Euclidean methods and Sen's entropy function

Pedro J. Silva
UAB-IFAE-CSIC

Valencia October 2007

work in collaboration O. Dias arXiv:0704.1405

On Sen's entropy function

On Sen's Entropy function

- To calculate the entropy S for extremal BH use only the Near-Horizon geometry (attractor mechanism). The NH is of the form $Ads_2 \otimes S^{D-2}$,

$$ds^2 = v_1 \left(-\rho^2 d\tau + \frac{d\rho^2}{\rho^2} \right) + v_2 d\Omega^2_{D-2} \,,$$
$$F^{(i)}_{\rho\tau} = e_i \,, \qquad \phi_s = u_s \,, \tag{1}$$

It is defined f and S such that

$$f(\vec{u}, \vec{v}, \vec{e}) = \int_{S^{D-2}} \sqrt{-g} \, (lagrangian) \,,$$
$$S = e_i \frac{\partial(2\pi f)}{\partial e_i} - 2\pi f \tag{2}$$

extremizing $\rightarrow S(q_i)$ with electric charges $q_i = \partial(2\pi f)/\partial e_i$

On Sen's Entropy function

- The attractor mechanism is used to fix the metric form, and extremization of entropy reproduce the usual BH entropy as a function of the conserved charges.

- This framework is powerful enough to include higher derivative corrections to the supergravity action (just use Wald generalization whenever is need it).

- But, what is the geometric origin or motivation for these definitions?, What is this f??

On Sen's Entropy function

- Notice that

  $q_i = \frac{\partial(2\pi f)}{\partial e_i}$ is defined as the conjugated variable to e_i and there is like a Legendre transformation between $(S, f)^*$.

- This reminds the usual manipulations of thermodynamics but we are working at zero temperature with extremal solutions

- There should be an underlying thermodynamic framework where to base the discussions for extremal BH, after all, this should be dual to ensembles in matrix quantum mechanics!!!!

*All the above can be generalized to include magnetic charges and rotation.

On GR thermodynamics

On GR thermodynamics

- Black Hole show Thermodynamic properties... like

$$I(\beta, \Omega, \Phi) = \beta E - \beta \Omega J - \beta \Phi Q - S(E, J, Q)$$

where (E, J, Q) are charges and (β, Ω, Φ) are potentials

- ...But in the extremal cases we do not write potentials

(E, Q, J) OK $\qquad (\beta =?, \Omega = 1, \Phi = 1)$ NOT OK

- In particular starting from non-extremal cases, taking the extremal limit we get

$$\beta \to \infty \quad i.e. \quad T \to 0$$

On GR thermodynamics

- Yes, it is possible to extend the usual BH thermodynamics to the extremal case!!!

- This is done by noticing that the near-extremal solution are such that,

$$\beta \to \infty\,, \quad \Omega \to \Omega_{ext} - \frac{w}{\beta} + O(\beta^{-2})\,, \quad \Phi \to \Phi_{ext} - \frac{\phi}{\beta} + O(\beta^{-2})$$

and

$$E \to E_{ext} + O(\beta^{-2})\,, \quad J \to J_{ext} + O(\beta^{-2})\,, \quad Q \to Q_{ext} + O(\beta^{-2})$$

- then,

$$I = \beta \left(E_{ext} - \Omega_{ext} J_{ext} - \Phi_{ext} Q_{ext} \right) + w J_{ext} + \phi Q_{ext} - S_{ext} + O(\beta^{-1})$$

- where the extremal condition reads

$$E_{ext} - \Omega_{ext} J_{ext} - \Phi_{ext} Q_{ext} = 0$$

- therefore we get

$$\boxed{I_{ext} = w J_{ext} + \phi Q_{ext} - S_{ext}}$$

where $I_{ext}(w, \phi)$ and $S(J_{ext}, Q_{ext})$

On GR thermodynamics

- In fact all the statistical mechanics is recovered

- It is useful to find phase transitions and curves of marginal stability for all extremal BH

- The above analysis has a dual CFT picture in 5D for asymptotic AdS BH and for D1/D5/P BH

- Since there is a NH AdS_2 geometry, the analysis should tell information on the unknown dual quantum mechanics...

Entropy function and Euclidean action

Entropy function and Euclidean action

- comparison between both equations tell us that,

$$2\pi f = I_{ext}$$

$$e_i = (w, \phi, etc)$$

- but how? well, put the BH in a box at $r = r_+$, then it is easy to show

$$I = \underbrace{\beta(\Phi_{ext} - \Phi)Q - S}_{r = r_h} + \underbrace{\beta(E - \Phi_{ext}Q)}_{r_+} .$$

Entropy function and Euclidean action

- Taking the extremal limit

$$\lim_{ext\ limit} \beta(\Phi_{ext} - \Phi)Q - S = \phi Q_{ext} - S_{ext} \quad \text{NH region}$$

$$\lim_{ext\ limit} \beta(E - \Phi_{ext}Q) = 0 \quad \text{Asymp region}$$

- and we have

$$\int dt \longrightarrow 2\pi$$
$$\int dr \longrightarrow |_{r_h}$$

that makes clear the identification of $2\pi f$ and I_{ext}

Entropy function and Euclidean action

- regarding the NH electric fields e_i and the potentials (w, ϕ) we have

$$\phi = \frac{\partial \Phi}{\partial \epsilon} \qquad \text{where} \qquad \Phi = A|_{r_h} - A|_{r_+}$$
$$e = \frac{\partial A}{\partial r}|_{r_h}$$

but the variation with respect to the near-extremality parameter ϵ translates into a variation on the position of the event horizon r_+ and hence both expressions agree!!!

Concluding remarks

- We have defined the extension of Euclidean methods to extreme BH
- We understood sen's entropy functional within this framework of "BH statistical mechanics"
- We have found, using this framework, a rich phase diagram with phase transitions for AdS BH
- Opens up a avenue to study AdS/CFT duality.