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Quantum Cosmology from Superstrings

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In collaboration with:

- Tristan Catelin-Jullien, Hervé Partouche, Nicolaos Toumbas,
[In heterotic Superstrings at finite Temperature and broken SUSY]

- Hervé Partouche, [In effective supergravities from Strings]

- Nicolaos Toumbas and Jan Troost,
[In Suprerstrings with broken SUSY]

1. Introduction

Is superstring theory able to describe the basic features of our Universe?

It is necessary to develop a string theoretic framework for studying cosmology.

Despite considerable effort over the last few years, still very little is known about the dynamics of string theory in time-dependent, cosmological settings.

Our aim is to provide, in superstring theory,

a new class of non-trivial, Quantum and Thermal cosmological solutions.

At the classical string level, it seems difficult to obtain exact cosmological solutions.

After extensive studies in the framework of superstring compactifications, without or with fluxes, the obtained results appear to be unsuitable for cosmology.

Most of the time the classical ground states correspond to stationary Anti-de Sitter or flat backgrounds but not to cosmological ones.

The same situation appears to be true in the effective supergravity theories.

Naively, these results lead to the conclusion that it is unlikely to find cosmological ground states in superstring theory.

From our viewpoint this conclusion cannot be correct for two reasons:

- **Cosmological solutions already exist which are even exact in all orders in α' and which are based on a 2d conformal field theory: Gauged WZW model at negative level $-|k|$:**

$$\left(\frac{SL(2,R)}{U(1)}\right)_{-|k|} \times M$$

Kounnas - Lust '92,
Nappi - Witten '92,
Kounnas - Toumbas - Troost '07.

- **In the classical stringy-supergravity regime the Quantum and Thermal corrections have been neglected.**

The exact in α' stringy-cosmological models was studied recently by Kounnas,Toumbas and Troost. It was shown how to define a stringy wave-function of the universe, extending the no-boundary proposal of Hartle and Hawking in superstrings.

Explicit calculable examples were given for small values of the level $|k|$.

However, in the absence of a semiclassical limit with $|k|$ arbitrarily large prevents us from obtaining a clean geometrical picture.

The second direction consists of studying

"Quantum and Thermal stringy-cosmologies" which are dynamically generated at the Quantum level and at finite Temperature.

Although this study looks to be hopeless and out of any systematic control, it turns out that in certain “physically” interesting cases both the quantum and thermal corrections are under control thanks to the special “no-scale structure” of the effective supergravity theory in its broken SUSY phase.

An effective field theory study has been recently performed by Kounnas and Partouche.

To see how the Quantum and Thermal Stringy cosmologies emerge, consider initially the case of a flat supersymmetric background.

At finite temperature the thermal fluctuations and the quantum corrections produce a non zero energy density which is calculable at the string level.

The back reaction on the
i) space-time metric, $g_{\mu\nu}$

ii) moduli fields, Φ_I

gives rise to a specific calculable cosmological evolution.

When T **is bellow the Hagedorn temperature** T_H**, the evolution of the universe is known to be radiation dominated.**

The interesting cases are those where SUSY is broken spontaneously by (non)-geometrical fluxes; Freely Acting Orbifolds. In that case both the thermal and SUSY breaking will correspond to a generalization in superstrings of the Scherk-Schwarz compactification.

• **The thermal corrections are implemented by introducing a coupling of the space-time fermion number Q_F to the string momentum and windings associated to the Euclidean time cycle S^1_T.**

• **The breaking of SUSY is generated by a similar coupling of an internal R-symmetry charge Q_R to the momentum and windings associated to an internal coordinate, say X_5 cycle S^1_M.**

Two very special mass scales appear associated with the breaking of supersymmetry.

The temperature scale $T = 1/(2\pi R_0)$

The SUSY breaking scale $M = 1/(2\pi R_5)$

The initially degenerate mass levels of bosons and fermions are shifted by an amount that is proportional to T and/or M according to the charges Q_F and Q_R.

This mass shifting is the signal of supersymmetry breaking and gives rise to a non-trivial free energy density and effective potential at the quantum level.

2. Thermal and Quantum corrections in the Heterotic superstring backgrounds

The one-loop effective action ,(in string frame)

$$S = \int d^4x\sqrt{|g|}\left(e^{-2\phi}(\frac{1}{2}R + 2\partial_\mu\phi\partial^\mu\phi + \ldots) - \mathcal{V}_{\rm String}\right)$$

ϕ **is the** $4d$ **dilaton field and the ellipses stand for the kinetic terms of other moduli fields.**

$\mathcal{V}_{\rm String}$ **is the one-loop effective potential, which is obtained from the one-loop Euclidean string partition function**

$$\frac{Z}{V_4} = -\mathcal{V}_{\rm String}$$

V_4 **denotes the** $4d$ **Euclidean volume.**

At finite temperature, the one-loop Euclidean partition function determines the free energy density and pressure

$$\frac{Z}{V_4} = -\mathcal{F}_{\text{String}} = P_{\text{String}}.$$

In order to determine the back-reaction of the thermal and quantum corrections, it is convenient to work in the Einstein frame where there is no mixing between the metric and the dilaton kinetic terms. We define as usual the complex field S

$$S = e^{-2\phi} + ia$$

After the Einstein rescaling of the metric, the one loop effective action becomes:

$$S = \int d^4x \sqrt{|g|} \left[\frac{1}{2} R - g^{\mu\nu} \; K_{I\bar{J}} \; \partial_\mu \Phi_I \partial_\nu \bar{\Phi}_{\bar{J}} \; \right]$$

$$\left[-\frac{1}{s^2} \; \mathcal{V}_{\text{String}}(\Phi_I, \bar{\Phi}_{\bar{I}}) \right]$$

$K_{i\bar{j}}$ **is the metric of the scalar manifold** $\{\Phi_I\}$**, which is parameterized by various compactification moduli including the field** S**.**

This manifold includes also the main moduli fields $T_I,\, U_I,\;\; I = 1, 2, 3$**, which are the volume and complex structure moduli of the internal coordinates.**

In the Einstein frame the effective potential, $\mathcal{V}_{\rm String}$**, is rescaled by a factor** $1/s^2$

$$s = \Re S = e^{-2\phi}$$

Taking this rescaling into account, we have

$$\mathcal{V}_{\rm Ein} = \frac{1}{s^2}\;\mathcal{V}_{\rm String}$$

The above relation turns out to be crucial.
We will always work in gravitational mass units, with $M_G = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ **GeV.**

What will be crucial in our analysis is some fundamental scaling properties of $\mathcal{V}_{\rm Ein}$ in the limit of $R_0, R_5 \gg 1$.

In this limit only the temperature scale R_0 and three of the main moduli fields $\{S, T_1, U_1\}$ appears in $\mathcal{V}_{\rm Ein}$. The contribution of all the others is exponentially suppressed.

$$\mathcal{V}_{\rm Ein} \simeq \frac{f\left(\frac{sR_0^2}{sR_0^2}, \dots\right)}{(stu)^2} + \mathcal{O}(e^{-c_0 R_0 - c_5 R_5})$$

Freezing all other moduli, the classical Kälher potential is of a no-scale type as was expected from the effective field theory approach:

$$K = -\log\ (S+\bar{S}) - \log\ (T_1+\bar{T}_1) - \log\ (U_1+\bar{U}_1)$$

$$\equiv -3\log\ (Z+\bar{Z}) + \dots$$

The classical superpotential is constant so that the gravitino mass is:

$$m_{3/2}^2 = \frac{c}{stu} = \frac{c}{z^3}$$

Freezing further the ImZ and defining the field Φ:

$$e^{2\alpha\Phi} = m_{3/2}^2 = \frac{8c}{(Z+\bar{Z})^3}$$

$$g_{\mu\nu}\ 3\frac{\partial_\mu Z \partial_\nu \bar{Z}}{(Z+\bar{Z})^2} = g_{\mu\nu}\ \frac{\alpha^2}{3}\ \partial_\mu \Phi \partial_\nu \Phi\ .$$

The choice $\alpha^2 = 3/2$ normalize canonically the kinetic terms of the no-scale modulus Φ.

$$\mathcal{V}_{\rm Ein} \simeq m_{3/2}^4\ f\left(\ \frac{sR_0^2}{sR_0^2}, \ldots\right) \simeq m_{3/2}^4\ f\left(\frac{m_{3/2}^2}{T^2}, \frac{m_Y^2}{T^2}\right)$$

The possible dependance on the SUSY mass scale m_Y^2 will be clear latter on when we will consider explicit string examples.

3. Heterotic SUSY backgrounds at finite T

We first consider the case of an exact SUSY background of heterotic string with maximal space-time supersymmetry ($N = 4$).

The Euclidean time as well as all nine spatial directions are compactified on T^{10}.

At zero temperature and in the absence of SUSY breaking the Euclidean string partition function is zero due to space-time SUSY

At finite temperature and non-vanishing SUSY breaking the result is a well defined finite quantity and is given in terms of the thermal partition function :

$$Z = \oint \frac{d\tau d\bar{\tau}}{4\mathrm{Im}\tau} \; \frac{1}{2} \sum_{a,b} \; \frac{(-)^{a+b+ab} \; \theta \left[{a \atop b}\right]^4}{\eta(\tau)^{12} \; \bar{\eta}(\bar{\tau})^{24}} \; \Gamma_{(5,21)}(R_I)$$

$$\times \; \Gamma_{(3,3)}(R_x = R_y = R_z)$$

$$\times \; \sum_{h_0, g_0} \; \Gamma\left[{h_0 \atop g_0}\right](R_0) \; (-)^{g_0 a + h_0 b + g_o h_o}$$

$$\times \; \sum_{h_5, g_5} \; \Gamma\left[{h_5 \atop g_5}\right](R_5) \; (-)^{g_5 a + h_5 b + g_5 h_5}$$

The non-vanishing of the partition function is due to the non-trivial coupling of the $\Gamma(R_0)$ and $\Gamma(R_5)$ lattices to the spin structures (a, b).

$a = 0$ for space-time bosons and $a = 1$ for space-time fermions.

$$\Gamma\left[{h \atop g}\right](R) = \sum_{m,n} R(\mathrm{Im}\tau)^{-\frac{1}{2}} \; e^{-\pi R^2 \; \frac{|2m+g+(2n+h)\tau|^2}{\mathrm{Im}\tau}}$$

In the limit of large $3{+}1$ dimensions and small SUSY beaking

$$R_x = R_y = R_z \equiv R \gg 1 \quad \text{and} \quad R \gg R_0, R_5 \gg 1.$$

The 3d spatial volume factorizes

$$\Gamma_{(3,3)} \cong R^3 \ \mathrm{Im}\tau^{-\frac{3}{2}} = \frac{V_3}{(2\pi)^3} \ \mathrm{Im}\tau^{-\frac{3}{2}}$$

- **The sector $(h_0, g_0) = h_5, g_5) = (0,0)$ gives zero contribution. This is due to the fact that we started with a SUSY background.**
- **In the odd winding sectors, $h_0 = 1$ and/or $h_5 = 1$, the partition function diverges when R_0 and/or R_5**

$$\frac{1}{R_H} < R_A < R_H$$

Hagedorn $R_H = (\sqrt{2}{+}1)/2$ and its dual $1/R_H$

The divergence is due to the winding states that become tachyonic.

- **In the regime $R_0, R_5 \gg R_H$, there is no tachyon**
  i) the winding sectors and
  ii) the oscillator states
  are both exponentially suppressed.
- **When $R_0, R_5 \gg 1$, the contributions of the internal $\Gamma_{(5,21)}(R_I)$ lattice states are also exponentially suppressed, provided that the moduli R_I are of order unity.**

For large R_0, R_5, only the sectors

$(h_0, g_0) = (0, g_T)$ and $(h_5, g_5) = (0, g_T)$ with $g_0 + g_5 = 1$

contributes significantly.

Using the identity:

$$\Gamma(R) = \Gamma\begin{bmatrix}0\\0\end{bmatrix} + \Gamma\begin{bmatrix}0\\1\end{bmatrix} + \Gamma\begin{bmatrix}1\\0\end{bmatrix} + \Gamma\begin{bmatrix}1\\1\end{bmatrix}$$

and neglecting the $h = 1$ sectors, we may replace

$$\Gamma\begin{bmatrix}0\\1\end{bmatrix} \to \Gamma(R) - \Gamma\begin{bmatrix}0\\0\end{bmatrix} = \Gamma(R) - \frac{1}{2}\Gamma(2R)$$

in the integral expression for Z.

We decompose the contribution in modular orbits:

$$(m_i, n_i) = (0,0) \text{ and } (m_i, n_i) \neq (0,0).$$

For $(m_i, n_i) \neq (0,0)$, the integration over the fundamental domain is equivalent with the integration over the strip but with $n_i = 0$.

The $(0,0)$ orbit gives zero contribution due to the initial SUSY.

So we are left with the integration over the whole strip:

$$Z = \frac{V_4}{(2\pi)^4} \int_{||} \frac{d\tau d\bar{\tau}}{4\mathrm{Im}\tau^{\frac{7}{2}}} \sum_{g_0, g_5} \frac{\theta\left[{1 \atop 1+g_0+g_5}\right]^4 \ \Gamma_{(5,21)}}{\eta(\tau)^{12} \ \bar{\eta}(\bar{\tau})^{24}}$$

$$\times \sum_{m_0, m_5} e^{-\pi R_0^2 \ \frac{(2m_0+g_0)^2}{\mathrm{Im}\tau}} \times \ R_5 \ e^{-\pi R_5^2 \ \frac{(2m_5+g_5)^2}{\mathrm{Im}\tau}}$$

The integral over τ_1 imposes the left-right level matching condition.
The left-moving part contains the ratio

$$\frac{\theta\left[{1 \atop 0}\right]^4}{\eta^{12}} = 2^4 + \mathcal{O}(e^{-\pi\tau_2}),$$

which implies that the lowest contribution is at the massless level.

Finally, after the integration over τ_1 ($\tau_2 \equiv t$),

$$Z = \frac{V_4 \left(2^4 \ D_0\right)}{(2\pi)^4} \int_0^\infty \frac{dt}{2t^{\frac{7}{2}}} \sum_{m_i} \sum_{g_0+g_5=1} R_5$$

$$e^{-\pi R_0^2 \frac{(2m_0+g_0)^2}{t} - \pi R_5^2 \frac{(2m_5+g_5)^2}{t}} ,$$

$2^4 \ D_0$ **is the multiplicity of the massless level.**

Changing the integration variable by setting

$$t = \pi \left(R_0^2 (2m_0 + g_0)^2 + R_5^2 (2m_5 + g_5)^2 \right) \ x$$

and integrating over x, we can write Z in terms of Eisenstein functions of order $k = 5/2$:

$$E_k(U) = \sum_{(m,n) \neq (0,0)} \left(\frac{\text{Im } U}{|m + nU|^2} \right)^k$$

Define the function

$$f(u) \equiv \sum_{m_1,m_2} \frac{u^4}{|(2m_1+1)iu+2m_2|^5}$$

$$f(u) = u^{\frac{3}{2}} \left(\frac{1}{2^{\frac{5}{2}}} E_{5/2}\left(\frac{iu}{2}\right) - \frac{1}{2^5} E_{5/2}(iu) \right)$$

with

$$u = \frac{R_0}{R_5} = \frac{M}{T}, \qquad M = m_{3/2}$$

Then the pressure in the Einstein frame

$$P \equiv \frac{Z}{V_4} = \; C_T \; T^4 \; f(u) + C_V \; M^4 \; \frac{f(1/u)}{u}$$

with

$$C_T = C_V = n^* \; \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{5}{2}}}.$$

$n^* = 8D_0$ **is the number of massless boson/fermion pairs**

In this particular example the coefficients C_T and C_V are equal due to the underlying gravitino mass/temperature duality.

For fixed u
the first term stands for the
thermal contribution to the pressure, P_{thermal}
the second term stands for
minus the effective potential, $-V_{\text{eff}}$

$$P = P_{\text{thermal}} \; - \; V_{\text{eff}}$$

- **The coefficient C_T is fixed and positive as it is determined by the number of all massless boson/fermion pairs in the initially SUSY theory.**
- **The coefficient C_V will depend on the way the SUSY-breaking operator Q_R couples to the left- and right- movers.**

- **In general $Q_F \neq Q_R$ and the temperature/ gravitino duality will be broken. C_V can be either positive or negative.**
- **For models with $N = 2$ initial SUSY and with Q_R acting only on the left-movers and $Q_R \neq Q_F$, the contribution of the twisted sector is negative**

$$(-)^{(Q_R - Q_F)} = (-)^H = -1.$$

The change of sign indicates that in the twisted sector the states that become massive are the bosons rather than the fermions.

4. Small SUSY mass scales from Wilson line deformations

A generic SUSY heterotic background contain in its spectrum massive supermultiplets whose mass is obtained by switching-on non-trivial, continuous Wilson-lines.

This is a stringy realization of the Higgs mechanism, breaking the initial gauge group G to a smaller smaller sup-groups.

For arbitrary and small Wilson line starting from a given SUSY background with
$R_I,\ I=4,6,\ldots,9$ of the order one.
In the zero winding sector, a Wilson line modifies the Kaluza-Klein momenta,

$$\frac{m_I^2}{R_I^2} \quad \longrightarrow \quad \frac{(m_I + \ y_I^a \ Q_a)^2}{R_I^2}$$

We distinguish two different situations according to the direction I:

i) $I = 5$ **where** R_5 **is large,**

ii) $I = 4, 6, \ldots, 9$ **where** R_I **is of order one.**

Disregarding the first case for simplicity, we can set in the second case, $I = 4, 6, \ldots, 9$**, the momentum and winding numbers to zero, since their contribution is exponentially suppressed.**

The relevant modification in the partition function is the insertion of the term:

$$e^{-\pi t\left(\frac{y_I^a Q_a^I}{R_I}\right)^2} \;\simeq\; 1 - \pi t\left(\frac{y_I^a Q_a^I}{R_I}\right)^2$$

Incorporating the effects of the Wilson lines up to quadratic order.

$$P = C_T\; T^4\; f_{\frac{5}{2}}(u) \;-\; D_T\; T^2\; M_Y^2\; f_{\frac{3}{2}}(u)$$

$$+\; C_V\; M^4\; \frac{f_{\frac{5}{2}}(1/u)}{u} \;-\; D_V\; M^2\; M_Y^2\; \frac{f_{\frac{3}{2}}(1/u)}{u}$$

$M_Y \sim y_I^a Q_a^I / R_I$ **introduces a new scale in the theory, which is qualitatively different than** T **and** M**.** M_Y **is an effective SUSY mass scale rather than a SUSY-breaking scale like** T **and** M**.**

5. Scaling properties of the thermal effective potential

The final expression for P contains three mass scales: M, T and M_Y. The first identity follows from the definition of P

$$\left(T\frac{\partial}{\partial T} + M\frac{\partial}{\partial M} + M_Y\frac{\partial}{\partial M_Y}\right) \; P = 4P$$

Writing P as

$$P \equiv T^4 \; I_4(u) \; + \; T^2 \; M_Y^2 \; I_2(u) = P_4 + P_2, \quad u = \frac{M}{T}$$

and using standard thermodynamic identities, we can obtain the energy density $\rho = \rho_4 + \rho_2$:

$$\rho \equiv T\frac{\partial}{\partial T}P - P = \rho_4 + \rho_2$$

$$\rho_4 = \left(3P_4 - u\frac{\partial}{\partial u}P_4\right) \qquad \rho_2 = \left(P_2 - u\frac{\partial}{\partial u}P_2\right)$$

6. Gravitational equations and the critical solution

In the sequel, investigate the back-reaction to the initially flat metric and moduli fields allowing the SUSY-breaking scales T and M to vary with time while fixing the SUSY mass scale M_Y and u.

The gravitino scale M is given in terms of the no-scale modulus Φ

$$M = e^{\alpha\Phi}, \qquad \alpha = \sqrt{\frac{3}{2}},$$

The fact that $-P$ play the role of the one loop effective potential we obtain the Φ-field equation

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\partial}{\partial\Phi}P = \alpha u \frac{\partial}{\partial u}P = -\alpha\left(\rho_4 - 3P_4 + \rho_2 - P_2\right)$$

Assuming that the space time metric to be homogeneous and isotropic

$$ds^2 = -dt^2 + a(t)^2 \; d\Omega_k^2, \qquad H = \left(\frac{\dot{a}}{a}\right)$$

we can derive the gravitational equations in terms of the no-scale modulus Φ and in terms of ρ and P

Ω_k denotes the three dimensional space with curvature k

$$3H^2 = \frac{1}{2}\dot{\Phi}^2 + \rho - \frac{3k}{a^2}$$

$$2\dot{H} + 3H^2 = -\frac{k}{a^2} - P - \frac{1}{2}\dot{\Phi}^2$$

We find useful to use the linear sum of the above two equations instead of the second.

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P)$$

The Critical Solution

The fundamental ingredients in our analysis are the scaling properties of the thermal effective potential $-P = -P_4 - P_2$ at finite temperature T.

Their structure suggests to search for a solution where the mass scales of the system, $M(\Phi)$, T and $(1/a)$ remain proportional during their evolution in time

$$e^{\alpha\Phi} \equiv M(\Phi) = \frac{1}{\gamma a} \quad \longrightarrow \quad H = -\alpha\dot{\Phi}, \quad M(\Phi) = u\,T$$

Our aim is thus to determine the constants γ and $u = 1/\xi$.

On the critical trajectory

$r_4 = \rho_4/T^4, \;\; p_4 = P_4/T^4, \;\; r_2 = \rho_2/T^2, \;\; p_2 = P_2/T^2$

remains constants

The compatibility of the Φ-equation and the gravity equation along the critical trajectory implies an identification of the coefficients of the monomials in M.

The quartic terms determines $\xi = 1/u$
The quadratic terms determine k

$$r_4 = \frac{6\alpha^2 - 1}{2\alpha^2 - 1}\, p_4$$

$$-2k\gamma^2 = \frac{2\alpha^2 - 1}{2}\,(r_2 - p_2)\,\xi^2 M_Y^2$$

The Friedman-Hubble equation in the background of the critical trajectory $\dot{\Phi}^2 = (H^2/\alpha^2)$

$$\left(\frac{6\alpha^2 - 1}{6\alpha^2}\right) \; 3H^2 = -\frac{3k}{a^2} + \rho = -\frac{3k}{a^2} + \rho_4 + \rho_2$$

The dilatation factor in front of $3H^2$ can be absorbed in the definition of $\hat{k}$ and r_4

$$3H^2 = -\frac{3\hat{k}}{a^2} + \frac{C_R}{a^4}$$

where

$$C_R = \frac{\xi^4}{\gamma^4} \; \frac{6\alpha^2}{6\alpha^2 - 1} \; r_4 \; = \frac{\xi^4}{\gamma^4} \; \frac{6\alpha^2}{2\alpha^2 - 1} \; p_4$$

and

$$3\hat{k} = -\frac{\xi^2 M_Y^2}{\gamma^2} \; \frac{6\alpha^2}{6\alpha^2 - 1} \left(\; \frac{3(2\alpha^2 - 1)}{4} (r_2 - p_2) + r_2 \; \right) .$$

Concluding Remark

The plausible existence of cosmological super-string solutions *(Inflationary or not)*

which are generated dynamically

at the quantum sting level

from a flat classical space-time and

spontaneously broken supersymmetry

(no-scale radiative-induced cosmology).