Non renormalisations theorems in superstring and maximal supergravity theories

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based on some work done in collaboration with M.B. Green and J.Russo

UV behaviour of gravity amplitudes

Gravity describes the interactions of a massless spin 2 particle with a dimensionfull coupling constant

$$[1/\kappa_{(D)}^2] = (1/length)^{D-2}$$

A L-loop n-point *pure* gravity amplitude in dimensions D has the *superficial* UV behaviour

$$[A_L^{(n)}] = \kappa_{(D)}^{2L-2+n}\,\Lambda^{(D-2)L+2}; \qquad \delta_L = (D-2)L+2$$

UV behaviour of gravity amplitudes

- The 4-graviton amplitude in $D = 4$ behaves as

$$\text{1-loop: } A_1^{(4)} \sim \kappa_{(4)}^4 \, \Lambda^4$$
$$\text{2-loop: } A_2^{(4)} \sim \kappa_{(4)}^6 \, \Lambda^6$$

which requieres the following counter-terms to

$$\delta_1 A_1^{(4)} \sim \kappa_{(4)}^4 \, R_{mnpq}^2$$
$$\delta_2 A_2^{(4)} \sim \kappa_{(4)}^6 \, R_{mnpq}^3$$

- Gravity is diverging at 1-loop when coupled to matter or at 2-loop for pure gravity 't Hooft/Veltman; Goroff/Sagnotti; van de ven

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- For $N \geq 1$ susy the R^3 counter-term is not allowed. Two-loop is finite.

Tomboulis; Grisaru

Superspace and counter-terms for Supergravity?

Howe, Lindstrom have constructed a possible counter-term at

- $L = 3$ loop order for $N \leq 4$ on-shell supersymmetries

$$\mathcal{L}_3 = \int d^{4N}\theta \, (\kappa_{(4)}^2 R_{\alpha\beta\gamma\delta} \, \theta^N)^4$$

- $L = N - 1$ loop order for $N \geq 4$ on-shell supersymmetries.

$$\mathcal{L}_{N-1} = \int d^{4N}\theta \, (\kappa_{(4)}^2 R_{\alpha\beta\gamma\delta}\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta)^N$$

- But no divergence at $L = 3$ are found in $D = 4$ for the 4 graviton amplitude for $N = 8$ Bern et al.

Superspace and counter-terms for Supergravity?

Berkovits' formalism gives a different way of contructing superinvariants

$$\mathcal{L}_g = \int d^{16}\theta_L d^{16}\theta_R \, \theta_L^{12-2g} \, \theta_R^{12-2g} \, \mathcal{W}_{\alpha\beta}^4$$

with the chiral superfield ($\alpha, \beta \in 16$ of $SO(10)$ for instance)

$$\mathcal{W}_{\alpha\beta} = F_{\alpha\beta} + \cdots + \theta_L^\gamma \theta_R^\delta \, W_{\alpha\beta\gamma\delta} + \cdots$$

entering the contribution of the vertex operators

$$V = \int d^2 z \, (G_{MN} \partial x^M \bar\partial x^N + \cdots + d_L^\alpha d_R^\beta \mathcal{W}_{\alpha\beta})$$

The measure of integration arise from the zero mode of a genus g string amplitude in the non-minimal pure spinor formalism.

Superspace and counter-terms for Supergravity?

$$\mathcal{L}_g = \int d^{16}\theta_L d^{16}\theta_R \, \theta_L^{12-2g} \, \theta_R^{12-2g} \, (\theta_L \theta_R \, W)^4$$

- Up to and including genus 5 $\mathcal{L}_g$ is a gravitational F-term
- $g \geq 6$ gives a D-term that is the *first* possible counter-term at 9 loops

$$\kappa_{(4)}^{20} \int d^{16}\theta_L d^{16}\theta_R \, (\theta_L \theta_R \, W)^4 \sim \kappa_{(4)}^{20} D^{12} R^4$$

- This superspace formalism would give

$$\delta_L = (D - 2)L - 18$$

Superspace and counter-terms for Supergravity?

All the superspace argument presented above gives

$$\delta_L = (D - 2)\, L - Cste <=> D \geq D_c = 2 + \frac{Cste}{L}$$

which means that only $D = 2$ supergravity would be finite.

Can we expect something better?

UV behaviour of gravity amplitudes

Considering a n-graviton one-loop amplitude a naive power counting argument indicate $2n$ powers of loop momenta in the numerator

$$\int d^D\ell \, \frac{\ell^{2n}}{\prod_{i=1}^{n}(\ell + K_{j_1\cdots j_i})^2} \sim \Lambda^D$$

Various computations of 4,5, ..., 7-graviton amplitudes at one-loop Bern et al.; Bjerrum-Bohr et al. indicate that

$$\int d^D\ell \, \frac{\ell^{n+4-\mathcal{N}}}{\prod_{i=1}^{n}(\ell + K_{j_1\cdots j_i})^2} \sim \Lambda^{D-n+4-\mathcal{N}}$$

where $0 \leq \mathcal{N} \leq 8$ is the number of supersymmetries counted in 4d
These cancellations can be tracked to the good high-energy behaviour of tree-level gravity amplitude Cachazo et al.; Bern et al.

UV behaviour of supergravity amplitudes

- $N = 8$ supergravity factorizes extra powers of the external momenta

$$[A_L^{(4)}] = \Lambda^{(D-2)L-6-2\beta_L}\,[D^{2\beta_L} R^4]$$

- At one-loop one has $\beta_1 = 0$ Green et al.

$$[A_1^{(4)}] = R^4\,\Lambda^{D-8}$$

- At two-loop one has $\beta_2 = 2$ Bern et al.

$$[A_2^{(4)}] = D^4\,R^4\,\Lambda^{2(D-7)}$$

- At higher loop we will except Green, Russo, Vanhove $\beta_L = L$ for $L \geq 2$

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we will discuss the validity of this relation

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Multiloop amplitudes in M-theory

- $D = 11$ four gravitons supergravity L-loop amplitudes

$$\begin{aligned} A_L^{(4)} &= S^{\beta_L} R^4 \,\mathcal{I}(s,t,u) + perms. \\ &\sim \Lambda^{9L-6-2\beta_L} D^{2\beta_L} R^4 + \cdots \end{aligned}$$

- Compactifying on a circle of radius R_{11} the derivative expansion

$$A_L = \sum_{w,\nu} \ell_P^{9(L-1)} \frac{\Lambda^{9L-6-2\beta_L-w}}{R_{11}^w} (R_{11}^2 D^2)^v \, D^{2\beta_L} R^4$$

$w \geq 0$ parametrizes the UV (sub-)divergences of the L-loop diagram

From M-theory parameters to string variables

The relation between the Type II string and M-theory parameters

$$\begin{aligned} R_{11}^3 &= (g_s^A)^2\,\ell_P^3 \\ ds^2_{M-th} &= \frac{\ell_P^3}{\ell_s^2}\,R_{11}^{-1}\,ds^2_{IIA} + R_{11}^2\,(dx^{11} - C_m dx^m)^2 \end{aligned}$$

The derivative are rescaled as

$$D^2 \longrightarrow R_{11}\,\frac{\ell_P^3}{\ell_s^2}\,D^2$$

Multiloop amplitudes in M-theory

$$A_h = \ell_s^{2k-2} \sum_{w,\nu} \Lambda^{\delta} c_h \, e^{2(h-1)\phi_A} \, D^{2k} R^4$$

$$\delta = 9L - 6 - 2\beta_L - w; \qquad h = 1 + k - \frac{w + 2\beta_L}{3}$$

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$$w \geq 0 <=> h \leq k + 1 - \frac{2\beta_L}{3}$$

- At one-loop $\beta_1 = 0$ and the only diverging term is R^4
- Since at higher-loop we have $\beta_L = 2$ for $L \geq 2$ then

$$h \leq k$$

Non renormalisation conditions of $D^{2k}R^4$ (or R^{4+k}) operators

- *There are* no contributions *with string loop genus* $h > k$.
- *The contributions with* $h = k$ *are* **exactly** *determined by the one-loop* $(L = 1)$ *diagram in eleven dimensions.*
- $\beta_L = L$ so $\delta_L = (D - 4)L - 6$

$$[A_L^{(4)}] = \Lambda^{(D-\mathbf{4})L-6}\,[D^{2L}R^4]$$

- Same critical dimension for divergences as in $N = 4$ SYM

$$D \geq D_c = \mathbf{4} + \frac{6}{L}$$

  If valid to all orders $D = 4$ would be UV finite

- $\beta_L = L$ gives an infinite set of non renormalisation theorems in string theory & sugra

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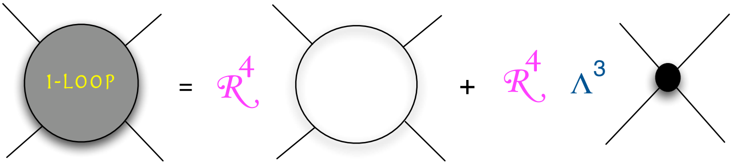
One loop amplitude



- The loop integral is the massless scalar φ^3 box diagram

$$\int \frac{d^D p}{p^2 (p-k_1)^2 (p-k_1-k_2)^2 (p+k_4)^2}$$

- There is no triangle or bubble loop integrals
- We have $\beta_1 = 0$ and the degree of divergence $\delta_1 = 3$ in $D = 11$

One loop in 11D

- Compactification on T^2 to 9D gives type IIA/IIB string on circle

$$\begin{aligned}
\mathcal{A}^{(4)}_{L=1} &= r_a \left[\frac{2\zeta(3)}{(g^a_s)^2} + \frac{4\zeta(2)}{r_a^2} + (\ell_P \Lambda)^3 \right] \mathcal{R}^4 \\
&+ r_a \sum_{n\geq 2} \frac{\zeta^*(2n-1)}{n!}\, r_a^{2(n-1)} (\alpha' \mathcal{D}^2)^n \mathcal{R}^4 \\
&+ r_a \sum_{n\geq 2} \frac{\zeta(2n-2)}{n!}\, e^{2(n-1)\phi} (\alpha' \mathcal{D}^2)^n \mathcal{R}^4 \\
&+ \text{non-anal.} + \text{non-pert.}
\end{aligned}$$

- Higher string genus contributions satisfying the bound $\beta_h = h$
- The one-loop expressions are not T-duality $r_a = 1/r_b$ invariant. To be cured by the higher-loop contributions
- Non analytic terms from the massless threshold, and non perturbative effects (D-branes)

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Renormalisation the one-loop amplitude

- T-duality invariance of the 4 gravitons amplitude at one string-loop

$$r_a \leftrightarrow r_b = \frac{1}{r_a}; \qquad \frac{r_a}{(g_s^a)^2} = \frac{r_b}{(g_s^b)^2}$$

$$A_{L=1} \quad = \quad \left(r_a \frac{2\zeta(3)}{(g_s^a)^2} + \frac{4\zeta(2)}{r_a} + r_a \left[(\ell_P \Lambda)^3 + c_1\right] \right) \mathcal{R}^4$$

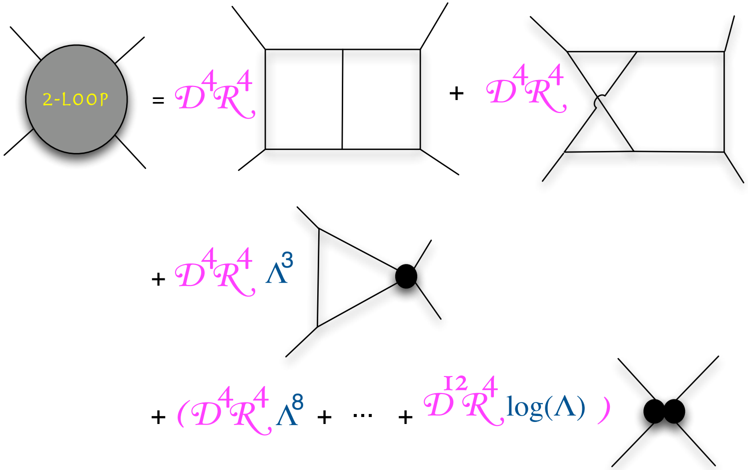
- RG scheme: we substract the Λ^3 divergence and leave a finite answer

$$4\zeta(2) = (\ell_P \Lambda)^3 + c_1$$

- The UV divergence is regulated by matching string perturbative answer.

Two loops in 11D

- $\mathcal{N} = 8$ susy implies that the amplitude factorizes $\mathcal{D}^4\,\mathcal{R}^4$ `Bern et al.`



- This has $\beta_2 = 2$ and the superficial UV behaviour is $\delta_2 = 8$

Renormalising the two loops

$$\delta A^{(L=2)} = c_8\, D^4 R^4 + c_6\, D^6 R^4 + \cdots + c_0\, D^{12} R^4$$

The $D^4 R^4$ factor is given by Green et al.

$$D^4 R^4\,(\Lambda^8 + \Lambda^3 \frac{E_{\frac{5}{2}}(\Omega)}{\mathcal{V}^{\frac{5}{2}}} + \frac{\zeta(3)\zeta(4)}{\mathcal{V}^4})$$

- The Λ^8 term does not have an interpretation in string theory

$$(\ell_P \Lambda)^8 + c_2 = 0$$

- The 2nd term decompactifies to a 10d contribution
- The 3rd term is a genus one contribution to 9d effective action restauring the T-duality invariance $r_a \to r_b = 1/r_a$

Renormalisation of M-theory

The set of counter-term needed to regularise the $L = 2$ amplitude is Green, Vanhove; Green,Russo,Vanhove

$$\delta_2 A_2 = \zeta(4)\, \ell_P^{12}\, D^6 R^4 + \zeta(6)\, \ell_P^{18}\, D^{12} R^4$$

together with the 1-loop counter we have the following correction to the M-theory effective action

$$S_{eff} = R_{(11)} + \ell_P^3 \zeta(2)\, R^4 + \zeta(4)\, \ell_P^{12}\, D^6 R^4 + \zeta(6)\, \ell_P^{18}\, D^{12} R^4$$

The series of R^{3m+1} corrections to the M-theory action compatible with the strong coupling limit of string theory Russo, Tseytlin

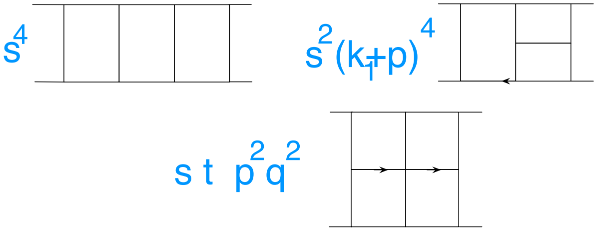
Two loops in 11D

The $L = 2$ amplitude on T^2 is compared with string theory on S^1 in 9D

$$
\begin{aligned}
A^{(L=2)} &= r_b \Big(g_s^{-1/2} E_{\frac{5}{2}}\, D^4 R^4 + g_s \mathcal{E}_{(0,1)}\, D^6 R^4 + \frac{g_s^2}{r_b^2}\, \mathcal{E}_{(2,0)}\, D^8 R^4 \\
&+ \frac{g_s^3}{r_b^4}\, \mathcal{F}_{(1,1)}\, D^{10} R^4 + \frac{g_s^4}{r_b^6}\, \mathcal{G}^{(x)}_{(3,0)}\, D^{12} R^4 + \frac{g_s^4}{r_b^6}\, \mathcal{G}^{(y)}_{(0,2)}\, D^{12} R^4 + \cdots \Big)
\end{aligned}
$$

- In 9d this gives terms that complete the $L = 1$ answer into T-duality invariant expressions

Three loops in 11D



- The ladder diagram has $\beta_3 = 4$
- Mondrian & Window diagrams have $\beta_3 = 2$
  No more reduction to pure φ^3 diagrams
- No triangles

Three loops amplitude

- Althougth the mondrian diagrams only have a D^4R^4 explicitly factorized the total amplitude starts with D^6R^4 at low-energy Bern et al. in agreement with the string predictions Green et al.
- The UV divergences up to this order is as for $N = 4$ SYM

$$\delta_L = (D-4)L - 6$$

- No $L = 3$ UV divergences in 4D Bern et al.

Protected couplings in type IIB superstring

- Some couplings are given by Eisenstein series

Green, Gutperle; Green, Vanhove; Green, Sethi

$$\alpha' \quad : \quad (\Delta - (\frac{1}{2} + g)(g - \frac{1}{2}))E_{\frac{1}{2}+g}(\Omega, \bar{\Omega}) = 0$$

- with a maximum genus g contribution
- Gives the correct string perturbation contributions to the R^4 (g=1) and D^4R^4 (g=2) couplings Green,Vanhove; D'Hoker,Phong; Berkovits

Protected couplings in type IIB superstring

- At higher order the differential equation changes structure Green, Vanhove

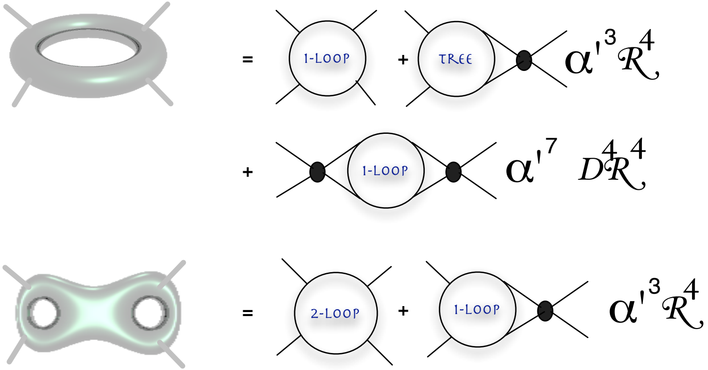
$$\alpha'^6 : (\Delta - (1+3)\times 3)\mathcal{E}_{(0,1)}(\Omega,\bar\Omega) = -6E_{\frac{3}{2}}^2$$

$$\Omega_2^{-1}\mathcal{E}_{(0,1)} = 4\zeta(3)^2\Omega_2^2 + 8\zeta(2)\zeta(3) + \frac{48}{5}\frac{\zeta(2)^2}{\Omega_2^2} + \frac{8}{9}\frac{\zeta(6)}{\Omega_2^4} + o(e^{-2\pi\Omega_2})$$

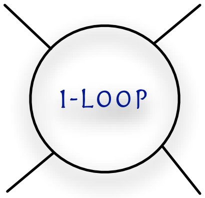
- The highest genus contribution is 3
- The highest contribution matches the values from the $L=1$ amplitude
- Exactness of the result no contributions from higher loop $L \geq 3$ contributions

Summary

Althougth we extrapolated from strong $R_{11} = \infty$ to weak string coupling $R_{11} = 0$ we could reconstructing the string theory S-matrix up to high order in derivative expansion including higher loop contributions, and derive some important non renormalisation theorems.
The resuts described in this talk *match* various result from string perturbation Green et al.



=
1-LOOP
+
TREE
$\alpha'^3 \mathcal{R}^4$
+
1-LOOP
$\alpha'^7 \; D^4\mathcal{R}^4$
=
2-LOOP
+
1-LOOP
$\alpha'^3 \mathcal{R}^4$



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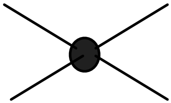
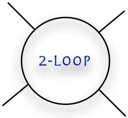
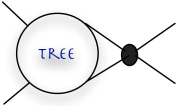


$\Lambda^3 \; \mathcal{R}^4$

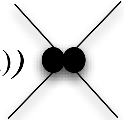


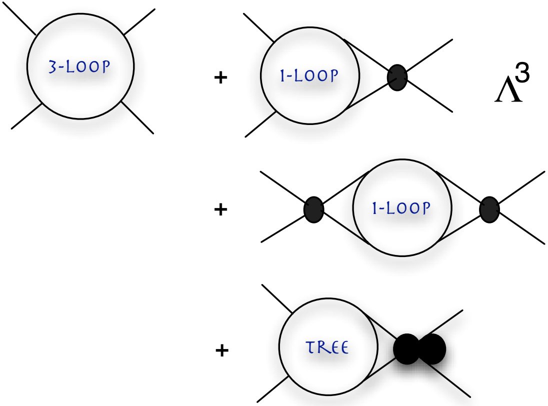
+



$\Lambda^3 \; \mathcal{R}^4$

$+ \; (\mathcal{D}^4\mathcal{R}^4\Lambda^8 + \cdots + \mathcal{D}^{12}\mathcal{R}^4 \; \log(\Lambda))$





$$\Lambda^3 \; \mathcal{R}^4$$

$$\Lambda^6 \; \mathcal{D}^8 \mathcal{R}^4$$

$$\Lambda^6 \; \mathcal{D}^6 \mathcal{R}^4$$