Higher-spin dynamics and Chern-Simons theories

Olaf Hohm

Based on:

- Johan Engquist, O.H.,
  Higher-spin Chern-Simons theories in odd dimensions, arXiv:0705.3714 [hep-th]
- Johan Engquist, O.H.,
  Geometry and dynamics of higher-spin frame fields, arXiv:0708.1391 [hep-th]

Valencia - October 2, 2007

The higher-spin problem:

Consistent interactions of Higher-Spin (HS) fields?

$\longrightarrow$ String/M-theory [Gross (1988), Witten (1988), Sundborg (2001), Sagnotti (2004)]

Free HS theories [Fronsdal (1978), de Wit, Freedman (1980)]

Massless spin-s field: totally symmetric $h_{\mu_1 \dots \mu_s}$ with gauge symmetry

$$\delta_\epsilon h_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_s)}$$

Gravity coupling $\partial \to \nabla$ violates HS symmetry $\longrightarrow$ Inconsistency!

Vasiliev's proposal:

non-vanishing cosmological constant $\Lambda \;\to\;$ no flat-space limit

Gauging of HS algebra $\mathfrak{ho}(D-1,2) \supset \mathfrak{so}(D-1,2)$

$$A_\mu \;=\; \underbrace{e_\mu{}^a P_a + \frac{1}{2}\omega_\mu{}^{ab} M_{ab}}_{\mathfrak{so}(D-1,2)} \;+\; \text{infinite HS sum}$$

Dynamics? $\longrightarrow$ 'unfolded formulation' [Vasiliev (1994)]

Chern-Simons theory

Chern-Simons theory in $D = 3$:

$$S = \int \mathrm{Tr}\left(A \wedge dA + \tfrac{2}{3} A \wedge A \wedge A\right)$$

$\longrightarrow$ Gravity [Witten (1988)], SUGRA [Achucarro & Townsend (1986)], HS [Blencowe (1989)]

e.o.m. $F_{\mu\nu} = 0 \quad \longrightarrow$ topological, no local d.o.f.

Chern-Simons theory in $D \geq 5$:

$$S = \int g_{abc} A^a \wedge dA^b \wedge dA^c + \ldots$$

Gauge algebra $\mathfrak{g} = \mathfrak{so}(2n, 2) \rightarrow$ Lovelock gravities in $D = 2n + 1$.
e.o.m.:

$$g_{abc} F^a \wedge F^b = 0$$

Background $\bar{F}^a = 0 \quad \longrightarrow$ no propagator, topological phase

KK background in $D = 5$: $\quad AdS_4 \times S^1 \quad \longrightarrow$ non-topological phase

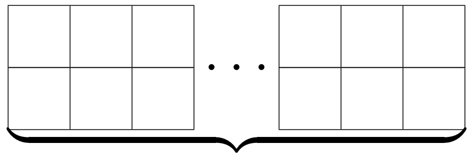
[Chamseddine (1990)]

Higher-spin Chern-Simons theory

Gauge algebra: Infinite-dimensional extension of $\mathfrak{so}(4,2)$ [Vasiliev (2003)]:

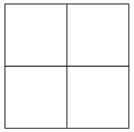
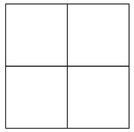
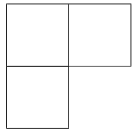
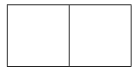
HS generator in definite Young tableau (spin-s):

$$Q_{A_1 \dots A_{s-1}, B_1 \dots B_{s-1}} :$$



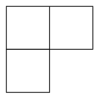
$s-1$

Linearisation around gravitational background for spin-3:

$Q_{AB,CD}$:  $\longrightarrow$ $Q_{ab,cd}$:  $Q_{ab,c}$:  Q_{ab} : 

gauge field in $\mathfrak{so}(4,1)$ basis

$$\mathcal{W}_\mu = \bar{e}_\mu{}^a P_a + \frac{1}{2}\bar{\omega}_\mu{}^{ab} M_{ab} + \kappa\Big(\frac{1}{2} e_\mu{}^{ab} Q_{ab} + \frac{1}{3}\omega_\mu{}^{ab,c} Q_{ab,c} + \frac{1}{12}\omega_\mu{}^{ab,cd} Q_{ab,cd}\Big)$$

Spin-3 field and symmetries (after gauge-fixing  to zero):

$$h_{\mu\nu\rho} = \bar{e}_{(\mu}{}^a \bar{e}_\nu{}^b e_{\rho)ab} \,, \qquad \delta_\epsilon h_{\mu\nu\rho} = \bar{D}_{(\mu}\epsilon_{\nu\rho)}$$

HS geometry: connections, curvature and torsion

Linearized field strength:

$$\mathcal{F}_{\mu\nu} = \bar{T}_{\mu\nu}{}^{a} P_a + \frac{1}{2}\bar{\mathcal{R}}_{\mu\nu}{}^{ab} M_{ab} + \kappa\Big(\frac{1}{2}T_{\mu\nu}{}^{ab} Q_{ab} + \frac{1}{3}T_{\mu\nu}{}^{ab,c} Q_{ab,c} + \frac{1}{12}R_{\mu\nu}{}^{ab,cd} Q_{ab,cd}\Big) + \ldots$$

with

$$\begin{aligned} T^{ab} &= \bar{D}e^{ab} + \omega^{ab,c}\wedge\bar{e}_c \\ T^{ab,c} &= \bar{D}\omega^{ab,c} + 3e^{\langle ab}\wedge\bar{e}^{c\rangle} + \omega^{ab,cd}\wedge\bar{e}_d\ , \\ R^{ab,cd} &= \bar{D}\omega^{ab,cd} + 4\omega^{\langle ab,c}\wedge\bar{e}^{d\rangle} \end{aligned}$$

Torsion constraints:

$$\begin{aligned} T^{ab} &= 0 \quad \Rightarrow \quad \omega^{ab,c} = \omega^{ab,c}(\partial e) \\ T^{ab,c} &= 0 \quad \Rightarrow \quad \omega^{ab,cd} = \omega^{ab,cd}(\partial\omega^{ab,c}) = \omega^{ab,cd}(\partial^2 e) \end{aligned}$$

[$\rightarrow$ hierarchy of de Wit–Freedman connections in metric-like formulation]

Spin-3 Riemann tensor: $R_{\mu\nu}{}^{ab,cd}$ 3rd derivative-order!

HS Dynamics:

How are e.o.m. of s-th derivative order consistent with Fronsdal eq.?

Damour-Deser identity:

$$(\text{Ric})_{\mu\nu|\rho\sigma} = 2\partial_{[\mu}\mathcal{F}_{\nu]\rho\sigma} = 0 ,$$

$\mathcal{F}_{\mu\nu\rho} = \Box h_{\mu\nu\rho} + \ldots$ HS-invariant 2$^{\text{nd}}$ order Fronsdal operator.

$\Rightarrow$ locally solvable (Poincaré lemma)

$$\mathcal{F}_{\mu\nu\rho} = \partial_\mu\partial_\nu\partial_\rho\alpha .$$

$\Rightarrow$ invariant under unconstrained HS transformations, provided

[Francia & Sagnotti (2002)]

$$\delta_\epsilon\alpha = \eta^{\mu\nu}\epsilon_{\mu\nu} .$$

$\Rightarrow$ gauge-fixing $\alpha = 0$ $\Rightarrow$ Fronsdal equations recovered!

[Bekaert & Boulanger (2003), Sagnotti, Sezgin & Sundell (2005)]

Generalization to arbitrary spin on AdS,
in particular: correct free field limit around AdS_4.

Summary & Outlook:

- We constructed consistent HS–gravity coupling $\longleftrightarrow$ consistent deformation of free HS symmetry $\delta_\epsilon h_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\epsilon_{\mu_2\ldots\mu_s)}$
- $AdS_4 \times S^1$ background: Correct free field equations despite higher-derivative nature of Ricci tensor
  $\longleftrightarrow$ contact with compensator formulation (trace-full generator)
- Dynamics for non-trivial HS torsion?
- Meaning of higher derivatives at non-linear level?
- Closed form of HS Lie algebra $\longleftrightarrow$ interactions?
- Organization into $\mathfrak{ho}(D-1,2)$ multiplets?
  No dynamics around AdS_5,
  but on AdS_4 off-diagonal KK components (scalar?)
- M theory as Chern-Simons theory of $Osp(1|32)$? [Horava (1999)]
  Recently: 'Non-critial M-theory' in $D = 3$
  $\longleftrightarrow$ HS Chern-Simons theory on $AdS_2 \times S^1$ [Horava & Keeler (2007)]