Consistent Kaluza-Klein reductions and susy AdS solutions

Oscar Varela

Imperial College London
100 years of living science

based on work with

Jerome Gauntlett, Eoin Ó Colgáin

3rd RTN ForcesUniverse Workshop

Valencia, October 2007

Outline

Consistent KK reductions

- A very useful way to construct solutions to sugra theories in a higher dimension D is to uplift solutions of simpler sugras in lower dimension d.
- For this uplift to be well defined, there must exist a consistent Kaluza-Klein (KK) reduction from the sugra in dimension D to the sugra in dimension d.
- To determine if such KK reduction is consistent is an interesting problem by its own.

Consistent KK reductions

- Upon compactification on an internal manifold M_{D-d}, the D-dimensional fields give rise to d-dimensional fields: a finite set of *light* L and a KK tower of *heavy* H fields.

- The D-dimensional e.o.m.'s can be rewritten in terms of these:

$$\Box L \sim a_{mn} L^m H^n$$

$$\Box H \sim b_{mn} L^m H^n$$

- A truncation keeping L and discarding H ($H = 0$) will be consistent only if $b_{m0} = 0$.

- Then, the fields L satisfy d-dimensional e.o.m.'s: $\Box L = 0$. Any solution to the d-dimensional theory can then be uplifted to D dimensions.

Consistent KK reduction

- Only in a few cases there is a group-theoretical argument behind the consistency of the truncation:
- (Toroidal) dimensional reductions,
- Compactifications on group manifolds.
- But in general, the compactification on arbitrary manifolds will be inconsistent.

$AdS \times Sphere$ compactifications

- Remarkable consistent compactifications of $D = 10, 11$ sugra are associated with the (maximally supersymmetric) solutions $AdS_7 \times S^4$, $AdS_4 \times S^7$ and $AdS_5 \times S^5$:

- The compactification of $D = 11$ sugra on S^4 can be consistently truncated to gauged maximal supergravity in $d = 7$ [Nastase, Vaman, van Nieuwenhuizen, hep-th/9905075, 9911238].

- The compactification of $D = 11$ sugra on S^7 can be consistently truncated to gauged maximal supergravity in $d = 4$ [De Wit, Nicolai, NPB 281 (1987) 211] .

- Similarly, the compactification of IIB sugra on S^5 is expected to consistently yield maximal gauged $d = 5$ sugra [Cvetic, Duff, Hoxha, Liu, Lu, Lu, Martinez-Acosta, Pope, Sati, Tran, hep-th/9903214; Lu, Pope, Tran, hep-th/9909203; Cvetic, Lu, Pope, Sadrzadeh, Tran, hep-th/0003103] .

A conjecture about consistency

- String/M-theory on all these backgrounds is dual, via the AdS/CFT correspondence, to a superconformal field theory (SCFT) in the boundary of AdS.
- Indeed, we would like to view the compactifications on those backgrounds as special cases of the following conjecture:
- For any supersymmetric $AdS_d \times_w M_{D-d}$ solution of $D = 10$ or $D = 11$ supergravity there is a consistent Kaluza-Klein truncation on M_{D-d} to a gauged supergravity theory in d-dimensions for which the fields are dual to those in the superconformal current multiplet of the $(d-1)$-dimensional dual SCFT [Gauntlett, OV, arXiv:0707.2315].

A conjecture about consistency

- Equivalently, the fields of the gauged supergravity are those that contain the d-dimensional graviton and fill out an irreducible representation of the superisometry algebra of the $D = 10$ or $D = 11$ supergravity solution $AdS_d \times_w M_{D-d}$.

- This is essentially a restricted version of the conjecture in [Duff, Pope, Nucl. Phys. **B255** (1985) 355].

- General arguments supporting it were subsequenty put forward in [Pope, Stelle, Phys. Lett. **B198** (1987) 151].

A conjecture about consistency

- For example, the $AdS_5 \times S^5$ solution of type IIB, which has superisometry algebra $SU(2,2|4)$, is dual to $N=4$ superYang-Mills theory in $d=4$.

- The superconformal current multiplet of the latter theory includes the energy momentum tensor, $SO(6)$ R-symmetry currents, along with scalars and fermions.

- These are dual to the metric, $SO(6)$ gauge fields along with scalar and fermion fields, and are precisely the fields of the maximally supersymmetric $SO(6)$ gauged supergravity in $d=5$.

A conjecture about consistency

- Here we will give evidence of this conjecture for the case of $AdS_5 \times_w M_6$ solutions in $D = 11$, which are dual to $N = 1$ SCFTs in 4 dimensions.

- These SCFTs all have a $U(1)$ R-symmetry and so we expect that $D = 11$ sugra on M_6 gives a $d = 5$ sugra with
  - $N = 1$ supersymmetry
  - a metric ds_5^2 (dual to the energy-momentum tensor of the SCFT)
  - and a $U(1)$ gauge field A (dual to the R-symmetry current).

- This is precisely the content of minimal $d = 5$ gauged sugra.

$AdS_5 \times_w M_6$ solutions of $D = 11$ sugra

The most general solution of the $D = 11$ sugra equations containing an AdS_5 factor in the metric was analysed by [Gauntlett, Martelli, Sparks, Waldram, hep-th/0402153] using G-structure techniques [Gauntlett, Martelli, Pakis, Waldram, hep-th/0205050].

The most general form of the $D = 11$ bosonic fields ds^2_{11}, $G_4 = dA_3$ containing AdS_5, compatible with $SO(4, 2)$ symmetry is

$$\begin{aligned} ds^2_{11} &= e^{2\lambda}[ds^2(AdS_5) + ds^2(M_6)] \ , \\ G_4 &\in \Omega_4(M_6, \mathbb{R}) \\ \lambda &\in \Omega_0(M_6, \mathbb{R}) \quad \text{(warp factor)} \end{aligned}$$

and subject to the field equations.

$AdS_5 \times_w M_6$ solutions of $D = 11$ sugra: $N = 1$ supersymmetry

- In order to have $N = 1$ supersymmetry, the solution must admit a Killing spinor ϵ, solution to the Killing spinor equation

  $$\mathcal{D}\epsilon = 0$$

  where $\mathcal{D}$ is the supercovariant derivative, involving the ordinary Riemannian covariant derivative and the $D = 11$ sugra fields.

$AdS_5 \times_w M_6$ solutions of $D = 11$ sugra: $N = 1$ supersymmetry

- The $D = 11$ Killing spinor splits as

$$\epsilon = \varepsilon \otimes e^{\lambda/2} \xi ,$$

where ε is a Killing spinor on AdS_5 and ξ is a (non-chiral) spinor on M_6.

- The Killing spinor equation also splits into
  - an equation for ε on AdS_5, immediately satisfied, and
  - equations for ξ on M_6, which specify a particular G-structure on M_6 (with $G = SU(2)$).

$AdS_5 \times_w M_6$ solutions of $D = 11$ sugra: the G-structure on M_6

The existence of ξ defines a G-structure on M_6, alternatively specified by a set of bilinears on ξ (e.g., $\tilde{K}^2_m = \frac{1}{2}\bar{\xi}\gamma_m\gamma_7\xi$):

$$\begin{aligned} K^1,\ \tilde{K}^2 &\in \Omega_1(M_6, \mathbb{R}) \\ J &\in \Omega_2(M_6, \mathbb{R}) \\ \Omega &\in \Omega_2(M_6, \mathbb{C}) \\ \cos\zeta &\in \Omega_0(M_6, \mathbb{R}) \end{aligned}$$

$AdS_5 \times_w M_6$ solutions of $D = 11$ sugra: the G-structure on M_6

- The Killing spinor equations for ξ translate into a set of differential and algebraic equations among these bilinear forms and the warp factor λ and four-form G_4.
- *e.g.*, $\nabla_{(\mu}\tilde{K}^2_{\nu)} = 0$, *i.e.*, $\tilde{K}^2 \equiv \cos\zeta K^2$ defines a Killing vector (related to the R-symmetry of the dual CFT)
- The equations among the bilinear forms constrain the internal geometry, *i.e.*, the metric on M_6, the flux G_4 and the warp factor λ.

Kaluza-Klein ansatz: the metric

- The 'KK ansatz' must express the $D = 11$ fields ds^2_{11}, $G_4 = dA_3$ in terms of the $d = 5$ fields ds^2_5, $F_2 = dA$. The KK ansatz must then satisfy the $D = 11$ field equations, provided ds^2_5, F_2 satisfy the $d = 5$ equations.

- It is natural to think of the $d = 5$ $U(1)$ gauge field A as arising from the $U(1)$ isometry of M_6 generated by $\tilde{K}^2$.

- Thus, we take the usual KK ansatz for the metric:

$$ds^2_{11} = e^{2\lambda}[ds^2_5 + ds^2(\hat{M}_6)]$$

where $\hat{M}_6$ denotes the deformation of M_6 parametrised by A as

$$K^2 \quad \longrightarrow \quad \hat{K}^2 = K^2 + \tfrac{1}{3}\cos\zeta A$$

Kaluza-Klein ansatz: the four-form

KK ansatz for the four-form:

$$G_4' = \hat{G}_4 + F_2 \wedge \hat{\beta}_2 + *_5 F_2 \wedge \hat{\beta}_1 .$$

Here,

- $F_2 = dA$
- hatted quantities are forms on $\hat{M}_6$ (*i.e.* with $K^2 \quad \longrightarrow \quad \hat{K}^2 = K^2 + \frac{1}{3}\cos\zeta A$)
- G_4 is the four-form on M_6 corresponding to the undeformed background $AdS_5 \times_w M_6$
- β_1, β_2 are forms on M_6 to be determined.

Consistent truncation

- Direct substitution shows that the KK ansatz satisfies the $D = 11$ field equations provided that
  - the $d = 5$ fields satisfy the equations of minimal $d = 5$ gauged supergravity, and
  - a set of differential and algebraic equations among β_1, β_2 and the warp factor λ and four-form G_4 is satisfied.

- These equations are actually of the same form than those among the bilinear forms defining the G-structure on M_6.

- Indeed β_1, β_2 have a solution in terms of some of the spinor bilinears on M_6:

$$\beta_1 = -\tfrac{1}{3} e^{3\lambda} \cos\zeta K^1$$

$$\beta_2 = \tfrac{1}{3} e^{3\lambda} \left(-\sin\zeta J + K^1 \wedge K^2\right) .$$

Consistent truncation

- To summarise, the $d = 5$ fields ds_5^2, F_2 can be embedded into the $D = 11$ fields ds_{11}^2, G_4 through the KK ansatz

$$ds_{11}^2 = e^{2\lambda}[ds_5^2 + ds^2(\hat{M}_6)] \ , \qquad \tilde{K}^2 \quad \longrightarrow \quad \hat{\tilde{K}}^2 = \tilde{K}^2 + A$$

$$G_4' = \hat{G}_4 + F_2 \wedge \tfrac{1}{3} e^{3\lambda}(-\sin\zeta J + K^1 \wedge \hat{K}^2) - *_5 F_2 \wedge \tfrac{1}{3} e^{3\lambda} \cos\zeta K^1$$

- This shows the consistency of the truncation, at the level of the bosonic equations [Gauntlett, O Colgain, OV, hep-th/0611219] .

- The $D = 11$ gravitino variations also reduce consistently to the $d = 5$ gravitino variation.

Further examples in $D = 11$

Other examples suport our conjecture about consistent KK reductions.

- The $D = 11$ solutions of the form $AdS_4 \times SE_7$, where SE_7 is Sasaki-Einstein are dual to 3d $N = 2$ SCFTs, and the reduction of $D = 11$ on M_7 consistently truncates to $d = 4$, $N = 2$ gauged sugra [Gauntlett, OV, arXiv:0707.2315] .
- The $D = 11$ solutions of the form $AdS_4 \times_w M_7$, corresponding to M5-branes wrapping SLAG 3 cycles [Gauntlett, Mac Conamhna, Mateos, Waldram hep-th/0605146] , also allow for a consistent reduction of $D = 11$ sugra on M_7 to $d = 4$, $N = 2$ gauged sugra [Gauntlett, OV, arXiv:0707.2315] .

Further examples in IIB

- IIB sugra on $d = 5$ Sasaki-Einstein spaces is dual to a 4d $N = 1$ SCFT, and consistently truncates to minimal $d = 5$ gauged sugra [Buchel, Liu, hep-th/0608002] .
- The IIB solutions of the form $AdS_5 \times_w M_5$ with $N = 1$ susy and all fluxes active [Gauntlett, Martelli, Sparks, Waldram, hep-th/0510125] also allow for a consistent reduction of IIB on M_5 to $d = 5$ minimal gauged sugra [Gauntlett, OV, arXiv:0707.2315] .

Conclusions and outlook

- All known consistent KK truncations on spheres could be recast in this language.
- The consistency of the KK truncation makes it manifest from the gravity side that SCFTs with type a IIB or $D = 11$ dual share common sectors. E.g., black hole solutions in lower d sugra should be relevant to any of the SCFTs to which these sugras are related.
- It would be interesting to prove the conjecture from the CFT side.