# Topology Change
# Spacetime Singularity Resolution
# Superconformal Quantum Theories

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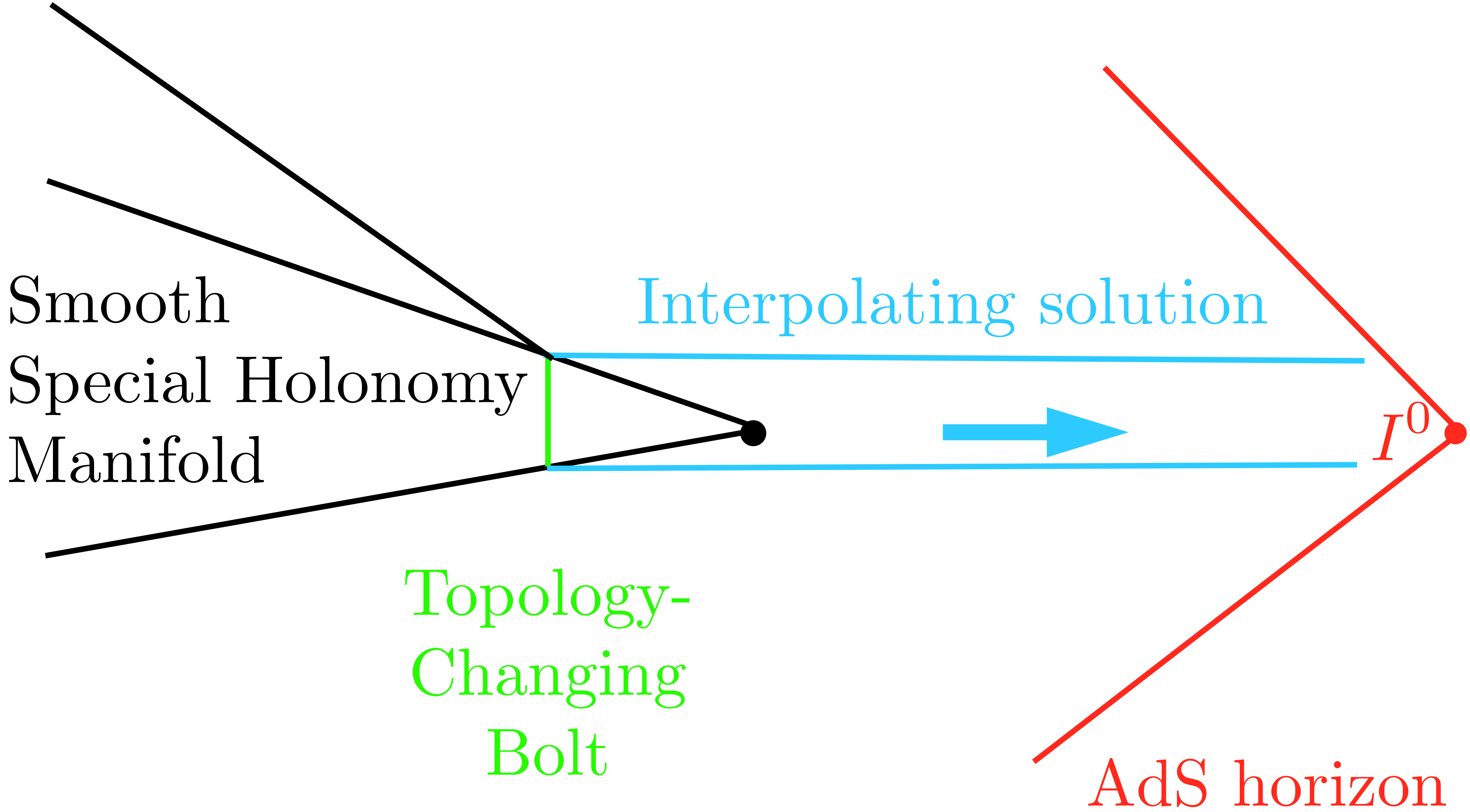



The Supergravity Description of a Wrapped Brane

# Special holonomy metrics of interest are all cones over homogeneous quotients $G/K$

| | | |
|---|---|---|
| $SO(n+2)/SO(n)$ | $SU(n+3)/U(n-1)$ | $SO(4)/SO(2)$ |
| $SU(n+1)$ | $Sp(n)$ | $SU(3)$ |
| Stenzel | Calabi | Candelas, de la Ossa |
| 1993 | 1979 | 1990 |

$SO(3)$

$SU(2)$

Eguchi, Hanson

1978

| | | |
|---|---|---|
| $SU(3)/(U(1)\times U(1))$ | $SU(2)\times SU(2)$ | $SO(5)/SO(3)$ |
| $G_2$ | $G_2$ | $\mathrm{Spin}(7)$ |
| BSGPP | BSGPP | BSGPP |
| 1989/90 | 1989/90 | 1989/90 |

All are foliated by principal orbits $G/K$ over a positive real line $\mathbb{R}_+$.

At some point in $\mathbb{R}_+$, the principal orbit collapses to a degenerate orbit, the bolt:

$$B = G/H,\ K \subset H \subset G.$$

The bolt is a calibrated cycle of the special holonomy manifold.

The supersymmetric $AdS$ M-theory solutions of interest are interpreted as the near-horizon limit of fivebranes wrapped on a calibrated cycle.

| | | |
|---|---|---|
| $\mathcal{N} = 2$ $AdS_5$<br>Kähler in SU(2)<br>Maldacena, Nuñez 2000 | $\mathcal{N} = 1$ $AdS_5$<br>Kähler in SU(3)<br>Maldacena, Nuñez 2000 | $\mathcal{N} = 2$ $AdS_4$<br>SLAG in SU(3)<br>Pernici, Sezgin 1985 |
| $\mathcal{N} = 1$ $AdS_4$<br>Associative in $G_2$<br>AGK 2000 | $\mathcal{N} = (2,1)$ $AdS_3$<br>CLAG in $Sp(2)$<br>Gauntlett, Kim 2001 | $\mathcal{N} = (2,0)$ $AdS_3$<br>Co-Associative in $G_2$<br>GKW 2000 |
| $\mathcal{N} = (2,0)$ $AdS_3$<br>Kähler in SU(4)<br>GKW 2000 | $\mathcal{N} = (1,1)$ $AdS_3$<br>SLAG in SU(4)<br>GKW 2000 | $\mathcal{N} = (1,0)$ $AdS_3$<br>Cayley in $Spin(7)$<br>GKW 2000 |

| | | |
|---|---|---|
| Eguchi-Hanson<br>Maldacena-Nuñez I | Resolved conifold<br>Maldacena-Nuñez II | Deformed conifold<br>Pernici-Sezgin |
| BSGPP I<br>AGK | Calabi<br>Gauntlett-Kim | BSGPP II<br>GKW I |
| SU(4) resolved conifold<br>GKW II | Stenzel<br>GKW III | BSGPP III<br>GKW IV |

Hitchin (2001) gave a construction of the special holonomy metrics as solutions of a dynamical system, with Hamilton's equations equivalent to the special holonomy condition:

$$\begin{aligned} \dot{x}_i &= \frac{\partial H}{\partial p_i}, \\ \dot{p}_i &= -\frac{\partial H}{\partial x_i} \end{aligned} \quad \Leftrightarrow \quad \begin{array}{l} \text{Special Holonomy metric} \\ \text{on } [t_0, \infty) \times (G/K), \end{array}$$

with time for the dynamical system identified with the radial coordinate $t$.

# The example: Eguchi-Hanson

$$\mathrm{d}s^2(\mathrm{EH}) = \left(1 - \frac{1}{R^4}\right)^{-1} \mathrm{d}R^2 + \frac{R^2}{4}\left[\mathrm{d}s^2(S^2) + \left(1 - \frac{1}{R^4}\right)(\mathrm{d}\psi - P)^2\right],$$

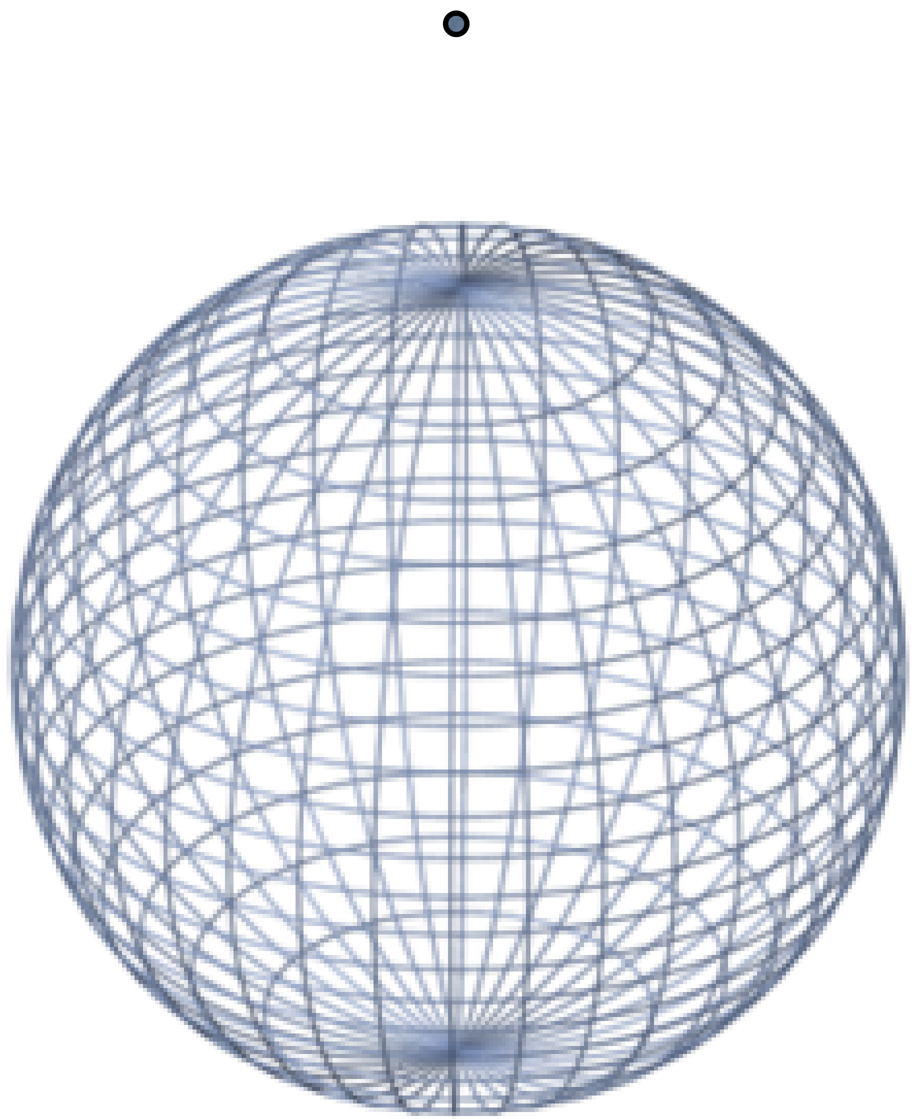
with $\psi \equiv \psi + 2\pi$, $\mathrm{d}P = \mathrm{Vol}[S^2]$. As $R \to \infty$,

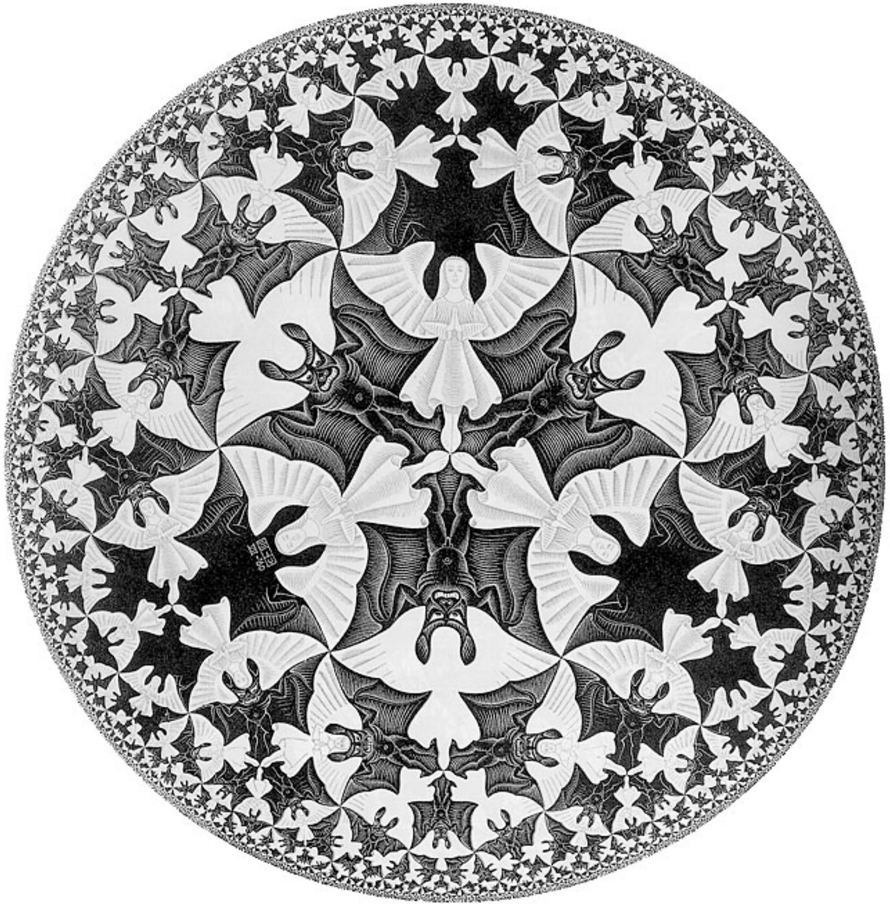
$$\mathrm{d}s^2 \to \mathrm{d}R^2 + \mathrm{d}s^2(S^3/\mathbb{Z}_2),$$

a cone over $S^3/\mathbb{Z}_2 = SO(3)$. As $R \to 1$,

$$\mathrm{d}s^2 \to \mathrm{d}u^2 + u^2(\mathrm{d}\psi - P)^2 + \frac{1}{4}\mathrm{d}s^2(S^2),$$

with the bolt $B = S^2 = SO(3)/SO(2)$.





After the hyperflop, the Hitchin Hamiltonian flow can be extended to $R = 0$. For $R > 1$ the metric is Eguchi-Hanson:

$$\mathrm{d}s^2(\mathrm{EH}) = \left(1 - \frac{1}{R^4}\right)^{-1} \mathrm{d}R^2 + \frac{R^2}{4} \left[\mathrm{d}s^2(S^2) + \left(1 - \frac{1}{R^4}\right)(\mathrm{d}\psi - P)^2\right].$$

For $R < 1$, the metric is

$$\mathrm{d}s^2(\mathcal{N}_\tau) = \left(\frac{1}{R^4} - 1\right)^{-1} \mathrm{d}R^2 + \frac{R^2}{4} \left[\mathrm{d}s^2(H^2) + \left(\frac{1}{R^4} - 1\right)(\mathrm{d}\psi - P)^2\right].$$

The $\mathcal{N}_\tau$ metric is singular at $R = 0$.

Now use an interpolating supergravity solution to excise the singularity, and replace it with an $AdS$ horizon at infinite proper distance.

An example of such a solution is for D3 branes at the tip of a Calabi-Yau cone:

$$\mathrm{d}s^2 = \left(1 + \frac{1}{r^4}\right)^{-1/2} \mathrm{d}s^2(\mathbb{R}^{1,3}) + \left(1 + \frac{1}{r^4}\right)^{1/2} (\mathrm{d}r^2 + r^2 \mathrm{d}s^2(\mathrm{SE}_5)).$$

$$r \to \infty, \qquad\qquad r \to 0,$$

$$\mathrm{d}s^2 \to \mathrm{d}s^2(\mathbb{R}^{1,3}) + \mathrm{d}s^2(\mathrm{CY}_3), \qquad \mathrm{d}s^2 \to \mathrm{d}s^2(AdS_5) + \mathrm{d}s^2(\mathrm{SE}_5)$$

The ansatz for the interpolating solution is

$$\begin{aligned} ds^2 &= L^{-1}\left[ds^2(\mathbb{R}^{1,3}) + F ds^2(H^2)\right] \\ &+L^2\left[F^{-1}\left(du^2 + u^2(d\psi - P)^2\right) + d\xi^2 + \xi^2 ds^2(S^2)\right] \end{aligned}$$

with $L, F$ arbitrary functions of $u, \xi$. This is required to be a solution of the 1/4 BPS Fayyazuddin-Smith (1999) equations for fivebranes on a two-cycle in a two-fold:

$$\begin{aligned} d\left(L^{-1/2}\Omega\right) &= 0, \\ \mathrm{Vol}[\mathbb{R}^3] \wedge d(LJ) &= 0, \\ d\left[L^2 \star_7 d\left(L^{-2}J\right)\right] &= 0. \end{aligned}$$

For the ansatz they reduce to

$$\begin{aligned} \frac{1}{\xi^2}\partial_\xi\left(\xi^2\partial_\xi F\right) &= -u\partial_u\left(\frac{F}{u}\partial_u F\right), \\ L^3 &= -\frac{1}{4u}\partial_u\left(F^2\right). \end{aligned}$$

There exist two 1/2-BPS solutions of these equations. One is

$$\mathrm{d}s^2 = \mathrm{d}s^2(\mathbb{R}^{1,6}) + \mathrm{d}s^2(\mathcal{N}_\tau).$$

The other is

$$F = \frac{u^2}{4t^2}\left(-1+\sqrt{1+t^2/u^4}\right) \equiv e^{2r},$$

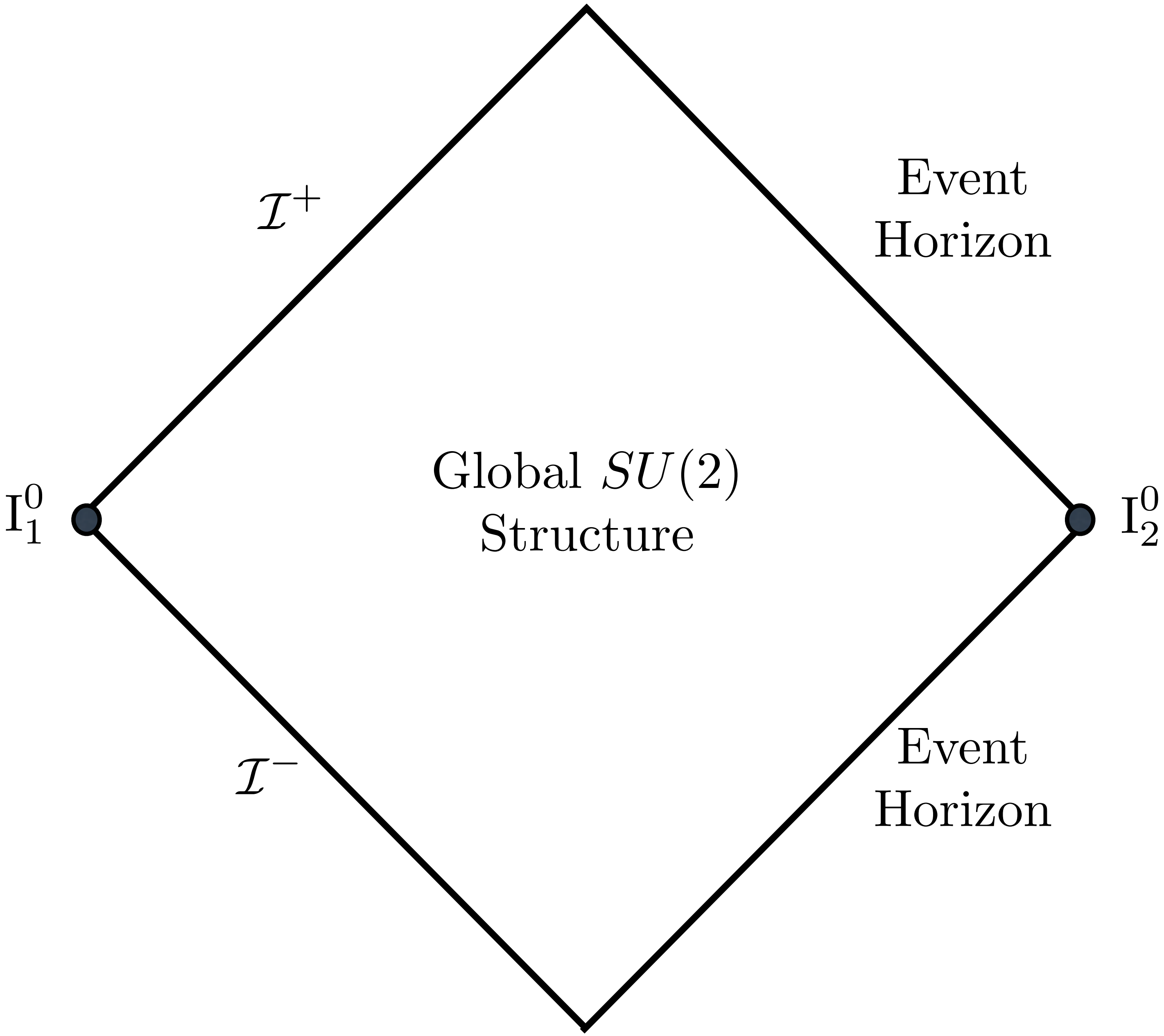
$$L^3 = \frac{u^2}{\sqrt{1+t^2/u^4}}\left(\frac{-1+\sqrt{1+t^2/u^4}}{4t^2}\right) \equiv \frac{8}{1+4\rho^2}e^{6r},$$

# inducing the metric

$$\begin{aligned} \mathrm{d}s^2 &= \frac{1}{\lambda}\Big[\mathrm{d}s^2(AdS_5) + \frac{1}{2}\mathrm{d}s^2(H^2) + (1-\lambda^3\rho^2)(\mathrm{d}\psi - P)^2 \\ &\quad + \frac{\lambda^3}{4}\left(\frac{\mathrm{d}\rho^2}{1-\lambda^3\rho^2} + \rho^2\mathrm{d}s^2(S^2)\right)\Big], \end{aligned}$$

$$\lambda^3 = \frac{8}{1+4\rho^2}.$$

# Conjecture: there exists a 1/4-BPS interpolating solution for $\mathcal{N}_\tau$ and Maldacena-Nuñez I.



The boundary conditions are

$$\begin{aligned} \mathrm{d}s^2|_{I_1^0} &= \mathrm{d}s^2(\mathbb{R}^{1,3}) + \Bigg[ \left(\frac{1}{R^4} - 1\right)^{-1} \mathrm{d}R^2 + \frac{R^2}{4}\Bigg[\mathrm{d}s^2(H^2) \\ &\quad + \left(\frac{1}{R^4} - 1\right)(\mathrm{d}\psi - P)^2\Bigg] + \mathrm{d}\xi^2 + \xi^2 \mathrm{d}s^2(S^2)\Bigg]_{R=1,\xi=\infty}, \end{aligned}$$

$$\mathrm{d}s^2|_{I_2^0} = \mathrm{d}s^2(\text{Maldacena-Nuñez})|_{r=\infty,\rho=0}.$$

The construction is qualitatively identical for all the other cases.

Things to be done:

Find the interpolating solutions or prove their existence

Understand the hyperflop in field-theoretic language

Construct the superconformally invariant quantum theories