Metastable vacua with F and D susy breaking in general supergravity theories

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Motivation

- Compactifications to four dimensions of string theory typically generate many moduli fields that should all be stabilized at a non-susy minimum with tiny cosmological constant.
- From a $4D$ effective Lagrangian approach these moduli fields are chiral superfields of a $\mathcal{N}=1$ supergravity theory and their dynamics are governed by the $4D$ scalar potential.
- For phenomenological/cosmological applications it is important to know when this $4D$ scalar potentials can give rise to realistic situations.
- Natural question: if we require that a general sugra theory has viable vacua, can one get some conditions that restrict the class of models with potential interest?

Chiral Models: Generalities

- A theory with n chiral multiplets Φ_i is specified in terms of a real Kähler potential K and a holomorphhic superpotential W.
- Depends only on the (Kähler invariant) function G:

$$G(\Phi_i, \Phi_i^\dagger) = K(\Phi_i, \Phi_i^\dagger) + \log W(\Phi_i) + \log \bar{W}(\Phi_i^\dagger)$$

 that is invariant under Kähler transformations

$$(K, W) \rightarrow (K + \Delta + \bar{\Delta}, e^{-\Delta} W)$$

- The scalar fields ϕ^i span a Kähler manifold whose metric is given by:

$$g_{i\bar{j}} = G_{i\bar{j}} = \frac{\partial G}{\partial \phi^i \partial \phi^{\bar{j}}}$$

 and can be use to lower and raise chiral indices.

Chiral Models: Generalities

- The e.o.m. for the auxiliary fields fix them to:

$$F^i = -e^{G/2} G^i$$

- Substituting back the expressions for the auxiliary fields into the Lagrangian, the scalar potential is found to be:

$$V = e^G \left(G_{i\bar{j}} G^i G^{\bar{j}} - 3 \right)$$

Cremmer,Julia,Scherk,Ferrara,Girardello,Van Nieuwenhuizen
Bagger,Witten

- If $G^i \neq 0$ at the vacuum, supersymmetry is spontaneously broken and the gravitino mass is:

$$m_{3/2} = e^{G/2}$$

and the direction given by the G^i parametrizes the Goldstino direction.

Chiral Models: Finding Viable Vacua

- The flatness condition ($V = 0$) fixes that at the vacuum:

$$g_{i\bar{j}} G^i G^{\bar{j}} = 3$$

- The stationary condition ($\nabla_i V = 0$) implies that:

$$G_i + G^k \nabla_i G_k = 0$$

- Finally, the stability condition requires that the matrix of second derivatives is positive definite

$$\begin{pmatrix} m^2_{i\bar{j}} & m^2_{\bar{i}\bar{j}} \\ m^2_{i j} & m^2_{\bar{i} j} \end{pmatrix} > 0$$

where $m^2_{i\bar{j}} = \nabla_i \nabla_{\bar{j}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$ and are given by:

$$\begin{aligned} m^2_{i\bar{j}} &= e^G \Big(G_{i\bar{j}} + \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}p\bar{q}}\, G^p G^{\bar{q}} \Big) \\ m^2_{ij} &= e^G \Big(\nabla_i G_j + \nabla_j G_i + \frac{1}{2} G^k \{\nabla_i, \nabla_j\} G_k \Big) \end{aligned}$$

Chiral Models: Finding Viable Vacua

- To see if the matrix m^2_{IJ} is positive definite one should check the behavior of the $2n$ eigenvalues.
- This is in general a complicated task and should be studied model by model. However it is possible to find simple necessary (but not sufficient) conditions:
  - Remark: If m^2_{IJ} is positive definite then all its upper left submatrices are also positive definite.
  - Necessary condition for the existence of viable vacua: the quadratic form $m^2_{i\bar{j}} z^i z^{\bar{j}} > 0$ for any vector z^i.
  - Strategy: Find simple conditions by looking at particular directions in field space!
- In this case there is only one special direction in field space: the Goldstino direction G^i. Looking in that direction:

$$m^2_{i\bar{j}} G^i G^{\bar{j}} = e^G \left(6 - R_{i\bar{j}p\bar{q}} G^i G^{\bar{j}} G^p G^{\bar{q}}\right)$$

- We define the rescaled variables:

$$f^i = -\frac{1}{\sqrt{3}} G^i$$

- The necessary conditions for the existence of non-susy Minkowski minima can then be written as:

Flatness: $g_{i\bar{j}} f^i f^{\bar{j}} = 1$

  - Fixes the amount of susy breaking

Stability: $R_{i\bar{j}p\bar{q}}\, f^i f^{\bar{j}} f^p f^{\bar{q}} < \frac{2}{3}$

  - Requires the existence of directions with $R < 2/3$ and constraints the direction of susy breaking to be aligned with it.

Gauge Invariant Models: Generalities

- A theory with n chiral multiplets Φ^i and m vector multiplets V^a is specified by three functions:
  - A real Kähler function $G = K(\Phi_i, \Phi_i^\dagger) + \log W(\Phi_i) + \log \bar{W}(\Phi_i^\dagger)$.
  - A set of holomorphic Killing vectors X_a^i .
  - A holomorphic gauge kinetic matrix H_{ab}.
- Gauge transformations of chiral and vector multiplets are:

$$\delta\Phi^i = \Lambda^a X_a^i \qquad \delta V^a = -i(\Lambda^a - \bar{\Lambda}^a)$$

- The function G should be invariant under the gauge transformations:

$$G_a = -i\, X_a^i\, G_i = i\, X_a^{\bar{i}}\, G_{\bar{i}}$$

Gauge Invariant Models: Generalities

- As before for the chiral indices the metric is $g_{i\bar{j}} = G_{i\bar{j}}$, and also now the real part the gauge kinetic matrix $h_{ab} = \mathrm{Re} H_{ab}$ acts as a metric for the vector indices $G_a = h_{ab} G^b$.

- The auxiliary fields are fixed from the Lagrangian by the e.o.m.:

$$F_i = -\, e^{G/2}\, G_i$$

$$D_a = -\, G_a = i\, X_a^i\, G_i = -i\, X_a^{\bar{i}}\, G_{\bar{i}}$$

- The vector auxiliary fields D_a are the Killing potentials:

$$X_a^i = -i\, \nabla^i D_a$$

Gauge Invariant Models: Generalities

- The vector auxiliary fields D^a induce a new contribution to the scalar potential (in addition to the standard one coming from the chiral auxiliary fields F^i):

$$V = e^G\left(G^k G_k - 3\right) + \frac{1}{2} G^a G_a$$

Cremmer,Ferrara,Girardello,Van Proeyen
Bagger

- As before, if $G^i \neq 0$ at the vacuum supersymmetry is spontaneously broken and the gravitino mass is:

$$m_{3/2} = e^{G/2}$$

- Gauge symmetries are also spontaneously broken and the m-dimensional mass matrix for the vector fields is:

$$M^2_{ab} = 2\, g_{i\bar{j}} X^i_a X^{\bar{j}}_b = 2\, g_{i\bar{j}}\, \nabla^i G_a \nabla^{\bar{j}} G_b$$

Gauge Invariant Models: Finding Viable Vacua

- The flatness condition ($V = 0$) fixes at the vacuum:

$$-3 + G^i G_i + \frac{1}{2} e^{-G} G^a G_a = 0$$

- The stationarity condition ($\nabla_i V = 0$) implies:

$$G_i + G^k \nabla_i G_k + e^{-G} \Big[G^a \Big(\nabla_i - \frac{1}{2} G_i \Big) G_a + \frac{1}{2} h_{abi} G^a G^b \Big] = 0$$

- The stability condition requires in this case the slightly weaker condition:

$$m_{IJ} = \begin{pmatrix} m^2_{i\bar{j}} & m^2_{ij} \\ m^2_{\bar{i}\bar{j}} & m^2_{\bar{i}j} \end{pmatrix} \geq 0$$

where $m^2_{i\bar{j}} = \nabla_i \nabla_{\bar{j}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$.

- The equality sign takes care of the flat directions associated with the m scalars that are absorbed by the gauge fields and get a positive mass.

Gauge Invariant Models: Finding Viable Vacua

- The two different *n*-dimensional blocks of the mass matrix are given by:

$$
\begin{aligned}
m^2_{i\bar{j}} &= e^G\Big[G_{i\bar{j}} - R_{i\bar{j}p\bar{q}}G^pG^{\bar{q}} + \nabla_i G_k \nabla_{\bar{j}} G^k\Big] - \frac{1}{2}\Big(G_{i\bar{j}} - G_i G_{\bar{j}}\Big)G^a G_a \\
&+\Big(G_{(i}h_{ab\bar{j})} + h^{cd}h_{aci}h_{bd\bar{j}}\Big)G^aG^b - 2\,G^aG_{(i}\nabla_{\bar{j})}G_a \\
&-2\,G^a h^{bc}h_{ab(i}\nabla_{\bar{j})}G_c + h^{ab}\nabla_i G_a \nabla_{\bar{j}} G_b + G^a \nabla_i \nabla_{\bar{j}} G_a
\end{aligned}
$$

$$
\begin{aligned}
m^2_{ij} &= e^G\Big[2\,\nabla_{(i}G_{j)} + G^k\nabla_{(i}\nabla_{j)}G_k\Big] - \frac{1}{2}\Big(\nabla_{(i}G_{j)} - G_iG_j\Big)G^aG_a \\
&+\Big(G_{(i}h_{abj)} + h^{cd}h_{aci}h_{bdj} - \frac{1}{2}h_{abij}\Big)G^aG^b - 2\,G^aG_{(i}\nabla_{j)}G_a \\
&-2\,G^a h^{bc}h_{ab(i}\nabla_{j)}G_c + h^{ab}\,\nabla_i G_a \nabla_j G_b
\end{aligned}
$$

Gauge Invariant Models: Finding Viable Vacua

- Aim: study the constraints imposed in this case by the flatness condition and the stability condition.
- Strategy: same as before, find simple conditions by looking at particular directions in field space!
- In this case, there exist two types of special complex directions one could look at: G^i and X_a^i.
  - From G^i we get the condition $m^2_{i\bar{j}} G^i G^{\bar{j}} \geq 0$, which simplifies to:

$$
\begin{aligned}
R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}} \quad \leq \quad & 6 + e^{-G}\Big[-2\, G^a G_a + h^{c\,d} h_{a\,c\,i} h_{b\,d\,\bar{j}}\, G^i G^{\bar{j}} G^a G^b\Big] \\
& + e^{-2G}\Big[M^2_{ab}\, G^a G^b - \frac{3}{2}(\nabla^i G_a) h_{b\,c\,i}\, G^a G^b G^c \\
& -\frac{1}{2}\Big(G^a G_a\Big)^2 + \frac{1}{4} h_{a\,b}{}^{k} h_{c\,d\,k}\, G^a G^b G^c G^d\Big]
\end{aligned}
$$

  - From X_a^i we get the condition $m^2_{i\bar{j}} X_a^i X_a^{\bar{j}} \geq 0$, one finds a complicated expression:

no extra useful condition!

Gauge Invariant Models: Constraints

- We introduce the rescaled variables

$$f_i = \frac{1}{\sqrt{3}} \frac{F_i}{m_{3/2}} = -\frac{1}{\sqrt{3}} G_i \,, \qquad d_a = \frac{1}{\sqrt{6}} \frac{D_a}{m_{3/2}} = -\frac{1}{\sqrt{6}} e^{-G/2} G_a$$

- In terms of the rescaled variables f^i and d^a, the flatness and stability conditions take then the following form:

$$\begin{cases} f^i f_i + d^a d_a = 1 \\ R_{i \bar{j} p \bar{q}} f^i f^{\bar{j}} f^p f^{\bar{q}} \leq \frac{2}{3} + \frac{4}{3} \left(\frac{M^2_{ab}}{m_{3/2}} - h_{a\,b} \right) d^a d^b + 2 h^{c\,d} h_{a\,c\,i} h_{b\,d\,\bar{j}} f^i f^{\bar{j}} d^a d^b \\ \qquad -(2\, h_{a\,b} h_{c\,d} - \frac{1}{2} h^i_{a\,b} h_{c\,d\,i})\, d^a d^b d^c d^d \\ \qquad -3 (G^i d_a + 2 \nabla^i d_a) h_{b\,c\,i} d^a d^b d^c \end{cases}$$

- But now there is the additional complication coming from the fact that f^i and d^a are not independent of each other.

- The fields f^i and d^a are related in several ways:
  - As a consequence of gauge invariance:

$$d^a = -\frac{i\,X_i^a}{\sqrt{2}m_{3/2}}\,f^i \qquad \Longrightarrow \qquad |d_a| \leq \frac{1}{2}\,\frac{M_{aa}}{m_{3/2}}\sqrt{f^i f_i}$$

  - Projecting the stationarity condition along the directions X_i^a (valid only at the stationary points of the potential):

$$i\,\nabla_i X_{a\bar{j}}\,f^i f^{\bar{j}} - \sqrt{\frac{2}{3}}m_{3/2}\Big(3f^i f_i - 1\Big)\,d_a - \frac{M_{ab}^2}{\sqrt{6}\,m_{3/2}}\,d^b - 2\,i\,X_a^i h_{b\,c\,i}\,d^b d^c = 0$$

Kawamura

- The f^i represent the basic qualitative seed for susy breaking whereas the d^a provide additional quantitative effects.

Analysis of the Constraints

- To see the implications of the constraints we restrict to constant and diagonal gauge kinetic function of the form:

$$h_{ab} = g_a^{-2}\delta_{ab}$$

  - We can rescale the vector variables so that the metric becomes just δ_{ab} by including a factor g_a for each index a.

- Using this the flatness and stability conditions take the following form:

$$\begin{cases} f^i f_i + \sum_a d_a^2 = 1 \\ R_{i\bar{j}p\bar{q}}\, f^i f^{\bar{j}} f^p f^{\bar{q}} \leq \dfrac{2}{3} + \dfrac{4}{3}\displaystyle\sum_a \left(2\, m_a^2 - 1\right) d_a^2 - 2\sum_{a,b} d_a^2 d_b^2 \end{cases}$$

- As before the flatness condition fixes the amount of susy breaking and the stability condition fixes the direction.

Analysis of the Constraints

- The relations between f^i and d^a read:

$$d_a = i\, m_a v_a^i f_i \quad \Longrightarrow \quad |d_a| \leq m_a \sqrt{f^i f_i}$$

$$d_a = \sqrt{\frac{3}{2}}\, \frac{m_a\, q_{a\, i\bar{j}}\, f^i f^{\bar{j}}}{m_a^2 - 1/2 + 3/2\, f^i f_i}$$

where:

$$v_a^i = \frac{\sqrt{2} X_a^i}{M_a}\,, \qquad q_{a\, i\bar{j}} = \frac{i\, \nabla_i X_{a\bar{j}}}{M_a}$$

and we also define the quantity:

$$m_a = \frac{M_a}{2\, m_{3/2}}$$

measuring the hierarchies between scales

Interplay Between F and D Breaking Effects

- To see the interplay between the *F* and *D* breaking effects we introduce variables:

$$z^i = \frac{f^i}{\sqrt{1 - \sum_a d_a^2}}$$

- Using these variables the conditions can be rewritten as:

$$\begin{cases} z^i z_i = 1 \\ R_{i\bar{j}p\bar{q}}\, z^i z^{\bar{j}} z^p z^{\bar{q}} \leq \frac{2}{3} K(d_a^2, m_a^2) \end{cases}$$

where:

$$K(d_a^2, m_a^2) = 1 + 4\,\frac{\sum_a m_a^2 d_a^2 - \left(\sum_a d_a^2\right)^2}{\left(1 - \sum_b d_b^2\right)^2}$$

Interplay Between F and D Breaking Effects

- And the relations between auxiliary fields:

$$
\begin{aligned}
d_a \frac{1 + m_a^2 - 3/2 \sum_b d_b^2}{1 - \sum_b d_b^2} &= \sqrt{\frac{3}{2}}\, m_a\, q_{a\,i\,\bar{j}}\, z^i z^{\bar{j}} \\
\frac{d_a}{\sqrt{1 - \sum_b d_b^2}} &= i\, m_a v_a^i z_i \\
\frac{|d_a|}{\sqrt{1 - \sum_b d_b^2}} &\leq m_a
\end{aligned}
$$

- In the limit $d_a << 1$:

$$
d_a \simeq \sqrt{\frac{3}{8}} \frac{1}{1 + m_a^2}\, q_{a\,i\,\bar{j}}\, z^i\, z^{\bar{j}}
$$

and $z^i \simeq f^i$.

Interplay Between F and D Breaking Effects

- At first order In the limit $d_a << 1$:

$$K \simeq 1 + \frac{3}{2} \sum_a \left(\frac{m_a^2}{1 + m_a^2} \right) q_{a\, i\, \bar{j}}\, q_{a\, p\, \bar{q}}\, z^i z^{\bar{j}} z^p z^{\bar{q}}$$

- Therefore we can write the flatness and stability conditions as:

$$\begin{cases} z^i z_i = 1 \\ \hat{R}_{i\bar{j}p\bar{q}}\, z^i z^{\bar{j}} z^p z^{\bar{q}} \leq \dfrac{2}{3} \end{cases}$$

where:

$$\hat{R}_{i\bar{j}p\bar{q}} = R_{i\bar{j}p\bar{q}} - \sum_a \left[\frac{m_a^2}{1 + m_a^2} \right]^2 q_{a\, i\, (\bar{j}}\, q_{a\, p\, \bar{q})}$$

- The net effect in this case is to change the curvature felt by the chiral multiplets. Not necessarily a small effect!

Interplay Between F and D Breaking Effects

- For larger values of d_a one can find an upper bound to K:

$$K \leq 1 + \frac{3}{2} \sum_a \left[\frac{m_a^2 \, (1 + \sum_b m_b^2)}{1 + m_a^2 + (m_a^2 - \frac{1}{2}) \sum_b m_b^2} \right]^2 q_{a\,i\,\bar{j}} \, q_{a\,p\,\bar{q}} \, z^i z^{\bar{j}} z^p z^{\bar{q}}$$

- So in this general case we get as well that the effect of vector multiplets can be encoded into an effective curvature:

$$\hat{R}_{i\bar{j}p\bar{q}} = R_{i\bar{j}p\bar{q}} - \sum_a \left[\frac{m_a^2 \, (1 + \sum_b m_b^2)}{1 + m_a^2 + (m_a^2 - \frac{1}{2}) \sum_b m_b^2} \right]^2 q_{a\,i\,(\bar{j}} \, q_{a\,p\,\bar{q})}$$

Some Examples: Simple Scalar Geometries

- In certain situations the conditions for flatness and stability can be solved exactly.

One chiral field and one isometry

$$K = -n \,\mathrm{Log}(\Phi + \bar{\Phi}) \quad \Longrightarrow \quad \begin{cases} R = \dfrac{2}{n} \\ X = i\xi \end{cases}$$

- The flatness condition can be solved by parametrizing $|f|^2 = \cos^2 \delta$ and $|d|^2 = \sin^2 \delta$, and the stability condition is:

$$n > \frac{3}{1 + 4\,|d/f|^6}$$

- For example for the dilaton field $n = 1$, so the *D-term* should contribute significantly to susy breaking.

Two chiral fields and one isometry

$$K = -n_1 \,\mathrm{Log}(\Phi_1 + \bar{\Phi}_1) - n_2 \,\mathrm{Log}(\Phi_2 + \bar{\Phi}_2) \implies \begin{cases} R_i = \dfrac{2}{n_i} \\ X = i\,(\xi^1, \xi^2) \end{cases}$$

- Parametrizing $|f_1|^2 = \cos^2\theta \cos^2\delta$, $|f_2|^2 = \sin^2\theta\cos^2\delta$, $|d|^2 = \sin^2\delta$ we solve the flatness condition, and from the stability condition we derive the bound ($f = \sqrt{|f1|^2 + |f2^2|}$):

$$n_1 + n_2 \geq 3 \begin{cases} \dfrac{1 - |d/f|^2}{1 - |d/f|^2 + |d/f|^4}\,, & \text{if} \quad |d/f| \leq 1/2 \\ \dfrac{1}{1 + 4\,|d/f|^6}\,, & \text{if} \quad |d/f| > 1/2 \end{cases}$$

- Also solvable for other relevant models, as for Kähler potentials of the type $K = -\sum_i n_i \,\mathrm{Log}(\Phi_i + \bar{\Phi}_i - \sum_{a_i=1}^{N_i-1} X_{a_i} X^\dagger_{a_i})$ whose Kähler manifold is given by the coset spaces $\frac{SU(N_i,1)}{SU(N_i)\times U(1)}$.

Conclusions

- In a general $\mathcal{N} = 1$ supergravity theory with chiral and vector multiplets there are strong necessary conditions for the existence of phenomenologically viable vacua.
- These necessary conditions severely constrain the geometry of the scalar manifold as well as the direction of susy breaking and the size of the auxiliary fields (relevant for soft terms).
- When susy breaking is dominated by the *F*–terms the conditions restrict the Kähler curvature.
- When the *D*–terms participate also to susy breaking the net effect is to alleviate the constraints through a lower effective curvature (although restrict the theory as well!).
- These conditions should be useful to identify phenomenologically viable theories.