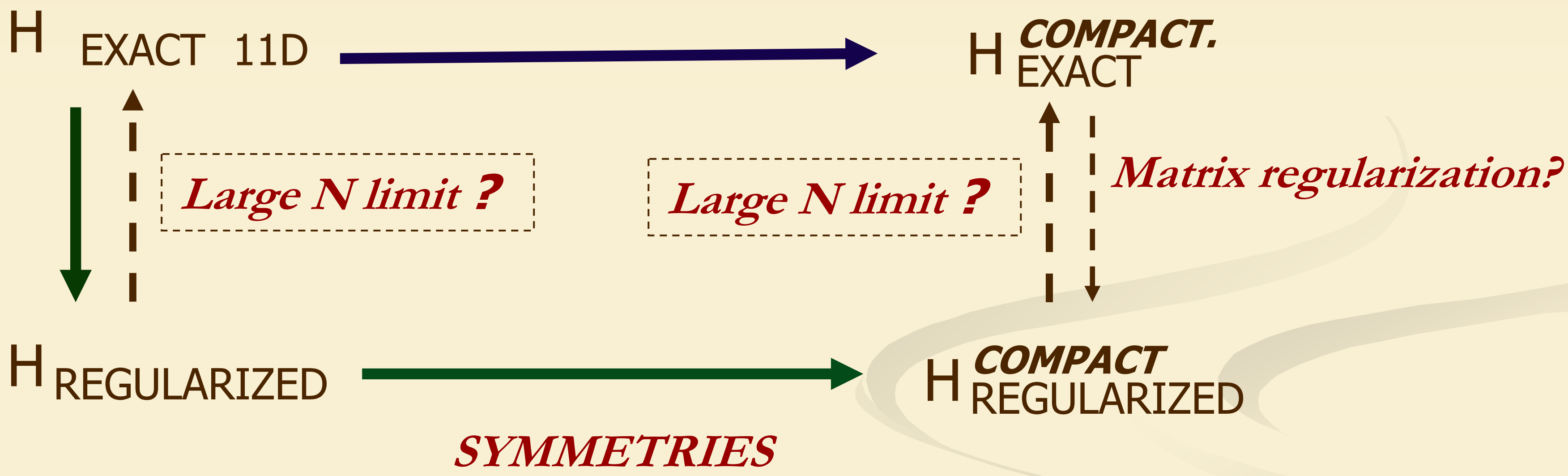
$$\phi_1 := d\left(\frac{p_M}{\sqrt{W}} dX^M\right) = 0$$

$$\phi_2 := \oint_{C_s} \frac{P_M}{\sqrt{W}} \quad dX^M = 0,$$

MATRIX MODELS

Original Point of View: *Halpern; Hoppe ,De Wit+Hoppe+Nicolai*

Integrate the spatial dependence of the original action and obtain a susy Quantum mechanics of matrices.



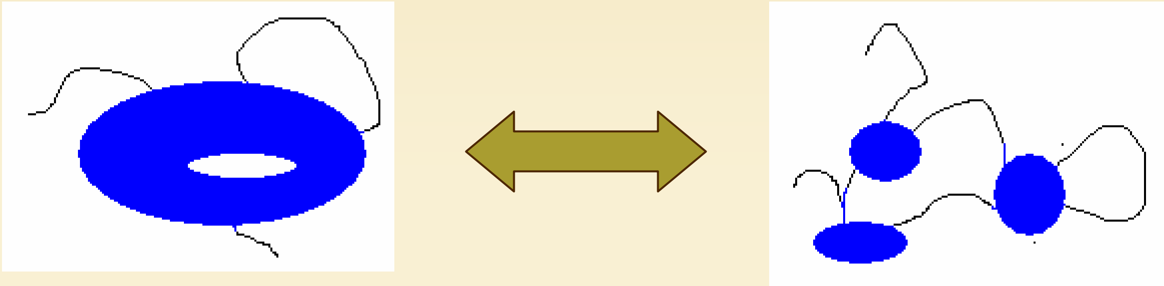
Present Point of View

BFSS/IKKT CONJECTURE: D0 or D-1 action is considered **FUNDAMENTAL**
Impose SYMMETRIES: BMN MODEL ETC..

11D Quantization Supermembrane

Historical Approach

CLASSICALY IS UNSTABLE. Topology nor number of Particles is preserved



String-like Spikes

•**QUANTUM ANALYSIS : MATRIX MODELS** *De Wit,Marquard, Nicolai*

Bosonic sector: Purely **DISCRETE** *Luscher 86; G.M,Navarro,Perez, Restuccia;06*

SUSY Sector: **CONTINUOUS** *De Wit+Luscher+Nicolai*

•**CONCLUSIONS: 2nd QUANTIZED THEORY**

Compactifications generically do not change behaviour *De Wit, Peeters, Plefka 96*

Picture of interacting D0's or Strings. Former interpretation as fundamental object seemed to be discarded

4. THE SUPERMEMBRANE WITH CENTRAL CHARGES

Martin, Restuccia, Torrealba; Martin,Ovalle,Restuccia;MPGM,Restuccia(1); Bellorin,Restuccia, ***96-07***
Boulton,MPGM.,Restuccia(3),Boulton,MPGM.,Martin, Restuccia,Boulton(2),MPGM+R,

$M_9 \times S^1 \times S^1$ **&** **TOPOLOGICAL CONDITION**

The 11D hamiltonian of M2 in the L.C.G. Indices (+ ,- , M, N =1,..,9)

H= $$\int_\Sigma \sqrt{W}\left(\frac{1}{2}\left(\frac{P_M}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^M, X^N\}^2 + \text{ Fermionic terms}\right)$$

Subject to the previous constrains + compactification conditions

$$\oint_{C_i} \mathrm{d}X^r = 2\pi S_i^r R^r, \qquad r = 1, 2,$$

$$\oint_{C_i} \mathrm{d}x^m = 0 \qquad m = 3, \ldots, 9$$

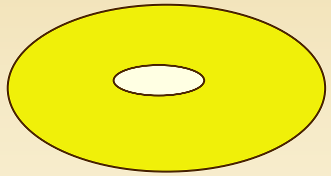
&

CENTRAL CHARGE CONDITION:

$$Z_{rs} = \int_\Sigma \mathrm{d}X^r \wedge \mathrm{d}X^s = \epsilon^{rs}(2\pi^2 R_1 R_2)n,$$

QUALITATIVE PROPERTIES

SU(N) Spectrum:



CLASSICALLY: **NOT** STRING-LIKE SPIKES (1)

QUANTUM: BOSONIC → PURELY DISCRETE SPECTRUM (2)

FERMIONIC → PURELY DISCRETE SPECTRUM (3)

BOSONIC EXACT LEVEL ***Boulton, MPGM, Restuccia 06***

Infinite number of Configuration Space fields: Canonical quantization Scheme

Pure discreteness of Spectrum of the M2 with central Charges in the 11D algebra.

4D COMPACTIFICATION

We perform a minimal inmersion:

$$\Sigma g \longrightarrow \mathbf{M}_4 \quad \times \quad \mathbf{T}^7$$

$$(\sigma_1,\sigma_2,\tau) \longrightarrow \mathbf{X}^{\mathbf{m}}(\sigma_1,\sigma_2,\tau) \quad \mathbf{X}^{\mathbf{r}}(\sigma_1,\sigma_2,\tau)\,,\ \mathbf{X}^{7}(\sigma_1,\sigma_2,\tau)$$

With the maps satisfying the compactification condition:

$$\oint_{C_s} dX^r = 2\pi S^r_s R^r \quad r,s = 1,\ldots,6$$

$$\oint_{C_s} dX^7 = 2\pi L_s R$$

$$\oint_{C_s} dX^m = 0 \quad m = 8,9$$

Together with the central charge condition:

$$I^{rs} \equiv \int_\Sigma dX^r \wedge dX^s = (2\pi R^r R^s)\omega^{rs}$$

$$M = \begin{pmatrix} 0 & 1 & & & & \\ -1 & 0 & & & & \\ & & 0 & 1 & & \\ & & -1 & 0 & & \\ & & & & 0 & 1 \\ & & & & -1 & 0 \end{pmatrix}.$$

1 Step: Compactifying to 5D

Bellorin+Restuccia

We perform the following decomposition on the multivalued fields

$$\mathrm{d}X^r = 2\pi R^r l^r \mathrm{d}\hat{X}^r + \delta^r_s \mathrm{d}A_s.$$

MPGM+Restuccia 00', Martin+Restuccia '00

The only dynamical degrees of freedom to quantized are (X's, A's): are univalued

$$H = \int \sqrt{W} \mathrm{d}\sigma^1 \wedge \mathrm{d}\sigma^2 \left[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^r}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2 + \Lambda(\{P_m, X^m\} + \mathcal{D}_r \Pi^r) \right] + \text{ Ferm. term.}$$

$$\mathcal{D}_r X^m = D_r X^m + \{A_r, X^m\} \qquad \mathcal{F}_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\}$$

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With $D_r = 2\pi R^r \frac{\varepsilon^{ab}}{\sqrt{W}} \partial_a \widehat{X}^r \partial_b$

A N=1 2+1 symplectic NCSYM coupled to scalars proceeding from NCSYM 10D reduccion=N=1 Supermembrane with Z

2 Step: Compactifying from 5D to 4D

MPGM+Pena +Restuccia

Due to the condition: $\oint_{C_s} dX = RL_s$

We decompose: $dX^7 = RL_s d\widehat{X}^s + d\widehat{\phi}$

Where the decomposition is performed on the harmonic forms of the circle

The 4D hamiltonian of the M2 :

$$H_d = \int \sqrt{w} d\sigma^1 \wedge d\sigma^2 [\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{2}(\frac{\Pi^r}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathscr{D}_r X^m)^2 + \frac{1}{4}(\mathscr{F}_{rs})^2 + \frac{1}{2}(F_{ab}\frac{\varepsilon^{ab}}{\sqrt{W}})^2 + \frac{1}{8}(\frac{\Pi^c}{\sqrt{W}}\partial_c X^m)^2 + \frac{1}{8}[\Pi^c \partial_c(\widehat{X}_r + A_r)]^2] + \Lambda(\{\frac{P_m}{\sqrt{W}}, X^m\} - \mathscr{D}_r \Pi^r - \frac{1}{2}\Pi^c \partial_c(F_{ab}\frac{\varepsilon^{ab}}{\sqrt{W}})) + \lambda \partial_c \Pi^c]$$

4. Spectral Analysis

5D CASE: $$V = \langle \mathcal{D}_r X^m \mathcal{D}_r X^m + \frac{1}{4}\mathcal{F}_{rs}\mathcal{F}_{rs}\rangle.$$

CLASSICALLY: NO-STRING-LIKE SPIKES *MPGM+ A. Restuccia*

$$V(X^m, A_r) = 0 \Leftrightarrow X^{mB} = 0, A_r^B = 0$$

QUANTUM LEVEL: BOSONIC SECTOR *Boulton+MPGM+Martin+Restuccia*

$$V(X^m, A_r) \to \infty \quad \text{whenever} \quad (X, A) \to \infty$$

QUANTUM LEVEL FERMIONIC SECTOR *Boulton+MPGM+Restuccia*

$$\forall v_k(x) \to \infty \quad \text{when} \quad |x| \to \infty \quad \Rightarrow H^N_{susy} \quad discrete$$

Eigenvalues of V(X) $\mu \equiv \Delta + V_B \mathcal{I} + V_F$

$$X^{mB} \equiv R\phi^{mB} \quad A_r^B \equiv R\Psi^{rB}$$

$$det((\frac{\lambda - V_B}{R}\mathcal{I}) - M(\Psi, \Phi)) = 0$$

$$\lambda = V_B(R\phi, R\phi) + R\widehat{\lambda}$$

4D CASE: *MPGM+Pena+Restuccia*

The piece of the potential added is:

$$V_7 = \langle (\mathscr{D}_r X^7)^2 + \{X^m, X^7\}^2 \rangle = \left\langle (L_s \mathscr{D}_r \widetilde{X}^s)^2 + (\mathscr{D}_r \phi)^2 + \{X^m, X^7\}^2 \right\rangle$$

Classically : NO string –like Spikes

V(X, A, Φ)=0 $\Longleftrightarrow$ **X= A= Φ=0** **No Flat directions**

Quantically: **Spectrum Purely Discrete**

$$H_N^B \geq c_N H_{sc,N}^B$$ & **Fermion contribution subdominant**

N=1 SUPERSYMMETRY

The hamiltonian preserves the whole supersymmetry when compactified in T^7

However the central condition charge: **THE VACUUM SOLUTION: BPS**

Dirac monopole solutions

$$\Psi = 0$$
$$X^m = 0$$
$$X_i^r = \widehat{X}_i^r$$

The invariance under the spinor implies the breaking of supersymmetry. $(1/2)^3$

The action is invariant under the whole Susy and Kappa symmetry when the variations on $\delta X^r=0$ are imposed. The central charge condition is invariant under Susy transformations. Classically there is a whole class of minima connected through susy transformations.

$$\Psi = \varepsilon_1 + \varepsilon_2$$
$$X^r = \widehat{X}^r + i\bar{\varepsilon}_2\Gamma\varepsilon_1$$
$$X^M = i\bar{\varepsilon}_2\Gamma^m\varepsilon_1 .$$

Quantically the system chooses one: **SPONTANEOUS BREAKING OF SUSY**

MODULI

There are two types of moduli in this set-up:

a. Moduli associated to the scalar fields V(X) ⟹ No Flat Directions

b. Moduli associated to the parameters Ri of the tori.

Hip: **Tori Isotropic Ri=R** ⟹ The dependence of the potential with the radius

$$V = A + BR + CR^2 + DR^4$$

With the coefficients:

$$A = \int_\Sigma \sqrt{W}\left[\tfrac{1}{4}\{X^m, X^n\}^2 + \tfrac{1}{4}\{X^m, \phi\}^2 + \tfrac{1}{2}\{A_r, X^m\}^2 + \tfrac{1}{2}\{A_r, \phi\}^2 + \{A_r, A_s\}^2\right]$$

$$C = \tfrac{1}{2}\int_\Sigma \sqrt{W}\left[(D_r X^m)^2 + (D_r\phi)^2 + (D_r A_s)^2 + \{X^m, \widehat{X}^s L_s\}^2\right]$$

$$D = \tfrac{1}{4}\int_\Sigma \sqrt{W}\{\widehat{X}^r, \widehat{X}^s\}^2$$

There exist a global minimum

$$\frac{d^2V}{dR^2} = 2C + 12DR^2 > 0$$

CONCLUSIONS

- It has been constructed the 4d hamiltonian of the supermembrane.
- The 4d supermembrane is classically stable.
- Its regularized quantum supersymmetric spectrum is discrete.
- The theory has N=1 supersymmetry
- There are no flat directions associated to the scalar fields parametrizing the transverse posititon of the supermembrane.
- For the case of the isotropic 7-tori, the potential has a global minimum with respect to the radius.

The End

MATRIX MODELS

D=11 SUPERMEMBRANES

$$H = T\int \sqrt{W} d\sigma^2 \left[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^m, X^n\}^2 + \bar{\Psi}\Gamma_-\Gamma_M\{X^M, \Psi\}\right]$$

10D SYM

$$H_{ym} = \int_{\Sigma_2} \frac{g^2}{2}(\Pi^i)^2 + \frac{1}{4g^2}F^a_{ij}F^a_{ij} - \mathcal{D}_i\Pi^{ai}A_0^{ai}$$

MATRIX REGULARIZATION

$$T^{2/3}H$$

$$\frac{g^{2/3}}{V_m^{1/3}}H$$

DIMEN. REDUCCION 0+1

$$H^{SU(N)} = tr\left[\frac{1}{2}(\Pi^i)^2 + \frac{1}{4}(f^{abc}A_i^bA_j^c)^2 - [A_i, \Pi^i]^a A_0^{ia}\right](t)$$

LARGE N LIMIT ?

MATRIX REGULARIZATION IN COMPACTIFIED SPACES?

PROBLEM OF CLOSED HARMONIC FORMS

De Wit+Peeters+Plefka

TOPOLOGICAL INFORMATION?