Q–branes in IIB Supergravity

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Based on

hep-th/0612072 Bergshoeff, Hartong, Ortín, Roest

Q7–branes 0708.2287 [hep-th] Bergshoeff, Hartong, Sorokin

Q–instantons (to appear) Bergshoeff, Hartong, Ploegh, Sorokin

Q–strings (in progress) Bergshoeff, Hartong, Sorokin

Outline

- Isometries and the quantum moduli space $PSL(2,\mathbb{Z})\backslash PSL(2,\mathbb{R})/SO(2)$
- Q7–branes and Q–‘orientifolds’
- Q–strings

IIB Supergravity

- Type IIB supergravity has two scalars:
  1. the RR 0-form χ
  2. the dilaton ϕ
- Together they form the complex axidilaton field

$$\tau = \chi + ie^{-\phi}$$

- τ parametrizes the coset space $PSL(2,\mathbb{R})/SO(2)$

Duality

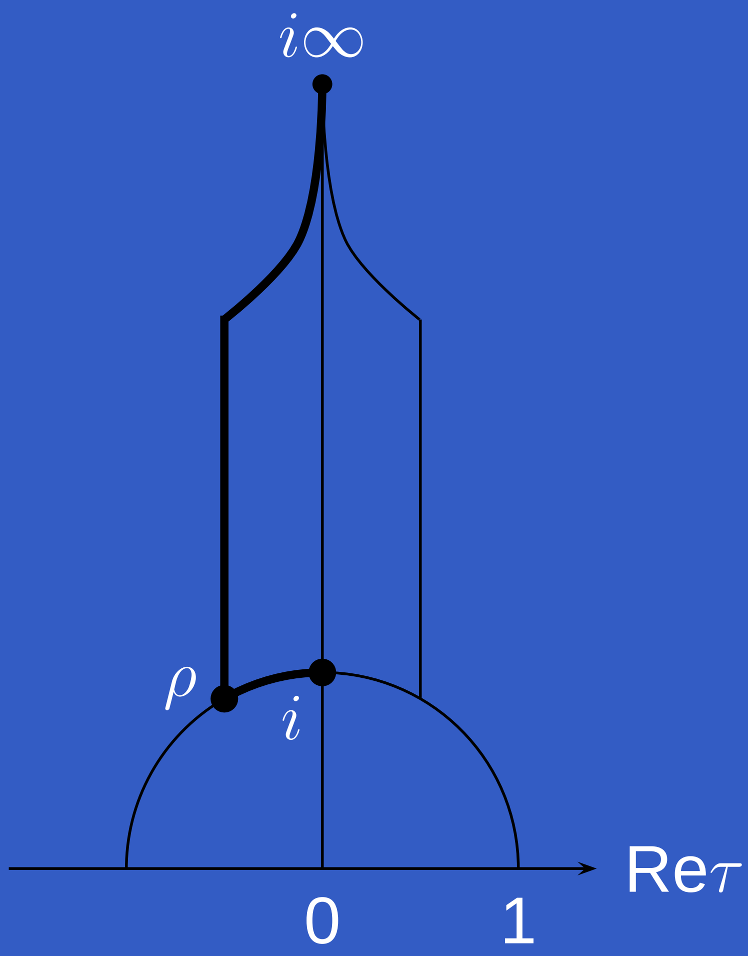
- The $PSL(2,\mathbb{R})$ duality of τ:

$$\tau \to \frac{a\tau + b}{c\tau + d} \equiv \Lambda\tau \quad \text{with} \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \det\Lambda = 1 \quad \Lambda \sim -\Lambda$$

- Instead of 4 constrained parameters a, b, c, d with $ad - bc = 1$ use 3 unconstrained parameters p, q, r

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = e^{Q} \quad \text{with} \quad Q = \begin{pmatrix} r/2 & p \\ -q & -r/2 \end{pmatrix}$$

Quantum moduli space



- Fundamental domain F of $PSL(2,\mathbb{Z})\backslash PSL(2,\mathbb{R})/SO(2)$
- Orbifold points τ_0:
  $e^Q\tau_0 = \tau_0$
  $\tau_0 = -\frac{r}{2q} + \frac{i}{|q|}\sqrt{\text{det}\, Q}$
- Special cases $\tau_0 = i\infty, i, \rho$
- Metric on moduli space:
  $\frac{d\tau d\bar{\tau}}{(\text{Im}\,\tau)^2}$
- The isometries are rotations around τ_0

Quantum moduli space II

- j-function expanded around the orbifold points:

$$j(\tau) = \begin{cases} e^{2\pi i\tau} & \text{around } \tau_0 = i\infty \\ \left(\frac{\tau-\tau_0}{\tau-\bar{\tau}_0}\right)^{\pi/\sqrt{\det Q}} \equiv e^{2\pi i\mathcal{T}} & \text{around } \tau_0 = i, \rho \end{cases}$$

- When $\tau \to e^Q\tau$ with $\det Q > 0$ we have

$$\left(\frac{\tau - \tau_0}{\tau - \bar{\tau}_0}\right)^{\pi/\sqrt{\det Q}} \to e^{2\pi i} \left(\frac{\tau - \tau_0}{\tau - \bar{\tau}_0}\right)^{\pi/\sqrt{\det Q}}$$

- Isometries:
  - $\text{Re}\,\tau \to \text{Re}\,\tau + 1$ for $\tau_0 = i\infty$
  - $\text{Re}\,\mathcal{T} \to \text{Re}\,\mathcal{T} + 1$ for $\tau_0 = i, \rho$

IIB actions adapted to orbifold points

- Define a new complex scalar field $\mathcal{T}$ by

$$\mathcal{T} \equiv \chi' + \frac{i}{4\sqrt{\det Q}} \log \frac{T + 2\sqrt{\det Q}}{T - 2\sqrt{\det Q}}$$

- IIB Lagrangian adapted to the orbifold point τ_0

$$\mathcal{L} = \left(\star 1 R - \frac{dT \wedge \star dT}{2(T^2 - 4\det Q)} - \frac{1}{2}(T^2 - 4\det Q) d\chi' \wedge \star d\chi' + \dots \right)$$

- Dualization of χ' into C_8':

$$(T^2 - 4\det Q) \star d\chi' = F_9' \qquad \text{and} \qquad F_9' = dC_8' + \dots$$

Q7–branes I

- Q7–brane: magnetic w.r.t. χ' and electric w.r.t. C_8'
- At leading order the 1/2 BPS Q7–brane action is [Bergshoeff, de Roo, Kerstan, Ortín, Riccioni 2006] :

$$S = -\int d^8\xi T\sqrt{-g_{(8)}} - \int C_8'$$

$$T = \frac{q}{2\,\mathrm{Im}\,\tau}\left(|\tau-\tau_0|^2 + |\tau-\bar{\tau}_0|^2\right)$$

- $q > 0$: positive tension Q7–brane and $q < 0$: negative tension Q–'orientifold'

Q7–brane Wess–Zumino term

- Demand that the Q7-brane WZ term satisfies:
  1. gauge invariance
  2. invariance under $\chi' \to \chi' + c$
  3. when det$Q \to 0$ it reduces to the (p,q) 7-brane WZ term of [Bergshoeff, de Roo, Kerstan, Ortín, Riccioni 2007]
- Q7 WZ term is given by [Bergshoeff, Hartong, Sorokin 2007]

$$-C_8' + \ldots + \left[b(T)\, e^{-4i\sqrt{\det Q}\,\chi'} \frac{i}{(\mathrm{Im}\tau_0)^2} \left(-\mathcal{G}_2 + \tau_0 \mathcal{F}_2\right)^4 + \text{c.c.} \right]$$

- 2 BI vectors: $\mathcal{G}_2 = C_2 + dV_C$ and $\mathcal{F}_2 = B_2 + dV_B$.

SUSY 7–brane solutions

- The metric: $ds^2 = -dt^2 + d\vec{x}_7^2 + (\text{Im}\,\tau)|f|^2 dz d\bar{z}$ [Greene, Shapere, Vafa, Yau 1989], [Gibbons, Green, Perry 1995]

$$j(\tau) = \frac{P(z)}{Q(z)}$$

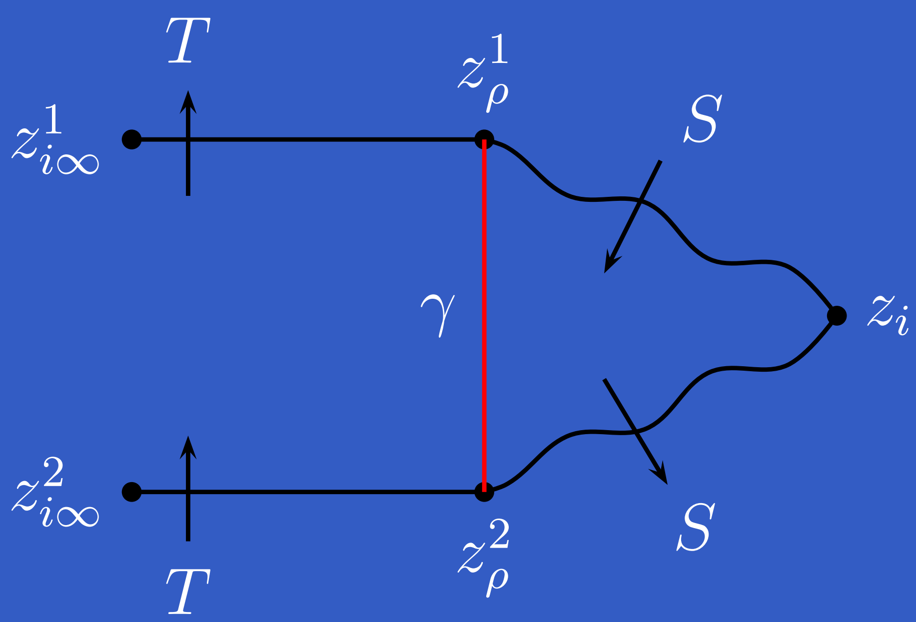
- General structure of f [Bergshoeff, Hartong, Ortín, Roest 2006]:

$$f = \eta^2 \underbrace{(z - z_{i\infty})^{-1/12}}_{\text{D7}} \ldots \underbrace{(z - z_\rho)^{-1/6}}_{\text{Q7 at } \tau_0 = \rho} \ldots \underbrace{(z - z_i)^{-1/4}}_{\text{Q7 at } \tau_0 = i} \ldots$$

$$\underbrace{(z - z'_\rho)^{1/3}}_{\text{Q–O7 at } \tau_0 = \rho} \ldots \underbrace{(z - z'_i)^{1/4}}_{\text{Q–O7 at } \tau_0 = i}$$

$$j = \infty \quad \text{at} \quad z_{i\infty}\,, \qquad j = 0 \quad \text{at} \quad z_\rho\,, \qquad j = 1 \quad \text{at} \quad z_i$$

Q–strings I



- $T\tau = \tau + 1$ and $S\tau = -1/\tau$
- 2 D7's at $z_{i\infty}^{1,2}$ and 2 Q7's at $z_\rho^{1,2}$
- No Q–'orientifolds'
- τ_∞ arbitrary
- red line: Q–string stretched along γ between 2 Q7's

- Mass of a stretched Q–string inspired by [Sen 1996]

$$m_{Q1} = \int_\gamma b(T) ds$$

$b(T)$ is the tension of the Q–string

Q–strings II

- If we take for the tension: $b(T) = (T^2 - 4\det Q)^{1/4}$ then
$$m_{Q1} = q^{1/2} \int_\gamma |(\tau - \tau_0)^{1/2} (\tau - \bar{\tau}_0)^{1/2} f dz|$$
and
$$(\tau - \tau_0)^{1/2} (\tau - \bar{\tau}_0)^{1/2} f$$
is regular everywhere along γ. In particular near z^1_ρ
$$(\tau - \tau_0)^{1/2} (\tau - \bar{\tau}_0)^{1/2} \sim (z - z^1_\rho)^{1/6} \qquad \text{and} \qquad f \sim (z - z^1_\rho)^{-1/6}$$
    1. The stretched Q–string can be BPS
    2. Q–strings can only end on positive tension Q7's
    3. The Q–string tension is close to zero near $\tau = \tau_0$

Q–strings III

- Consider a Q–string whose tension is $(T^2 - 4\det Q)^{1/4}$.
- Construct a WZ term with the properties:
  1. It is gauge invariant
  2. It is invariant under $\chi' \to \chi' + 1$
  3. It reduces to the (p,q) string WZ term when $\det Q \to 0$
- Such a WZ term is given by

$$\int \left(a(T) e^{-i\sqrt{\det Q}\,\chi'} \left(-\mathcal{G}_2 + \tau_0 \mathcal{F}_2\right) + \text{c.c.} \right)$$

- $\mathcal{G}_2 = C_2 + dV_C$ and $\mathcal{F}_2 = B_2 + dV_B$ with C_2 the RR and B_2 the NSNS 2-forms and V_C and V_B two BI vectors.

Q–strings IV

- The Q–string source term is proposed to be

$$S = \int d^2\xi (T^2 - 4\text{det}\, Q)^{1/4} \sqrt{-g_{(2)}} +$$
$$\int \left(a(T) e^{-i\sqrt{\text{det}\, Q}\, \chi'} \left(-\mathcal{G}_2 + \tau_0 \mathcal{F}_2 \right) + \text{c.c.} \right)$$

- This source term is 1/2 BPS provided

$$q(T^2 - 4\text{det}\, Q)^{1/2} = 2a^2 \left(T + \left(T^2 - 4\text{det}\, Q \right)^{1/2} \right)$$

- The projector is given by

$\gamma_{\underline{0}}\gamma_{\underline{1}}(\epsilon_1 - i\epsilon_2) = e^{i\varphi}(\epsilon_1 + i\epsilon_2)$ where φ is a function of T and χ'

Questions

- Can we define a Q7–brane as that brane on which a Q–string is ending?
  - The Q–string is a source for two BI vectors.
  - The gauge invariant Q7–brane WZ term also requires 2 BI vectors.
- Does there exist a kappa-symmetric Q–string action?
- What do Q–branes say about IIB near the orbifold points $\tau_0 = i, \rho$? Is IIB perturbative near $\tau_0 = i, \rho$ w.r.t. the Q–string tension?