New solution for thin rotating black ring in higher dimensions

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Mainly based on:

arXiv:0708.2181 with R. Emparan, V. Niarchos, N.A. Obers & M.J. Rodriguez

See also Roberto's lectures tomorrow

Also based on:

hep-th/0310259 (PRD)

hep-th/0701022 (CQG review) with V. Niarchos & N.A. Obers

arXiv:0706.3645 (PRD) with O. Dias, R. Myers & N.A. Obers

Motivation:

In 4D General Relativity:
Only one stationary & neutral black hole available $\rightarrow$ The Kerr black hole

Does this uniqueness extends to higher dimensional General Relativity?

Recent years of research gives the answer: No!

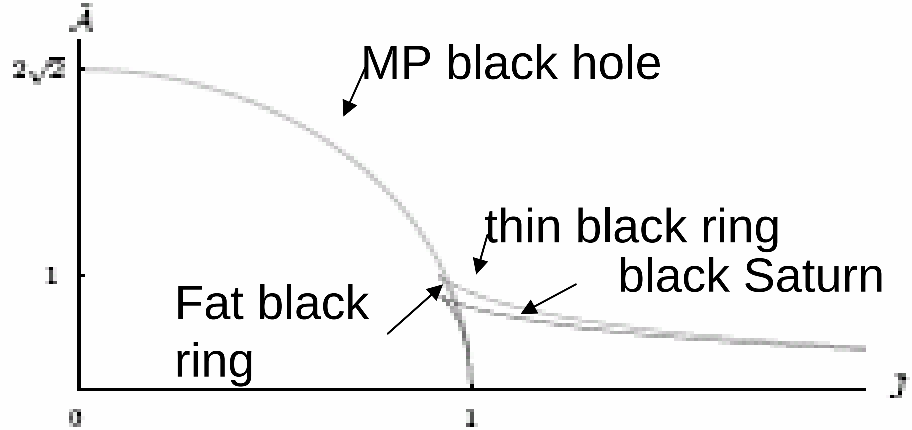
$\rightarrow$ E.g. for 5D asymptotically flat black hole solutions we have the Myers-Perry black hole, black rings, black saturn, black di-rings etc.

Why study black holes in higher dimensions?

- ▶ String theory $\Rightarrow$ D > 4
- ▶ Phenomenologically viable scenarios with large extra dimensions
- ▶ Gauge/string dualities:
  Phases of black holes D > 4 $\Leftrightarrow$ Phases of thermal gauge theory D=4

Great progress in 5D:

Consider asymptotically flat
stationary & neutral blackfolds



Several exact solutions found:

- ▶ Myers-Perry (MP) black hole
- ▶ Black ring (Emparan & Reall)
- ▶ Black saturn (Elvang & Figueras)
- ▶ Di-rings (Igushi & Mishima)

Reason: High degree of symmetry
A particular feature for asymptotically flat solutions in D=4,5
# Commuting Killing vectors = D-2 $\Rightarrow$ Einstein Eqs. reduces ← Weyl. Papapetrou. Emparan & Reall. TH
D=4: Theorem: All stationary solutions have two Killing vectors
D=5: Theorem (Hollands, Ishibashi & Wald): At least two Killing vectors

Asymptotically flat solutions for D $\geq$ 6?

Only known exact stationary & neutral solution is the Myers-Perry (MP) black hole

Solutions asymptoting to $\mathcal{M}^{D-1} \times S^1$ (D $\geq$ 5):

Exact solution: Black string
Other solutions: Localized black hole and non-uniform black string
Found by perturbative & numerical techniques

How to find new solutions? What are the possible objects with event horizons?

We can find solutions perturbatively in the small mass limit M $\rightarrow$ 0

Zeroth order: Ignore equilibrium, self-interaction
$\Rightarrow$ To this order any shape/topology will do

First order: Equilibrium from Newtonian/Special Relativity analysis

Equilibrium $\Longleftrightarrow$ Regularity

Second order: Gravitational self-interaction.
Deformation of event horizon.
Regularity of event horizon.

} Existence

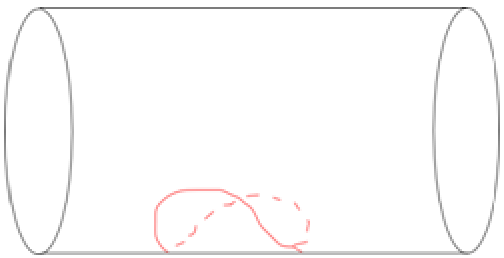
<u>Method: Matched Asymptotic Expansion</u>

TH
Kol & Gorbonos

Step 1: Write metric near the horizon for M $\rightarrow$ 0 limit.
Use conserved quantities to connect properties to asymptotic region
Step 2: Write down linearized solution away from the object
Step 3: Find from Step 2 linearized solution near the object (overlap region)
Analyze equilibrium from requiring regularity
Step 4: Find solution from overlap region all the way to horizon
If regular, this proves existence of regular solution

► One can go on to find higher order corrections

Localized black holes on Kaluza-Klein space:



Matched Asymptotic Expansion method first developed for localized black holes on Kaluza-Klein space $\mathcal{M}^{D-1} \times S^1$
i.e. black holes on cylinders $\mathbb{R}^{D-2} \times S^1$

TH
Kol & Gorbonos
Karasik et al
Chu, Goldberger & Rothstein

Using this method the metric for a small localized black hole ($M \to 0$ limit) was found

Takes into account self-interaction across the cylinder

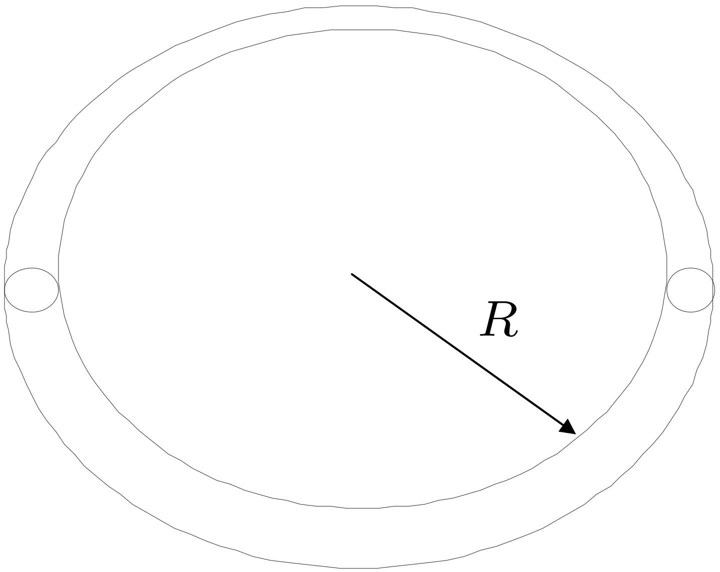
Method recently used to find metric for multi-black hole configurations on the cylinder (in $M \to 0$ limit)

Dias, TH, Myers & Obers

Reviews on the phases of black holes and strings on Kaluza-Klein space:

Class. Quant. Grav. 24: R1-R90, 2007. TH, Niarchos & Obers
Phys. Rept. 422: 119-165, 2006. Kol

Rotating black rings:



- ▶ For all dimensions $D \geq 5$
- ▶ Asymptotically flat and neutral
- ▶ Rotating in one plane, angular momentum J
- ▶ Horizon topology $S^{D-3} \times S^1$

Small mass limit $M \to 0 \Leftrightarrow$ Thin ring limit

Corresponds to: $$\frac{GM^{D-2}}{J^{D-3}} \ll 1$$ $\Leftrightarrow$ Ultraspinning regime!

Same as: $$R \gg r_0$$

R : radius of the S^1 of the ring
r_0 : radius of the S^{D-3} of the ring

Think of the black ring as a black string bent around in a circle

- ▶ Thin ring limit means the string is only a little bent
- ▶ Zeroth order approximation: A straight black string
- ▶ Rotation of the ring $\Rightarrow$ Boosted straight black string
- ▶ First order perturbation: Bending of the string

Metric for straight boosted black string:

$$ds^2 = \left(1 - \cosh^2\alpha \frac{r_0^{D-4}}{r^{D-4}}\right) dt^2 - 2\frac{r_0^{D-4}}{r^{D-4}} \cosh\alpha \sinh\alpha\, dt dz$$
$$+ \left(1 + \sinh^2\alpha \frac{r_0^{D-4}}{r^{D-4}}\right) dz^2 + \left(1 - \frac{r_0^{D-4}}{r^{D-4}}\right)^{-1} dr^2 + r^2 d\Omega_{D-3}^2$$

Corresponds to the following energy/momentum sources:

$$T_{tt} = \frac{r_0^{D-4}}{16\pi G}((D-4)\cosh^2\alpha + 1)\,\delta^{D-2}(r)$$
$$T_{tz} = \frac{r_0^{D-4}}{16\pi G}(D-4)\cosh\alpha \sinh\alpha\,\delta^{D-2}(r)$$
$$T_{zz} = \frac{r_0^{D-4}}{16\pi G}((D-4)\sinh^2\alpha - 1)\,\delta^{D-2}(r)$$

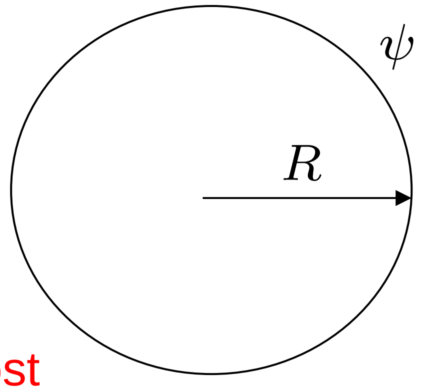
z: S^1 direction of ring

Angular coordinate for S^1:

$$\psi = \frac{z}{R}$$

General condition for equilibrium (Carter):

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$



Gives for the ring case: $\frac{T_{zz}}{R} = 0 \;\Rightarrow\; \sinh^2\alpha = \frac{1}{D-4}$

Critical boost

Determines physical quantities to leading order:

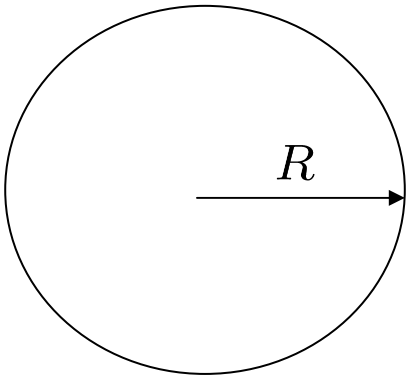
$$M = \frac{\Omega_{D-3}}{8G} R r_0^{D-4}(D-2)$$
$$J = \frac{\Omega_{D-3}}{8G} R^2 r_0^{D-4}\sqrt{D-3}$$
$$A = \Omega_{D-3} 2\pi R r_0^{D-3}\sqrt{\frac{D-3}{D-4}}$$

$$R = \frac{D-2}{\sqrt{D-3}}\frac{J}{M}$$

Step 2: Find linearized solution for $r \gg r_0$

Easily done for general $T_{tt},\ T_{t\psi}$

Gives first order black ring metric for $r \gg r_0$



Step 3: Consider the overlap region $r_0 \ll r \ll R$

We are near the ring, but sufficiently far away for linearized gravity to be valid

A. Find appropriate flat space coordinate system in overlap region

Demand r coordinate to be constant at scalar equipotential surfaces for the ring source

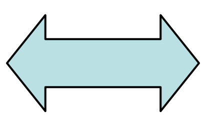
Demand $r = 0$ to correspond to a curve of constant extrinsic curvature $\frac{1}{R}$

B. Find linearized solution in overlap region for generic sources $T_{tt},\ T_{tz},\ T_{zz}$

Includes a dipole perturbation proportional to $\cos\theta$

$$d\Omega^2_{D-3} = d\theta^2 + \sin^2\theta d\Omega^2_{D-4}$$

C. Result: linearized solution in overlap region only regular for $T_{zz} = 0$

Equilibrium  Regularity

Step 4: Find solution near the horizon $r_0 \leq r \ll R$

Done by considering dipole perturbation of straight boosted black string

$$g_{\mu\nu} = g_{\mu\nu}^{\text{string}} + \frac{\cos\theta}{R} a_{\mu\nu}(r)$$

Impose: 1) Regularity at horizon $r = r_0$

2) Solution for $r_0 \ll r \ll R$ asymptotes to the one found in Step 3

We find one solution fitting all constraints

Solution can be written explicitly in terms of hypergeometric functions

Gives full regular solution for $r_0 \leq r \ll R$

Proves regularity of horizon of thin rotating black rings
$\Rightarrow$ Important part of proof of existence of rotating black rings

Combined with Step 2:
We have found the full solution for a thin rotating black ring

For D=5: Reduces to ultraspinning limit of exact black ring solution
found by Emparan and Reall

For D $\geq$ 6:

Myers-Perry black hole exists in the ultraspinning regime: $\frac{J^{D-3}}{GM^{D-2}} \gg 1$
(Unlike for D=5)

Myers-Perry black hole shown to be unstable in this regime Emparan & Myers

Entropy: $S_{\text{bh}}(M,J) \propto J^{-\frac{1}{D-4}} M^{\frac{D-2}{D-4}}$ $\leftarrow$ in ultraspinning regime

Thin rotating black ring $\Leftrightarrow$ Black ring in ultraspinning regime

Entropy: $S_{\text{br}}(M,J) \propto J^{-\frac{2}{D-5}} M^{\frac{D-2}{D-5}}$ $\leftarrow$ in ultraspinning regime

We see that: $S_{\text{br}}(M,J) \gg S_{\text{bh}}(M,J)$

For fixed M and J, with J being sufficiently large

The black ring is entropically favored over the Myers-Perry black hole in the ultraspinning regime!

Fits with Myers-Perry black hole being unstable in ultraspinning regime

However the thin black ring suffers from a Gregory-Laflamme instability

Outlook:

- ▶ Non-uniqueness also for D $\geq$ 6 (pure gravity)

- ▶ See Roberto's talk tomorrow for more on phase structure for D $\geq$ 6

- ▶ The technique of constructing new blackfold solutions:

  Used for: Localized black holes on Kaluza-Klein space (static & neutral)
  Thin rotating black rings (asymptotically flat & neutral)

  Future applications: Other topologies (e.g. T^2), other asymptotics (e.g. AdS, dS)

  A way to search for new possible horizon topologies

- ▶ Implication: Many new possible phases, one can add more angular momenta, charges, etc.

- ▶ Important relationship for blackfolds in General Relativity:

  Equilibrium $\Longleftrightarrow$ Regularity

  General condition for equilibrium (Carter): $K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$