From ten to four and back again: how to generalize the geometry

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$$\epsilon_2 = \zeta_+ \otimes \eta_\mp^{(2)} + \zeta_- \otimes \eta_\pm^{(2)} \ ,$$

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- Study 4d effective theory

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d_H\left(e^{4A-\Phi}\mathrm{Re}\,\Psi_1\right) &= \mp e^{4A}\alpha(\star_6 F)\,, \\
d_H\left(e^{3A-\Phi}\Psi_2\right) &= 0\,, \\
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for Minkowski.

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$$\begin{aligned} d_H\left(e^{4A-\Phi}\mathrm{Re}\,\Psi_1\right) &= (3/R)\, e^{3A-\Phi}\mathrm{Re}\left(e^{i\theta}\Psi_2\right) \mp e^{4A}\alpha(\star_6 F)\,, \\ d_H\left(e^{3A-\Phi}\Psi_2\right) &= (2/R) i\, e^{2A-\Phi} e^{-i\theta}\mathrm{Im}\,\Psi_1\,, \\ d_H(e^{2A-\Phi}\mathrm{Im}\,\Psi_1) &= 0\,, \end{aligned}$$

for AdS: $\nabla_\mu \zeta_- = \pm \frac{e^{-i\theta}}{2R}\gamma_\mu \zeta_+$.

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$$\Rightarrow \Psi_+ = -ic^{-1}e^{iJ}, \quad \Psi_- = \Omega$$

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  without sources:
  Lüst, Tsimpis (IIA) *Gauntlett, Martelli, Sparks, Waldram* (IIB)
  with sources: *PK, Tsimpis*

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  - Conformal supergravity formalism
  - Gauge-fixing local Weyl transformation leads to standard $N = 1$ sugra in Einstein frame

Superpotential

- Generalize the GVW superpotential:

$$\mathcal{W} = \int_M e^{3A-\Phi}\Omega \wedge G\,,$$

Superpotential

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$$\mathcal{W} = \int_M \langle e^{3A-\Phi}\Psi_2, F\rangle \,,$$

with the Mukai pairing $\langle \phi_1, \phi_2 \rangle = \phi_1 \wedge \alpha(\phi_2)|_{\text{top}}$.

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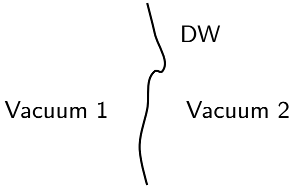

DW: generalized calibrated *PK, Martucci*

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The second holomorphic variable we find from the susy D-brane instanton action:

$$S_{\text{E}}(\mathcal{T}) = \int_\Sigma \underbrace{(e^{-\Phi}\text{Re}\,\Psi_1 - iC)}_{\mathcal{T}}|_\Sigma \wedge e^{\mathcal{F}}$$

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Kähler potential

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- It is more complicated when considering the deformations of the two pure spinors together for the $N = 1$ theory. Nevertheless

$$\mathcal{K} = -2\log\left(i \int_M e^{2A-\Phi} \langle \Psi_1, \bar{\Psi}_1 \rangle\right) - \log\left(i \int_M e^{-4A} \langle \mathcal{Z}, \bar{\mathcal{Z}} \rangle\right) .$$

Effective theory

Superpotential

$$\mathcal{W} = \int_M \langle \mathcal{Z}, F_0 + i d_H \mathcal{T} \rangle \,.$$

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Check on $\mathcal{W}$ and $\mathcal{K}$

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  This latter equation can be interpreted as a D-flatness condition

Non-perturbative corrections

- Add:
$$\mathcal{W}_{\mathsf{np}} = \mathcal{A}\, \exp\Big(- \frac{2\pi}{n} \int_\Sigma \mathcal{T}|_\Sigma \wedge e^{\mathcal{F}} \Big)\,,$$
from D-brane instantons $(n = 1)$ or gaugino condensation.

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    $\Rightarrow$ classically disallowed

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- $\bar{\partial}(\partial \log \mathcal{W}_{\text{np}}) = \frac{2\pi i}{n}\, \delta^{(2)}(\Sigma)$
- Solution: $\mathcal{W}_{\text{np}} = f^{1/n} \tilde{\mathcal{W}}_{\text{np}}$,
  with f holomorphic section line bundle associated to divisor Σ
  Ganor, Baumann, Dymarsky, Klebanov, Berg, Haack, Körs

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Conclusions

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- We can add $\mathcal{W}_{\mathsf{np}}$ $\Rightarrow$ non-perturbative correction to susy equations
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The end... The end... The end...

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