Axionic gaugings in $N = 4$ supergravities

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3rd RTN ForcesUniverse Workshop
Valencia, October 2007

based on work with Jean-Pierre Derendinger (Neuchâtel University)
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Highlights

Motivations and summary

Gauged supergravities and the embedding tensor

The non-unimodular gaugings

Higher-dimensional origin: generalized Scherk–Schwarz reduction

Outcome

Why gaugings and fluxes?

Usual caveats of string compactifications:

- Supersymmetry breaking: $N = 4$ or $8 \longrightarrow N = 1$ and 0
- Many massless neutral scalars: moduli stabilization
- Cosmological constant (?)

Tool for a better control of the situation: give vev's to antisymmetric-tensor fields such as NS–NS, R–R, spin connection [many groups – long literature – see M. Graña's '05 review]

From 4 to 10 dimensions

From the low-energy viewpoint

- Toroidal compactifications: $N = 8$ or $N = 4$ *ungauged* sugras with neutral scalars without potential
- Flux compactifications: $N = 8$ or $N = 4$ (or less) *gauged* sugras with charged scalars under (non-)Abelian gauge groups and moduli-dependent superpotential (and potential)

The method [Derendinger, Kounnas, Petropoulos, Zwirner '05]

- Start with phenomenologically relevant 4-dim gauged sugras
- Translate the gauging parameters into fluxes
- Reconstruct the fundamental theory

About the bottom-up programme

- Complementary to the 10-dim generalized-geometry approaches
- No systematic oxidation recipe
- Not all 4-dim gauged $N = 4$ ($N = 8$) sugras are heterotic, type-I or type-II-orientifold (M-theory) vacua
- Captures everything, including 4-dim remnants of non-geometric string backgrounds

Here

We focus on 4-dim $N = 4$ theories (seeds for "realistic" vacua)

- Remind the basics on the gauging procedure using the *embedding tensor* – outstanding tool
- Analyze the gauging of *axionic shifts and rescalings*
- Trace its 10-dim origin – not straightforward
  - requires a generalized Scherk–Schwarz with twist by the scaling symmetries
  - relies on a duality between massive vectors and massive two-forms

Highlights

Ungauged 4-dim $N = 4$ supergravity

Spectrum, interactions and symmetries

- 1 gravitational and n vector multiplets
- Bosonic content of the multiplets
  - gravitational multiplet: 1 graviton, 6 graviphotons, 2 real scalars combined into the *axion-dilaton* $\tau = \chi + i \exp -2\phi$
  - vector multiplet: 1 vector, 6 real scalars
- Gauge group: $U(1)^{6+n}$
- All scalars are neutral and non-minimally coupled to the vectors: interaction terms of the type $f(\text{scalars})\, F^2$
- There is *no* scalar potential
- The elimination of the auxiliary fields generates the *scalar manifold:*

$$\mathcal{M} = \frac{SL(2,\mathbb{R})}{U(1)} \times \frac{SO(6,n)}{SO(6) \times SO(n)}$$

- The $SL(2,\mathbb{R}) \times SO(6,n) \subset Sp(12+2n,\mathbb{R})$ is realized as a U-duality symmetry of the *full* theory [Gaillard, Zumino '81]
- The $SO(1,1) \times SO(6,n)$ is realized off-shell in heterotic
  - the $SO(1,1)$ does not mix electric and magnetic gauge fields
  - genuine electric–magnetic duality transformations relate different Lagrangians written in different "symplectic frames"

Putting electric and magnetic duals together

- The $2 \times (6+n)$ fields $\left(\left\{\mathbf{A}^{M+}\right\}, \left\{\mathbf{A}^{M-}\right\}\right)$, $M = 1, \ldots, 6+n$ form a $(\mathbf{2}, \mathbf{Vec})$ of $SL(2,\mathbb{R}) \times SO(6,n)$
- A Lagrangian exists that captures their dynamics without altering the number of propagating degrees of freedom – no kinetic terms for $\left\{\mathbf{A}^{M-}\right\}$, extra 2-form auxiliary fields dual to the scalars [de Wit, Samtleben, Trigiante '02 –]

Gauging: deformation compatible with supersymmetry

Promotion of a subgroup of the U-duality group to a local gauge symmetry supported by (part of) the existing $U(1)^{n+6}$ vectors

- The generators of the duality group are
  1. $T^{MN} = -T^{NM}$, $M, \ldots = 1, \ldots, 6+n$ generate the $SO(6,n)$
  2. $S^{\beta\gamma} = S^{\gamma\beta}$, $\beta, \ldots = +, -$ generate the $SL(2,\mathbb{R})$
- The generators of the *gauge algebra* are

$$\Xi_{\alpha L} = \frac{1}{2}\left(\Theta_{\alpha LMN}\, T^{MN} + \Theta_{\alpha L\beta\gamma}\, S^{\beta\gamma}\right)$$

  where $\{\Theta_{\alpha LMN}, \Theta_{\alpha L\beta\gamma}\} \in (\mathbf{2}, \mathbf{Vec} \times \mathbf{Adj}) + (\mathbf{2} \times \mathbf{3}, \mathbf{Vec})$ of $SL(2,\mathbb{R}) \times SO(6,n)$ is the *embedding tensor*
- At most $6+n$ Ξ's are independent and the Θ's are subject to *constraints*

Consistency constraints for the embedding tensor

Gauge invariance and supersymmetry [linear constraints]

This reduces the embedding tensor to $(\mathbf{2}, \mathbf{Ant}_{[3]}) + (\mathbf{2}, \mathbf{Vec})$:

$$\Xi_{\alpha L} = \frac{1}{2}\left(f_{\alpha LMN}\, T^{MN} + \eta_{LQ}\, \xi_{\alpha P}\, T^{QP} + \epsilon^{\gamma\beta}\, \xi_{\beta L}\, S_{\gamma\alpha}\right)$$

The fundamental of $Sp(12 + 2n, \mathbb{R})$ must contain the adjoint of the gauge algebra and the latter must close [quadratic constraints]

(i) $\eta^{MN}\, \xi_{\alpha M}\, \xi_{\beta N} = 0$

(ii) $\eta^{MN}\, \xi_{(\alpha M}\, f_{\beta) NIJ} = 0$

(iii) $\epsilon^{\alpha\beta}\left(\xi_{\alpha I}\, \xi_{\beta J} + \eta^{MN}\, \xi_{\alpha M}\, f_{\beta NIJ}\right) = 0$

(iv) $\eta^{MN}\, f_{\alpha MI[J}\, f_{\beta KL]N} - \frac{1}{2}\xi_{\alpha[J}\, f_{\beta KL]I} - \frac{1}{6}\epsilon_{\alpha\beta}\, \epsilon^{\gamma\delta}\, \xi_{\gamma I}\, f_{\delta JKL} + \frac{1}{2}\eta^{MN}\, \xi_{\alpha M}\, f_{\beta N[JK}\, \eta_{L]I} + \frac{1}{6} f_{\alpha JKL}\, \xi_{\beta I} = 0$ (Jacobi-like)

Important remarks

- f's and ξ's are the *gauging parameters* which determine
  - the algebra and its commutators
  - the charges and covariant derivatives
  - the scalar potential
  - the mass matrices
  - ...
- $f_{\alpha JKL}$ *are not* necessarily structure constants of some algebra

Examples

Gaugings with non-vanishing $f_{\alpha LMN}$ only

- Pure $SO(6, n)$ gaugings, extensively studied in the literature
- Large variety of gauge algebras as e.g. *flat algebras* related to unimodular Scherk–Schwarz reductions, de Roo–Wagemans phases, . . .

Gaugings with non-vanishing $\xi_{\alpha L}$

- Only some isolated examples have been studied that fall in this class [Villadoro, Zwirner '04; Schön and Weidner '06]
- Their systematic analysis is the subject of the next chapters

Lagrangian formulation – including electric and magnetic

An explicit Lagrangian is associated with any consistent gauging and its bosonic sector has three parts [Schön and Weidner '06]

- $\mathcal{L}_{\text{kin}}$ – kinetic terms for graviton, electric vectors and scalars
- $\mathcal{L}_{\text{top}}$ – auxiliary-field contributions (magnetic vectors and two-forms) necessary to maintain the correct number of propagating fields
- $\mathcal{L}_{\text{pot}} = -\frac{e}{16}\Big(f_{\alpha MNP}\, f_{\beta QRS}\, M^{\alpha\beta}\big(\frac{1}{3}M^{MQ}\, M^{NR}\, M^{PS} + \big(\frac{2}{3}\eta^{MQ} - M^{MQ}\big)\eta^{NR}\eta^{PS}\big) - \frac{4}{9}f_{\alpha MNP}\, f_{\beta QRS}\, \epsilon^{\alpha\beta}\, M^{MNPQRS} + 3\xi_{\alpha}^{M}\, \xi_{\beta}^{N}\, M^{\alpha\beta}M_{MN}\Big)$ – the scalar potential
    - $M^{\alpha\beta}$ are the components of $\frac{1}{\text{Im}\tau}\begin{pmatrix} 1 & -\text{Re}\tau \\ -\text{Re}\tau & |\tau|^2 \end{pmatrix}$
    - M^{MQ} and M^{MNPQRS} are constructed similarly with the remaining $6n$ scalars

Highlights

The axionic transformations and their gaugings

The axionic transformations are generated by the $SL(2,\mathbb{R})$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R})$ acts on the axion-dilaton as $\tau \longrightarrow \frac{a\tau+b}{c\tau+d}$

- $S^{--} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$ generates the *electric–magnetic duality*
- $S^{++} = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$ generates the *axionic shifts* $\tau \longrightarrow \tau + b$
- $S^{+-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ generates the *axionic rescalings* $\tau \longrightarrow a^2\tau$

Gauging the axionic symmetries [see the embedding tensor]

- requires an embedding tensor with $\xi_{\alpha M} \neq 0$
- is necessarily accompanied by a partial gauging of $SO(6,n)$

Aim: gauge the axionic shifts S^{++} and rescalings S^{+-} but not the electric–magnetic duality transformation S^{--} – "electric gaugings"

- We must set $\xi_{-I} = 0$ [see embedding tensor]
- Our further choice: $f_{-LMN} = 0$ (not compulsory)
- The quadratic constraints for ξ_I, f_{LMN} are simpler ("+" index dropped)

A solution: non-unimodular gaugings captured by $\{\lambda_i, i = 1, \ldots, 6\}$

- We focus on $n = 6$: 12 vectors in total
- Light-cone-like convention: $\{I\} \equiv \{i, i'\}$, $\eta = \begin{pmatrix} 0 & \mathbb{I}_6 \\ \mathbb{I}_6 & 0 \end{pmatrix}$
- $\xi_i = \lambda_i$, $\xi_{i'} = 0$ and $f_{iji'} = -\lambda_{[i}\,\delta_{j]i'} \Rightarrow f_{ij}{}^{j} = -\frac{5}{2}\lambda_i$

Remarks

- Here *all* other f's vanish: $f_{ijk}, f_{ii'j'}, f_{i'j'k'}$
- The f's are *not* Lie-algebra structure constants

The gauge algebra [see embedding tensor] has 8 independent generators out of 2×12*:* $\{\Upsilon, \Xi, \Xi_{i'}\}$

- $\Xi_{-i} = -\frac{\lambda_i}{2} S^{++} \equiv \lambda_i \Xi$
- $\Xi_{-i'} = 0$
- $\Xi_{+i} = -\frac{\lambda_i}{2} \left(T^j{}_j + S^{+-} \right) \equiv \lambda_i \Upsilon$
- $\Xi_{+i'} = -\lambda_j T^j{}_{i'} \equiv \Xi_{i'}$

Axionic symmetries are gauged along with 6 $\{\Xi_{i'}\} \subset SO(6,6)$

Commutation relations for $\{\Upsilon, \Xi, \Xi_{i'}\} \subset SL(2,\mathbb{R}) \times SO(6,6)$

- $[\Xi_{i'}, \Xi_{j'}] = 0$
- $[\Xi, \Xi_{j'}] = 0$
- $[\Xi_{i'}, \Upsilon] = \Xi_{i'}$
- $[\Xi, \Upsilon] = -\Xi$

More remarks and summary

- $\{\Upsilon, \Xi, \Xi_{i'}\}$ is *non-flat* in contrast to the algebras obtained by standard Scherk–Schwarz reductions [see latter]
- $\{\Upsilon, \Xi\}$ is the non-semi-simple subalgebra $A_{2,2} \subset SL(2,\mathbb{R})$ of axionic rescalings and axionic shifts
- $\{\Upsilon, \Xi, \Xi_{i'}\} = \{\Upsilon\} \ltimes \{\Xi, \Xi_{i'}\}$
- non-Abelian extensions exist (with e.g. $f_{ijk} \neq 0$)

The dynamics of non-unimodular gaugings

The 12 vectors

4 inert and $2+6$ embedded in $SL(2,\mathbb{R}) \times SO(6,6)$ as generators of local symmetries – enter in covariant derivatives acting on scalars

The $36 = 21 + 15$ *scalars of* $\frac{SO(6,6)}{SO(6)\times SO(6)}$

- The usual coset parameterization is

$$M^{MN} = \begin{pmatrix} h^{ij} & -h^{ik}\, b_{kj} \\ b_{ik}\, h^{kj} & h_{ij} - b_{ik}\, h^{k\ell}\, b_{\ell j} \end{pmatrix}$$

- The gauging at hand generates $\mathcal{L}_{\rm pot}$

$$\frac{1}{16} e^{2\phi} \lambda_i \left(8h^{ij} - h^{ij}\, h^{k\ell}\, b_{\ell m}\, h^{mn}\, b_{nk} + 2h^{ik}\, b_{km}\, h^{mn}\, b_{nr}\, h^{rj} \right) \lambda_j$$

(positive definite in analogous 5- to 4-dim reduction [VZ '04])

The axion-dilaton

- The kinetic term is

$$e^{-1}\mathcal{L}_{\text{kin:axion-dilaton}} = -D_\mu \phi D^\mu \phi - \frac{1}{4} e^{4\phi} D_\mu \chi \, D^\mu \chi$$

  - $D_\mu \phi = \partial_\mu \phi - \frac{1}{2} Y_\mu$
  - $D_\mu \chi = \partial_\mu \chi + X_\mu + Y_\mu \chi$

- Physical vectors involve electric *and* magnetic potentials:

$$Y_\mu = \lambda_i \, A_\mu^{i+} \quad X_\mu = \lambda_i \, A_\mu^{i-}$$

  - $Y_\mu \leftrightarrow \Upsilon$ (axion rescalings: $\chi \to a^2 \chi$, $\phi \to \phi - \log a$)
  - $X_\mu \leftrightarrow \Xi$ (axion shifts: $\chi \to \chi + b$)

- The axion can be gauged away – X_μ acquires a mass in this process via its Stückelberg coupling to χ and is traded for a massive two-form $C_{\nu\rho}$

The final bosonic content of the non-unimodular gauging

- The dilaton
- $4 + 1 + 6$ vectors with Abelian algebra
  - 4 inert
  - 1 associated with the axionic rescalings of $SL(2, \mathbb{R})$
  - 6 associated with MASA transformations of $SO(6, 6)$
- 1 massive two-form
- 36 scalars
  - with scalar potential
  - and minimal couplings to the $1 + 6$ vectors

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Outcome

Heterotic 10-dim pure supergravity

The $SO(1,1)$ symmetry

- Action for the bosonic sector ($H = \mathrm{d}B$)

$$\int_{M_4} \mathrm{d}x \int_{K_6} \mathrm{d}y \, \sqrt{-G} \, \mathrm{e}^{-\Phi} \left(R + G^{MN} \partial_M \Phi \, \partial_N \Phi - \frac{1}{12} H_{MNK} H^{MNK} \right)$$

- Invariance under $SO(1,1)$

$$\Phi \to \Phi + 4\lambda, \quad G_{MN} \to \mathrm{e}^{\lambda} G_{MN}, \quad B_{MN} \to \mathrm{e}^{\lambda} B_{MN}$$

Dimensional reduction

- K_6 is compact: infinitude of 4-dim modes
- Reduction: effective theory on M_4 for a *finite* subset
- Data: K_6 plus an ansatz for the y-dependance of all fields
- Necessary consistency condition: $\mathcal{L}$ independent of y

Ordinary vs. Scherk–Schwarz reduction

Standard reduction on flat torus

- Ansatz: no y-dependance
- Bosonic spectrum:
  - 1 graviton
  - $6 + 6$ Abelian vectors
  - 36 scalars
  - 1 dilaton
  - 1 axion (dual to the NS–NS form)

  all massless and neutral

Ordinary vs. Scherk–Schwarz reduction

Scherk–Schwarz reduction [Scherk, Schwarz '79; long literature]

- Ansatz: y-dependance compatible with internal symmetries
- Introduction of geometric (spin connection) fluxes $\gamma^i{}_{jk}$
  - $\mathrm{d}\theta^i = -\gamma^i{}_{jk}\,\theta^j \wedge \theta^k$
  - Bianchi–Jacobi $\gamma^i{}_{j[k}\,\gamma^j{}_{\ell m]} = 0$
  - $f_{jk}{}^i = 2\gamma^i{}_{jk}$ structure constants of a locally group manifold
  - $\gamma^i{}_{ij} = 0$: *unimodularity* property (truncation consistency)
- Results: non-Abelian vectors, massive scalars and vectors, spontaneous breaking of supersymmetry – gauging
- Note: unimodularity captures semi-simplicity or flatness
- Example: *twisted tori* leading to gaugings in $SO(6,6)$

External Scherk–Schwarz reduction

Using the "duality" $SO(1,1)$ 10-dim symmetry

- Ansatz: $\Phi(x,y) = \Phi(x) + 4\lambda_i y^i$ $G_{MN}(x,y) = \mathrm{e}^{\lambda_i y^i} G_{MN}(x)$ $B_{MN}(x,y) = \mathrm{e}^{\lambda_i y^i} B_{MN}(x)$
- Usual decomposition:
  - $G_{MN} \to g_{\mu\nu}, A_{\mu k}, h_{ij}$
  - $B_{MN} \to B_{\mu\nu}, B_{\mu k}, b_{ij}$
  - $\phi = \Phi - \frac{1}{2} \log \det \mathbf{h}$
- The ansatz is consistent: the y-dependence drops

External Scherk–Schwarz reduction

Various couplings emerge

- $A_{\mu k}$ and $B_{\mu k}$ carry Abelian gauge symmetry
- h_{ij} charged under $A_{\mu k}$ with charges λ_k
- b_{ij} charged under $B_{\mu k}$ and Stückelberg-coupled to $A_{\mu k}$
- ϕ Stückelberg-coupled to $A_{\mu k}$ with charges λ_k
- scalar potential for h_{ij} and b_{ij}

"duality-twisted tori"

Contact with axionic gaugings

After field redefinitions and integrations one vector drops and the two-form becomes massive due to the Stückelberg couplings – indicative of the gauging of a shift symmetry

- The reduced theory *is* the gauged $N = 4$ supergravity studied in the last chapter: exact matching of the Lagrangians
- The specific choice of generalized Scherk–Schwarz allows to
  1. turn on the 4-dim gauging parameters ξ_i as 10-dim $SO(1,1)$--shift parameters λ_i along the torus one-cycles and therefore gauge the 4-dim $SL(2,\mathbb{R})$ axionic shifts and rescalings
  2. evade unimodularity (here $\gamma^j{}_{ij} \equiv \frac{1}{2} f_{ij}{}^j = -\frac{5}{4}\lambda_i \Rightarrow$ non-unimodular geometric fluxes)

All this elegantly demonstrates the power of the gauging procedure for describing diverse flux compactifications

Highlights

The outcome

This analysis closes the chapter of characterizing a whole class of heterotic gaugings in terms of NS–NS and spin-connection fluxes

What are the geometrical features of the fundamental theory on the top that translate into the consistency constraints imposed to the embedding tensor from the bottom?

It calls for further investigation of other classes of gaugings related to the previous by duality transformations

Last slide

Further gaugings further fluxes

- f_{+IJK}, ξ_{+L}: 232 electric parameters
  - f_{+ijk} NS–NS, $f_{+ijk'}$ spin-connection [studied here in relation with axionic symmetries; Kaloper, Myers '99 in the unimodular case; ...]
  - $f_{+ij'k'}$ T-dual NS–NS, $f_{+i'j'k'}$ T-dual spin-connection: "non-geometric" [Hull *et al.* '05; Shelton, Taylor, Wecht '05; ...]
- f_{-IJK}, ξ_{-L}: 232 magnetic-dual parameters
  - f_{-ijk} NS–NS, $f_{-ijk'}$ spin-connection
  - $f_{-ij'k'}$ T-dual NS–NS, $f_{-i'j'k'}$ T-dual spin-connection

The number of degrees of freedom does not change – the algebra, its $SL(2,\mathbb{R}) \times SO(6,n)$ *embedding and the higher-dimensional setup do*
What is precisely the higher-dimensional setup?