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Mesons in marginally deformed AdS/CFT

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Outline

1) Exactly marginal deformations & SUGRA duals

2) Adding flavor to the AdS/CFT duality

3) Flavor on *deformed* backgrounds

4) The mesonic spectrum

5) Dual field theory picture

6) Conclusions

Exactly Marginal Deformations of $\mathcal{N} = 4$ SYM

- Superpotential of $\mathcal{N} = 4$ superconformal theory:

$$g\,\mathrm{Tr}(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2)$$

- Exactly marginal γ–deformation $\longrightarrow$ Superpotential of the $\mathcal{N} = 1$ theory:

$$h\,\mathrm{Tr}(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2)$$

[Leigh, Strassler '95]

- This $\mathcal{N} = 1$ theory is superconformal invariant in the large N limit at all loop orders if

$$g^2 = |h|^2$$

[Mauri, Penati, Santambrogio, Zanon '05]

- Global symmetries of the original theory:

$$U(1)_1: \ (\Phi_1, \Phi_2, \Phi_3) \rightarrow (\Phi_1, e^{i\alpha_1}\Phi_2, e^{-i\alpha_1}\Phi_3)$$
$$U(1)_2: \ (\Phi_1, \Phi_2, \Phi_3) \rightarrow (e^{-i\alpha_2}\Phi_1, e^{i\alpha_2}\Phi_2, \Phi_3)$$

- γ–deformation and noncommutativity:

$$f * g = exp\left[i\pi\gamma(Q_f^1 Q_g^2 - Q_g^1 Q_f^2)\right] fg$$

The Supergravity Dual

- Start from $AdS_5 \times S^5$ $\longrightarrow$ Select a T^2 in S^5 $\longrightarrow$ Perform a TsT (T-duality, shift, T-duality).

- TsT on each of the 3 "natural" tori of S^5 with different $\hat{\gamma}_i$ $\Rightarrow$ Non–supersymmetric *deformation*:

[Lunin, Maldacena '05, Frolov '05]

$$ds^2 = \frac{u^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{u^2}\left(d\rho^2 + \rho^2 d\theta^2 + G_{mn}dX^m dX^n\right)$$

$$u^2 = \rho^2 + X_5^2 + X_6^2 \quad \rho_1^2 = \frac{X_5^2 + X_6^2}{u^2} \quad \rho_2^2 = \frac{\rho^2 c_\theta^2}{u^2} \quad \rho_3^2 = \frac{\rho^2 s_\theta^2}{u^2}$$

$$G_{\phi_2\phi_2} = G(1 + \hat{\gamma}_2^2\rho_1^2\rho_3^2)\rho_2^2 u^2 \qquad G_{\phi_3\phi_3} = G(1 + \hat{\gamma}_3^2\rho_1^2\rho_2^2)\rho_3^2 u^2$$

$$G_{\phi_2\phi_3} = G\hat{\gamma}_2\hat{\gamma}_3\rho_1^2\rho_2^2\rho_3^2 u^2 \qquad \hat{\gamma}_i = R^2\gamma_i$$

$$G_{\phi_2 X_5} = -G\hat{\gamma}_1\hat{\gamma}_2\rho_2^2\rho_3^2 X_6 \qquad G_{\phi_2 X_6} = G\hat{\gamma}_1\hat{\gamma}_2\rho_2^2\rho_3^2 X_5$$

$$G_{\phi_3 X_5} = -G\hat{\gamma}_1\hat{\gamma}_3\rho_2^2\rho_3^2 X_6 \qquad G_{\phi_3 X_6} = G\hat{\gamma}_1\hat{\gamma}_3\rho_2^2\rho_3^2 X_5$$

$$G_{X_5X_5} = 1 - X_6^2\left[1 - G\left(1 + \hat{\gamma}_1^2\rho_2^2\rho_3^2\right)\right] / \left(u^2\rho_1^2\right)$$

$$G_{X_6X_6} = 1 - X_5^2\left[1 - G\left(1 + \hat{\gamma}_1^2\rho_2^2\rho_3^2\right)\right] / \left(u^2\rho_1^2\right)$$

$$G_{X_5X_6} = X_5X_6\left[1 - G\left(1 + \hat{\gamma}_1^2\rho_2^2\rho_3^2\right)\right] / \left(u^2\rho_1^2\right)$$

$$e^{2\phi} = e^{2\phi_0}G \qquad G^{-1} = 1 + \hat{\gamma}_1^2\rho_2^2\rho_3^2 + \hat{\gamma}_2^2\rho_1^2\rho_3^2 + \hat{\gamma}_3^2\rho_1^2\rho_2^2$$

- There are also non–vanishing $B(\hat{\gamma}_i)$, $C_2(\hat{\gamma}_i)$ in the internal directions.

- Finally $C_4 = C_4(\text{Minkowski}) + C_4(\hat{\gamma}_i)$ and $C_6 = C_4 \wedge B$

Adding flavor to AdS/CFT

[Karch, Katz '02]

- Holographic dual of $4d$ SYM with fundamental matter $\rightarrow$ near–horizon limit of a (N)D3–(N_f)D7 system ($N \gg N_f$)

$$\Downarrow$$

Type IIB SUGRA on $AdS_5 \times S^5$ + DBI theory on $AdS_5 \times S^3$

- Open strings in this set–up:

D3–D3	$\mathcal{N} = 4$ SYM theory
D3–D7	$\mathcal{N} = 2$ fundamental hypermultiplets

- On the field theory side:

$$\mathcal{W} = \text{Tr}\left(\Phi_1[\Phi_2, \Phi_3]\right) + \text{tr}\left(\tilde{Q}\Phi_1 Q\right) + m\,\text{tr}\left(\tilde{Q}Q\right)$$

where the chiral superfields Q and $\tilde{Q}$ transform in the $(N, \bar{N}_f)$ and $(\bar{N}, N_f)$ of $SU(N) \times SU(N_f)$.

- Holographic dictionary:

$$\langle \Phi_1 \rangle \sim m \;\rightarrow\; L \equiv \text{distance between D3 and D7}$$

- Quenching & $L = 0 \Rightarrow$ Conformal theory.

Flavor in marginally deformed $AdS_5 \times S^5$

[Penati, M.P., Ratti '07]

- D7–brane action in $AdS_5 \times \tilde{S}^3$: $S = S_{DBI} + S_{WZ}$

$$S_{DBI} = -T_7 \int_{\Sigma_8} e^{-\phi} \sqrt{-\det\left(g_{ab} - b_{ab} + 2\pi\alpha' F_{ab}\right)}$$

$$S_{WZ} = T_7 \int_{\Sigma_8} P\left[\sum_q C_q \, e^{-B}\right] e^{2\pi\alpha' F}$$

- Non–supersymmetric $AdS_5(x_\mu, \rho) \times \tilde{S}^5(\theta, \phi_2, \phi_3, X_5, X_6)$:

$$(G_{MN}\,,\, e^{\phi}\,,\, B\,,\, C_q) \longrightarrow \text{functions of } \hat{\gamma}_{1,2,3}$$

- D7–brane worldvolume $(x_\mu, \rho, \theta, \phi_2, \phi_3)$ in the static gauge and consider the embedding

$$X^M = (x_\mu, \rho, \theta, \phi_2, \phi_3, X_5(\rho), X_6(\rho)) \qquad F = F(X^M)$$

$$\Downarrow$$

$$\{X_5 = L, \quad X_6 = 0, \quad F = 0\} \; :$$

$$S = -T_7 \int_{\Sigma_8} d\sigma^8 \rho^3 \sin\theta \cos\theta$$

Identical situation to the undeformed case!

$SUSY \leftrightarrow NON\text{–}SUSY$

- This configuration is stable (BPS):

1) no dependence on $\hat{\gamma}_i$,

2) no–force condition (energy independent of L),

3) $[\text{adding probes}, TsT] = 0$:

D3–D7 system + near–horizon + TsT
$\vert\vert\vert$
$AdS_5 \times S^5 + TsT +$ D7–probe

(for instance [Imeroni, Naqvi '07])

4) no tachyonic modes in the spectrum of small fluctuations around the ground state,

5) when $\hat{\gamma}_i = \hat{\gamma}$, it is supersymmetric.

[Mariotti '07]

Probe fluctuations

- Stability & mesonic spectrum → vibrations of the D7:

$$X_5 = L + 2\pi\alpha'\chi(\sigma_i), \qquad X_6 = 2\pi\alpha'\varphi(\sigma_i) \qquad A_a \neq 0$$

$$\Downarrow$$

$$S = \int_{\Sigma_8} d^8\sigma \{\mathcal{L}_0 + 2\pi\alpha'\mathcal{L}_1 + (2\pi\alpha')^2\mathcal{L}_2 + \cdots\}$$

- Introduce the undeformed induced metric $\mathcal{G}^{ac}$:

$$\mathcal{G}^{\mu\nu} = \frac{R^2}{L^2+\rho^2}\eta^{\mu\nu}, \quad \mathcal{G}^{\rho\rho} = \frac{L^2+\rho^2}{R^2}, \quad \mathcal{G}^{ij} = \frac{L^2+\rho^2}{R^2\rho^2}g^{ij}$$

and the deformation matrix $\mathcal{T}^{ac}$:

$$\mathcal{T}^{\phi_2\phi_2} = \hat{\gamma}_3^2 \qquad \mathcal{T}^{\phi_3\phi_3} = \hat{\gamma}_2^2 \qquad \mathcal{T}^{\phi_2\phi_3} = \mathcal{T}^{\phi_3\phi_2} = -\hat{\gamma}_2\hat{\gamma}_3$$

- Equations of motion from the quadratic $\mathcal{L}_2$:

$$\partial_a\left[\sqrt{-det(\mathcal{G})}\,\frac{R^2}{(L^2+\rho^2)}\left(\mathcal{G}^{ac} + \frac{L^2}{R^2(L^2+\rho^2)}\mathcal{T}^{ac}\right)\partial_c\chi\right] = 0$$

$$\partial_a\left[\sqrt{-det(\mathcal{G})}\,\frac{R^2}{(L^2+\rho^2)}\,\mathcal{G}^{ac}\,D_c\right] = 0$$

$$\partial_a\left[\sqrt{-det(\mathcal{G})}\,\mathcal{G}^{ac}\mathcal{G}^{bd}\,F_{cd}\right] - \frac{4\rho(L^2+\rho^2)}{R^4}\epsilon^{bjk}\partial_j A_k$$
$$- \quad \sqrt{-det(\mathcal{G})}\,\frac{L}{(L^2+\rho^2)}\,\left(\hat{\gamma}_2\,\partial_{\phi_3} - \hat{\gamma}_3\,\partial_{\phi_2}\right)\mathcal{G}^{bc}\,D_c = 0$$

where for simplicity $D_c \equiv \partial_c\Phi - \frac{L}{R^2}\left(\hat{\gamma}_2\partial_{\phi_3} - \hat{\gamma}_3\partial_{\phi_2}\right)A_c$ and

$$\Phi \equiv \varphi + \frac{L}{R^2}\left(\hat{\gamma}_2 A_{\phi_3} - \hat{\gamma}_3 A_{\phi_2}\right)$$

- In the conformal case $(L=0)$ or in the UV $(\rho \gg L)$ the effect of the deformation disappears.

The mesonic spectrum

- Expand χ, Φ, A_μ, A_ρ in scalar spherical harmonics on S^3:

$$X(\sigma_i) = x(\rho)\, e^{ikx}\, \mathcal{Y}_l^{m_2,m_3}(\theta, \phi_2, \phi_3)$$

where

$$\begin{aligned} \Delta_{\mathrm{S}^3}\, \mathcal{Y}_l^{m_2,m_3} &= -l(l+2)\mathcal{Y}_l^{m_2,m_3} \\ \frac{\partial}{\partial\phi_{2,3}} \mathcal{Y}_l^{m_2,m_3} &= i m_{2,3} \mathcal{Y}_l^{m_2,m_3} \end{aligned}$$

- And A_i in vector spherical harmonics $\nabla_i \mathcal{Y}_l^{m_2,m_3}$ and $\mathcal{Y}_i^{l;\pm}$.

1) The scalar mode χ and and the vector A_μ (in the Lorentz gauge $\partial^\mu A_\mu = 0$) are *decoupled* and have the same mass:

$$M_{\hat{\gamma}_i} = \frac{2L}{R^2}\sqrt{(n+l+1)(n+l+2) + \left(\frac{\hat{\gamma}_2 m_3 - \hat{\gamma}_3 m_2}{2}\right)^2}$$

2) If A_i are expanded with $\mathcal{Y}_i^{l;\pm}$, they are *decoupled* form Φ and A_ρ since in different representation of $SO(4) \Rightarrow M_{\hat{\gamma}_i}$.

3) If A_i are expanded with $\nabla_i \mathcal{Y}_l^{m_2,m_3} \Rightarrow$ 4 *coupled* equations for Φ, A_ρ, A_i: only 2 independent differential equations and 1 algebraic $\Rightarrow$ 2 scalars with the same mass

$$M_3 = ?$$

[in progress]

Analysis of the spectrum

- In general the deformation manifests itself in the meson masses:

$$M_{\hat{\gamma}_i} = \frac{2L}{R^2}\sqrt{(n+l+1)(n+l+2) + \left(\frac{\hat{\gamma}_2 m_3 - \hat{\gamma}_3 m_2}{2}\right)^2}$$

$$\Downarrow$$

Zeeman–like splitting

- The same $\hat{\gamma}_i$–behavior found in the bosonic fluctuations of giant gravitons (wrapped on $\tilde{S}^3$) in the LM–Frolov models!

[M.P. '06]

- For fixed $\hat{\gamma}_i$ the spectrum is discrete.

- It has the same mass gap of the undeformed case:

$$(n = l = m_{2,3} = 0) \quad \longrightarrow \quad M_{gap} = \frac{2\sqrt{2}\,L}{R^2}$$

- In the SUSY case $\Rightarrow$ mesons should fill massive $\mathcal{N} = 1$ multiplets: 3 scalars and 1 vector with $M_{\hat{\gamma}}$ and 2 scalars with M_3 $\Rightarrow$

$$\begin{aligned} \mathcal{V}_{\hat{\gamma}} &\equiv (spin-0, spin-1) \\ \mathcal{S}_{\hat{\gamma}} &\equiv (spin-0, spin-0) \\ \mathcal{S}_3 &\equiv (spin-0, spin-0) \end{aligned}$$

[in progress]

- What about the fermions?

The dual field theory

- Since $[\text{adding probe}, TsT] = 0 \Rightarrow$

$$[\text{adding fundamental}, *\text{–product}] = 0$$

- Start from the undeformed conformal superpotential

$$\mathcal{W} = \text{Tr}\left(\Phi_1[\Phi_2, \Phi_3]\right) + \text{tr}\left(\tilde{Q}\Phi_1 Q\right)$$

with $U(1)_1 \times U(1)_2$ global symmetries:

	Φ^1	Φ^2	Φ^3	Q	$\tilde{Q}$
Q_1	0	1	-1	0	0
Q_2	-1	1	0	$1/2$	$1/2$

- Next, modify with the $*$–product and introduce the mass term as a relevant operator:

$$\begin{aligned} \mathcal{W} &= \text{Tr}\left(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2\right) \\ &+ \text{tr}\left(\tilde{Q}\Phi_1 Q\right) + m\,\text{tr}\left(\tilde{Q}Q\right) \end{aligned}$$

- γ enters only in the adjoint sector $\rightarrow$ expected?

$$\langle\Phi_1\rangle \sim m \;\rightarrow\; L \equiv \text{as in the undeformed case}$$

- Non–supersymmetric case $\rightarrow$ work in components and implementing the $*$–product $\Rightarrow$ dependence on γ_1 and $(\gamma_2 - \gamma_3)$.

Summary

1) We find STABLE and EXACT D7–embedding in $AdS_5 \times \tilde{S}^5$.

2) INDEPENDENCE of the deformation parameters $\longrightarrow$ It remains BPS despite the values of $\hat{\gamma}_i$.

3) Exact solutions for the mesonic spectrum:

$$M_{\hat{\gamma}_i}^2 \sim M_0^2 + \left(\frac{\hat{\gamma}_2 m_3 - \hat{\gamma}_3 m_2}{2} \right)^2$$

4) FIELD THEORY dual in the SUSY and NON–SUSY case.

Future

♣ Study of the fermionic sector in the spectrum.

♢ Consider other SUSY embeddings.

(for instance [Mariotti '07])

♠ Backreaction & dual gauge operators.