



MAX-PLANCK-GESELLSCHAFT

Dimers & Orientifolds

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in collaboration
with

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based on

arXiv: 0707.0298



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Motivation

What we want:

The low-energy gauge theory of a bunch of D3-branes probing an arbitrary orientifolded toric CY singularity.



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- Orientifold projection crucial for D-instanton induced couplings.
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How to obtain it:

Extending dimer technics!



Recap of Dimers

Recall:

Low-energy gauge theory of a bunch of D3-branes probing flat space is N=4 SYM with superpotential:

$$W = \Phi_1 \Phi_2 \Phi_3 - \Phi_1 \Phi_3 \Phi_2$$

(Note: Traces are implicit in this talk!)



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Note:

Each field occurs exactly twice. Once in a positive and once in a negative term!



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This property is preserved under orbifolding and higgsing (i.e. blowdowns)!

=> Hence, is a characteristic of gauge theories arising from probes of toric CY singularities!



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=> Hence, is a characteristic of gauge theories arising from probes of toric CY singularities!

=> Allows to give isomorphisms between the superpotential and bipartite graphs!



Recap of Dimers

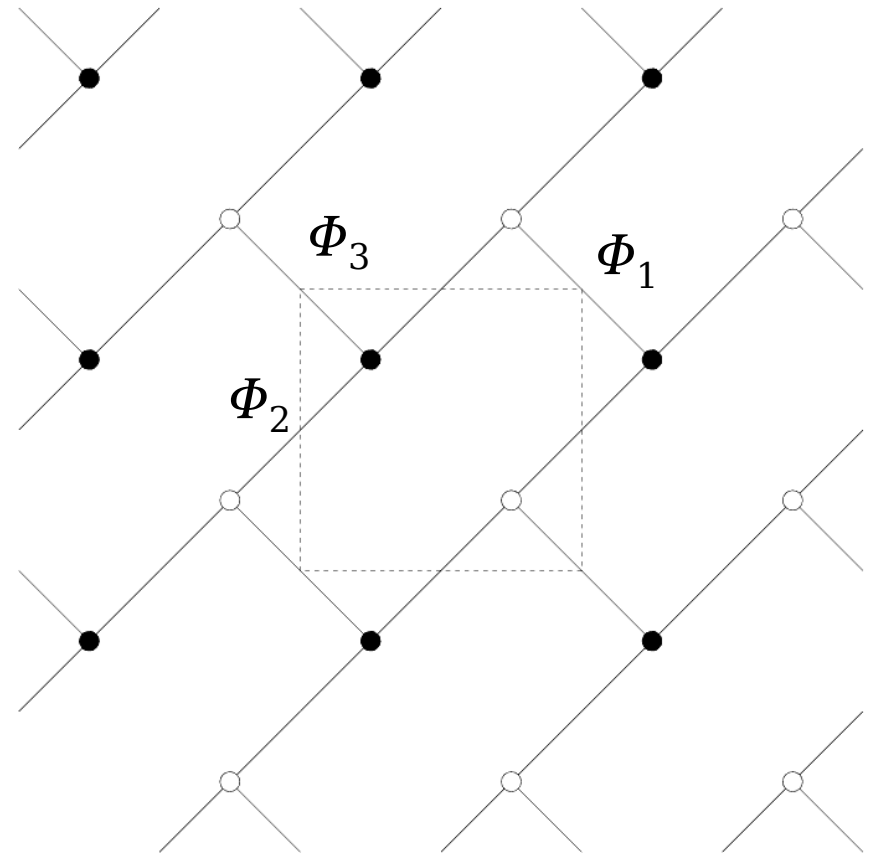
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The Dimer:

Bipartite graph on a torus:

Vertices : Superpotential terms
Edges : Fields
Faces : Gauge groups

[Franco, Hanany, Kennaway, Vegh
and Wecht '05]



Example: \mathbb{C}^3



Recap of Dimers

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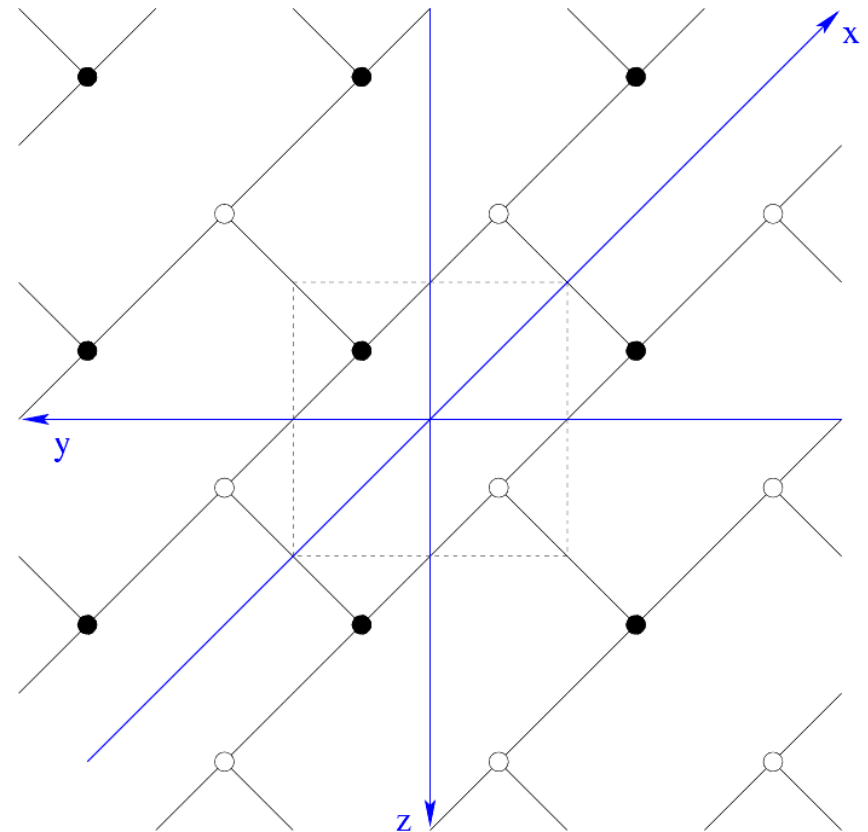
Bipartite graph on a torus:

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Mesonic operators:

Closed loops running through faces



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Recall:

The orientifold action is given by: $\omega \sigma (-1)^{F_L}$

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Appart from orbifolds and generalized conifolds, only a few other orientifolded D3-probe models were constructed via higgsing.

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TODO:

Let's find a better way via dimers!



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Translate known orientifolds of simple theories to the dimer, and hopefully there is some systematics!



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Simplest example:

Flat space! A possible geometric action:

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How can one infer the “qualitatively” corresponding action on the dimer ?

=> Use the mesonic operators corresponding to the coordinates !

(Note: Sign transformation will be more involved due to mixture with Chan-Paton action!)

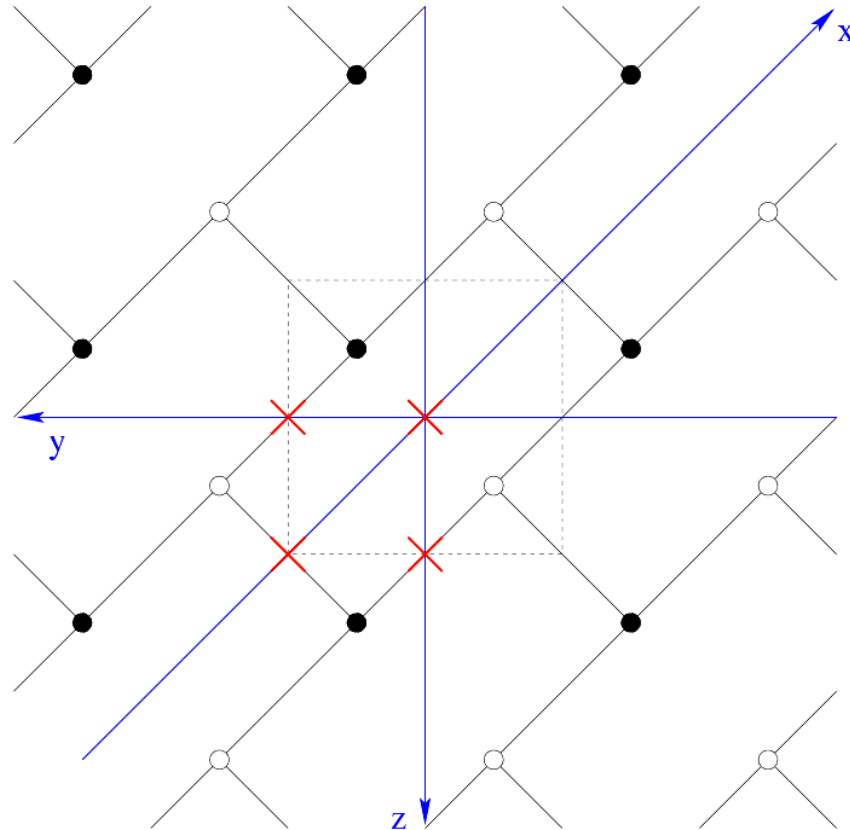


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Orientifolding Dimers

Action on the dimer:

Point reflection !





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Orientifolding Dimers

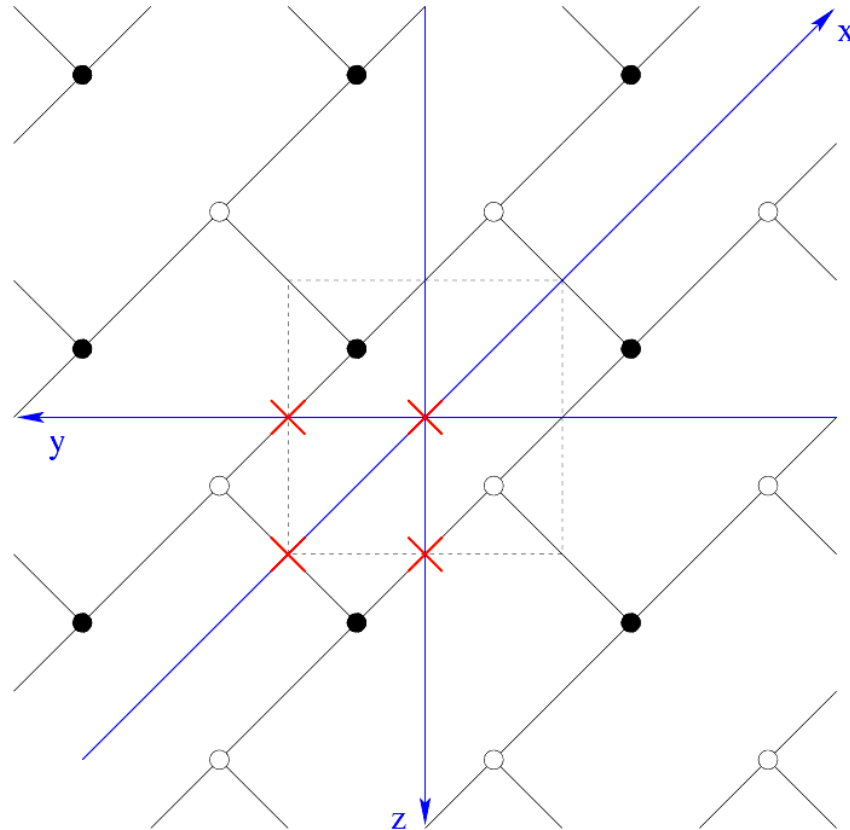
Action on the dimer:

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Under the reflection:

Faces mapped to themselves:
=> Enhanced gauge groups!

Edges mapped to themselves:
=> 2-tensors!





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Orientifolding Dimers

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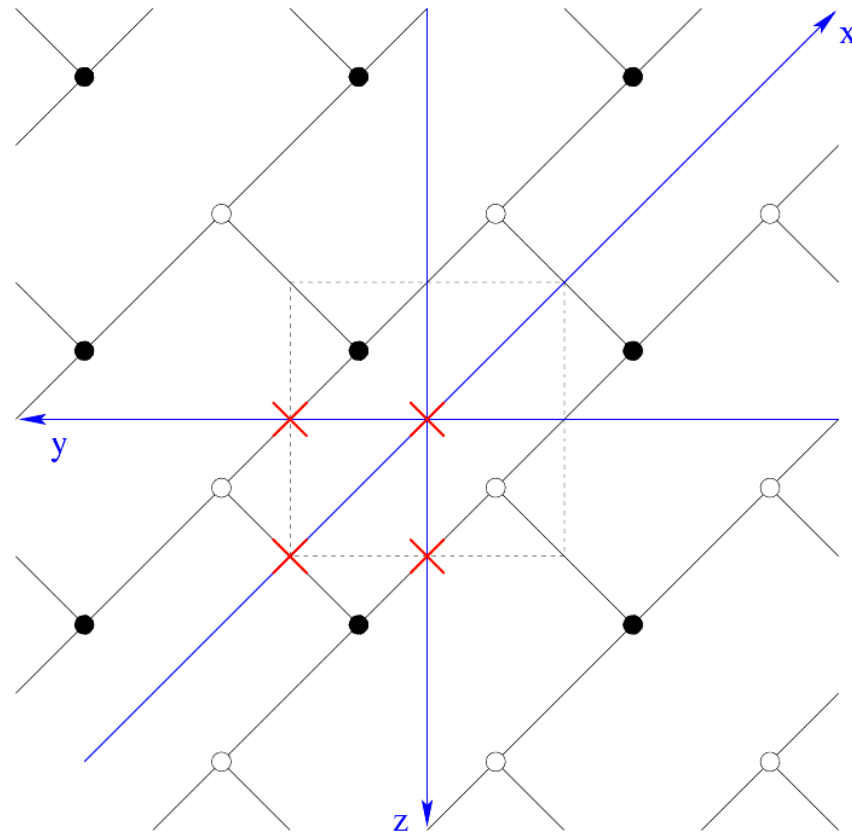
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Note:

We can have at most four fixed elements!





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Orientifolding Dimers

Chan-Paton action:

How to determine **SO** vs. **Sp** and **symmetric** vs. **anti-symmetric** ?



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(Note: They do not necessarily correspond to physical charges)



Orientifolding Dimers

Chan-Paton action:

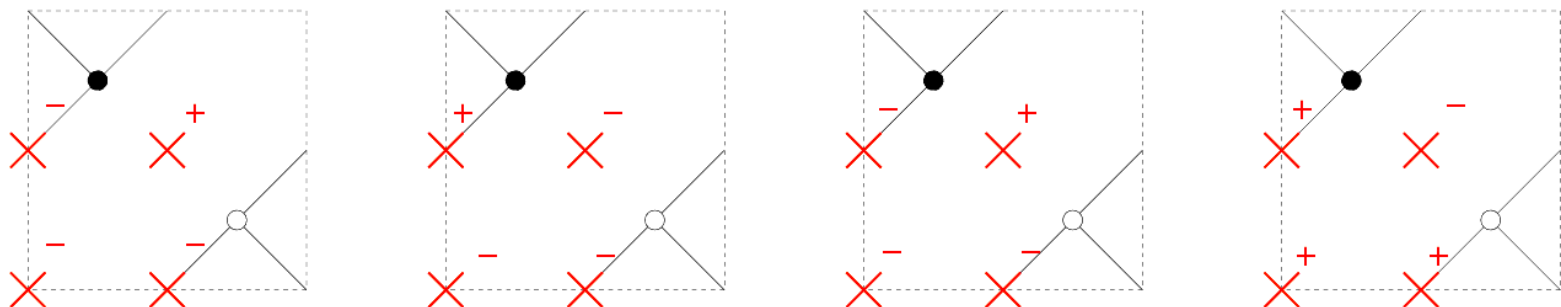
How to determine **SO** vs. **Sp** and **symmetric** vs. **anti-symmetric** ?

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Example:



Reproduce known SUSY orientifolds of flat space! => We are on the right track!



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Orientifolding Dimers

What about even sign setups ?

=> Non-susy models !



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Orientifolding Dimers

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For $2k$ superpotential terms, $k \bmod 2$ determines if an even or odd sign setup preserves susy!



Orientifolding Dimers

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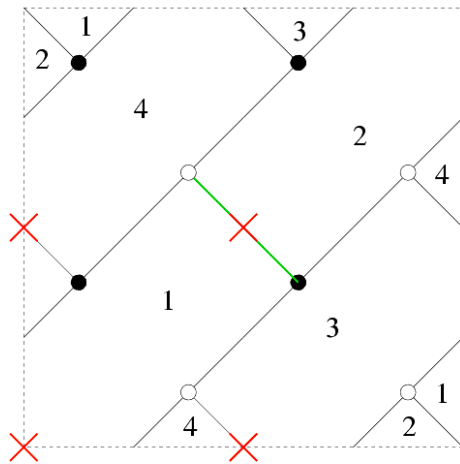
Why does this hold in general ?

The constraint is compatible with higgsing!

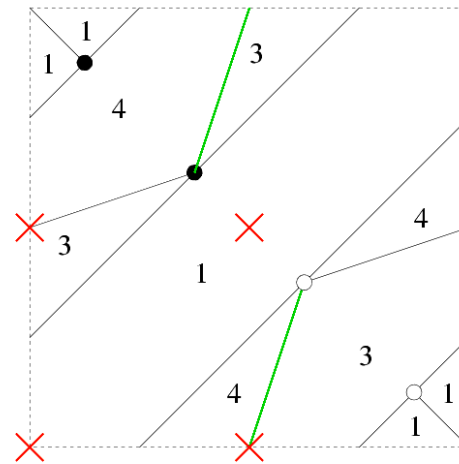


Orientifolding Dimers

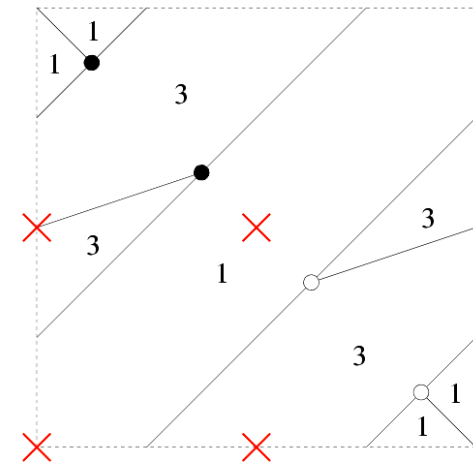
Some more examples:



$\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$



SPP



$\mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$

This almost trivially reproduces the orientifold models obtained originally via higgsing in [Park, Rabadan, and Uranga '00]



Orientifolding Dimers

What about other involutions?

It seems that the point reflections correspond to involutions sending all coordinates to themselves up to sign.

What about involutions exchanging coordinates?

For example in flat space:

$$\sigma(x, y, z) \rightarrow (x, z, y)$$



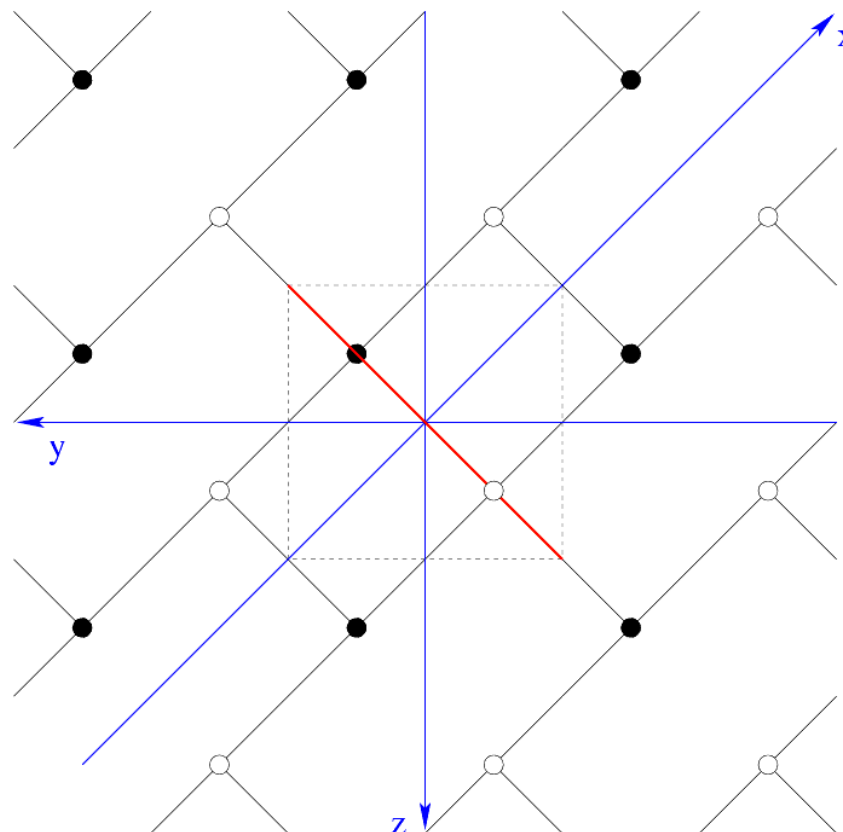
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Orientifolding Dimers

Action on the dimer:

Line reflection !

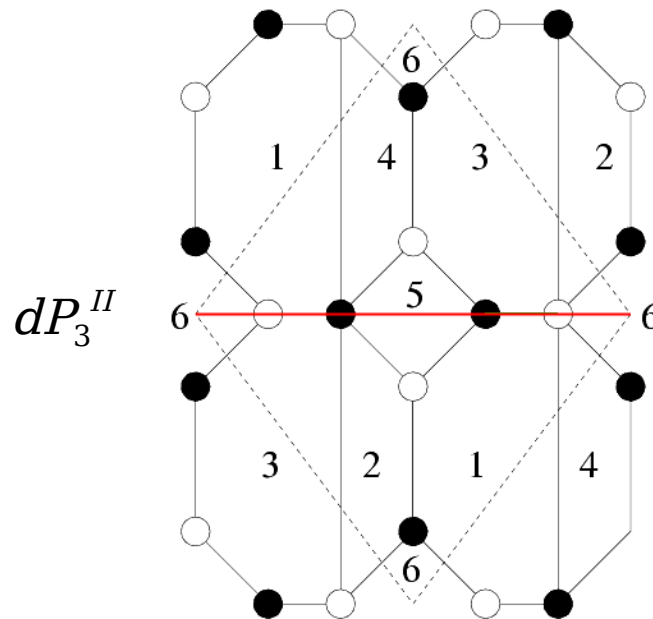
The rest is as before,
but there is no global sign
constraint !





Orientifolding Dimers

Another example:



Something to be aware of:
In some models anomaly cancellation might require the introduction of extra D7-branes!



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Orientifolding Dimers

Pre-conclusion:

- We have our recipe to obtain orientifolds of arbitrary toric CY 3-folds!
- One can recover in this way essentially all known orientifolded models + many more!



Orientifolding Dimers

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- One can recover in this way essentially all known orientifolded models + many more!

What I haven't told:

- We were also able to conjecture rules for the meson sign transformations!
- There is also a mirror IIA intersecting brane description a la
[Hori, Vafa '00] [Feng, He, Kennaway and Vafa '05]



Application I: DSB

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DSB models:

Recently, there were some attempts to construct known field theory models which feature DSB via branes at orientifolded singularities.

[Antebi and Volansky '07] [Wijnholt '07]



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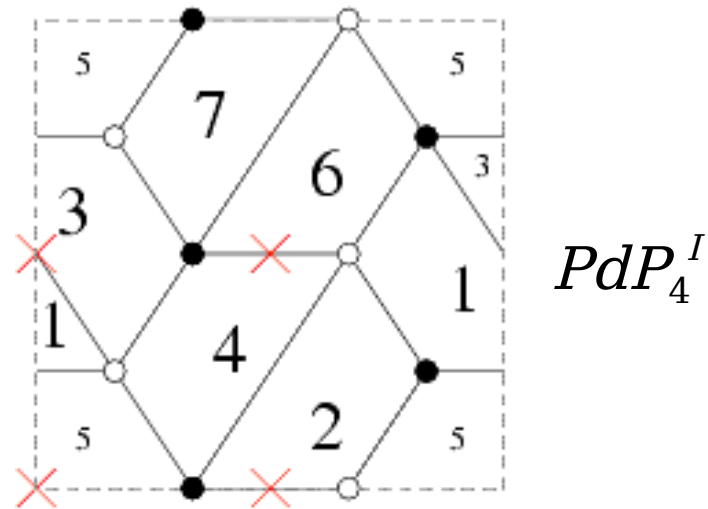
Indeed:

We can easily construct consistently such models.



Application I: DSB

Example:



Choosing,

$$n_2 = n_4 = 0, n_5 = N, n_1 = N + 4$$

we obtain the well-known SU(5) model with chiral 10+5 which features DSB
[Affleck, Dine and Seiberg '84]



Application II: D-instantons

Recall:

A contribution to the 4D superpotential can only be generated by a stringy D-instanton if it possesses 2 uncharged fermionic zero modes.

=> In absence of fluxes or other zero mode lifting mechanisms, only O(1) D-instantons can contribute!

[see Ibanez, Schellekens, Uranga '07 for a general discussion]



Application II: D-instantons

Recall:

A contribution to the 4D superpotential can only be generated by a stringy D-instanton if it possesses 2 uncharged fermionic zero modes.

=> In absence of fluxes or other zero mode lifting mechanisms, only $O(1)$ D-instantons can contribute!

[see Ibanez, Schellekens, Uranga '07 for a general discussion]

Integration over the charged zero modes will then induce a non-trivial superpotential contribution, if a suitable coupling between the charged zero modes and a 4D chiral operator is present!

[Ganor '96]



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Application II: D-instantons

=> To solve:

- Finding the invariant cycles which support proper D-instantons.
- Inferring if suitable couplings between the charged zero modes and 4D chiral fields are present.

Might the dimer be helpful for that too ?



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Dimers & D-instantons:

Yes. It becomes (almost) trivial to find stringy instanton contributions to the superpotential in local models.

Why?



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Application II: D-instantons

D-instantons in local models:

For obvious reasons, we consider only D-instantons with compact support:

Fractional D-instantons, i.e. bound states of D(-1), E1 and E3 instantons.



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=> Possible couplings are then easy to infer!

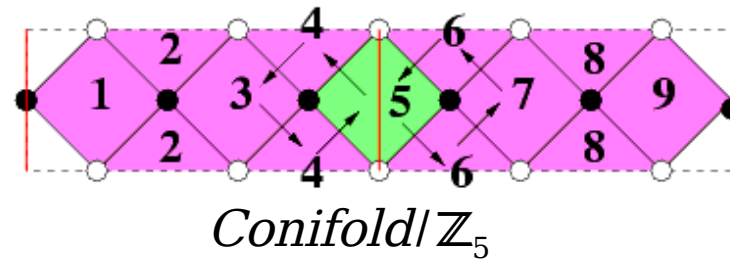
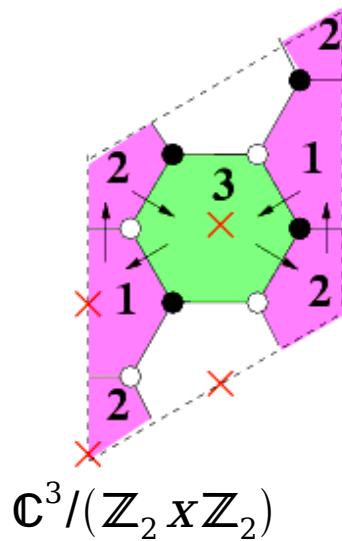


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Application II: D-instantons

Examples:

We can straight forwardly reproduce



[Argurio, Bertolini, Franco and Kachru '07]

[Argurio, Bertolini, Ferretti, Lerda and Peterson'07]



Conclusion

The message (you should remember):

Dimers present the so far most powerful (and simplest) tool to investigate the low-energy gauge theory of D3-branes probing an arbitrary (orientifolded) toric singularity!