World-sheet description of A and B branes revisited

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Outline

1. Motivation
    - Nonlinear σ-models in superspace
    - D-brane effective actions

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   - Derivative corrections

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Nonlinear σ-models

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- $N = (2,2)$ supersymmetry for general metric g and Kalb-Ramond field b (take ϕ constant) $\longrightarrow$ target space geometry is bihermitian/generalized Kähler
- Geometry of target space clarified by using $N = (2,2)$ superspace

Closed string superspace: $N = (2, 2)$

- $N = (2, 2)$ superspace action determined by Kähler potential

$$\mathcal{S}_{(2,2)} = \int d^2\sigma \, D_+ D_- \hat{D}_+ \hat{D}_- \, V(X, \bar{X})$$

BUT one needs constrained superfields!

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- Most general $N = (2, 2)$ superspace description in terms of chiral, twisted chiral and semi-chiral fields [Lindström, Roček, von Unge, Zabzine '05]

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- We will focus on chiral and twisted chiral superfields
- $\longrightarrow$ target space is Kähler manifold with metric $g_{\alpha\bar{\beta}} = \pm V_{\alpha\bar{\beta}}$

Boundaries?

First motivation

What happens when we include boundaries?

[Ooguri, Oz, Yin; Albertsson, Lindström, Zabzine, ...]

Completely local $N = 2$ superspace formulation still incomplete

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World-sheet description of A and B branes on Kähler manifolds

B branes: - holomorphic cycles
- holomorphic line bundle connection

A branes: - lagrangian cycles with flat connection
- coisotropic cycles with non-vanishing flux

D-brane effective actions

- Effective action for flat Dp-brane for slowly varying fields
  10d Born-Infeld action reduced to $p+1$ dimensions

$$\mathcal{S}_{BI} = -\tau_9 \int d^{10}x \sqrt{-\det h^{\pm}_{ab}}, \quad h^{\pm}_{ab} = \eta_{ab} \pm F_{ab}$$

deformation of Maxwell theory $\mathcal{S}_M = -\frac{1}{4}\int F^2$

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  deformation of Maxwell theory $\mathcal{S}_M = -\frac{1}{4}\int F^2$
- For holomorphic line bundle connection: $F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0$

$$\boxed{g^{\alpha\bar{\beta}}(\operatorname{arcth} F)_{\alpha\bar{\beta}} = 0}$$

  solves the BI equations of motion $\rightarrow$ Deformation of DUY stability condition

$$g^{\alpha\bar{\beta}} F_{\alpha\bar{\beta}} = 0$$

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$$\boxed{[D, D]F = [F, F]}$$

	$\alpha' = 0$ $\partial F = 0$	$\alpha' \neq 0$ $\partial F = 0$	$\alpha' \neq 0$ $\partial F \neq 0$
$[,] = 0$	Maxwell	Born-Infeld	!
$[,] \neq 0$	Yang-Mills	up to 4th order in α'	

$\rightarrow$ look at derivative corrections to BI

BI action + derivatives?

Second motivation

How to compute derivative corrections efficiently?

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Simple $N = 2$ superspace description of space-filling B brane in flat space with holomorphic line bundle connection

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Derivative corrections from β-function calculation in $N = 2$ superspace using supergraph techniques

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N=1 superspace

- Introduce a boundary → from $N = (1,1)$ to $N = 1$ [Koerber, Nevens, Sevrin '03]

$$\boxed{\begin{array}{ll} D = D_+ + D_- & \text{unbroken} \\ D' = D_+ - D_- & \text{broken} \end{array}}$$

$$D^2 = D'^2 = -\frac{i}{2}\partial_\tau\,, \quad \{D, D'\} = -i\partial_\sigma$$

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- $N = 1$ action

$$\mathcal{S}_{N=1} = -\int d^2\sigma\, D\left[D'\left(D_+X^a D_-X^b(g_{ab} + b_{ab})\right)\right]$$

Equivalent to bulk action up to boundary term:
$D_+D_- = -1/2\, DD' - i/2\, \partial_\sigma$

N=1 superspace: boundary conditions

- Boundary conditions
  - Dirichlet: $\delta X^a = 0$
  - Neumann: $D' X^a = b^a{}_b D X^b$

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  - Dirichlet: $\delta X^a = 0$
  - Neumann: $D' X^a = b^a{}_b D X^b$
- Mixed conditions: introduce projection operators

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm \mathcal{R})\,, \quad \mathcal{R}^2 = 1$$

$\mathcal{P}_+$: Neumann, $\mathcal{P}_-$: Dirichlet

$$\mathcal{P}_- \partial_\tau X = \mathcal{P}_- D X = 0 \rightarrow \mathcal{P}^d_{+b} \mathcal{P}^e_{+c} \mathcal{R}^a{}_{[d,e]} = 0$$

$\rightarrow$ $\mathcal{P}_+$ is integrable
$\rightarrow$ Brane wraps integrable submanifold

N=2 superspace

- From $N = (2,2)$ to $N = 2$: two choices

B-type:

$$D = D_+ + D_-, \qquad \hat{D} = \hat{D}_+ + \hat{D}_-,$$
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- But chiral with A-type = twisted chiral with B-type

N=2 superspace: Action

$N = 2$ action

$$\mathcal{S}_{N=2} = \int d^2\sigma\, D\hat{D}\left[D'\hat{D}'\, V(X,\bar{X})\right] + i\int d\tau\, D\hat{D} W(X,\bar{X})$$

with

$$D^2 = \hat{D}^2 = D'^2 = \hat{D}'^2 = -\frac{i}{2}\partial_\tau,$$
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Chiral with B-type	$=$	type B branes
$\updownarrow$	mirror symmetry	$\updownarrow$
Twisted chiral with B-type	$=$	type A branes

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Formalism

- Twisted chiral fields with B-type boundary

$$\begin{array}{c|c} \hat{D}X^{\mu} = iD'X^{\mu} & \hat{D}X^{\bar{\mu}} = -iD'X^{\bar{\mu}} \\ \hat{D}'X^{\mu} = iDX^{\mu} & \hat{D}'X^{\bar{\mu}} = -iDX^{\bar{\mu}}, \end{array}$$

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- Impose Dirichlet conditions

$$(\mathcal{P}_{-}\delta X)^{\mu} = 0 \rightarrow \delta X^{\mu} = \mathcal{R}^{\mu}{}_{\bar{\nu}}\,\delta X^{\bar{\nu}} + \mathcal{R}^{\mu}{}_{\nu}\,\delta X^{\nu}$$

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- We find additional projection operators $\pi_{\pm} : T_{\mathcal{M}}^{(1,0)} \to T_{\mathcal{M}}^{(1,0)}$
- Neumann conditions

$$\begin{aligned} (\pi_{+}\mathcal{P}_{+}D'X)^{\mu} &= \mathcal{R}^{\mu}{}_{\nu}D'X^{\nu} \\ (\pi_{-}\mathcal{P}_{+}D'X)^{\mu} &= 0 \end{aligned}$$

$\to$ non-degenerate F along π_{+} and $F = 0$ along π_{-}

Coisotropic branes

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  $\rightarrow \mathrm{Im}\pi_+$ is $4m$-dimensional, $m \in \mathbb{N}$
- In general this thus describes an $(n+2m)$-brane where $d = 2n$

Geometric interpretation

- If σ^a are coordinates along the brane $\mathcal{N}$ in π_- directions

$$T_{\mathcal{N}}^{\perp} = \{\partial/\partial\sigma^a\} \subset T_{\mathcal{N}} = \{\partial/\partial\sigma^a\} \oplus \mathrm{Im}\pi_+$$

$$\downarrow$$

Brane wraps coisotropic submanifold

[Kapustin, Orlov '01]
[Lindström, Zabzine '02]

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- Pullback of ω, F and $K = \omega^{-1}F$ only non-vanishing and non-degenerate on $T_{\mathcal{N}}/T_{\mathcal{N}}^{\perp} = \mathrm{Im}\pi_+$

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Brane wraps coisotropic submanifold

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- When $\pi_- = 1$, we find $T_{\mathcal{N}}^{\perp} = T_{\mathcal{N}} \rightarrow$ brane wraps lagrangian submanifold with flat connection

[Kapustin, Orlov '01]
[Lindström, Zabzine '02]

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- Lagrangian brane of any shape on T^2
  $\longrightarrow$ W determines the shape of the brane

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  $\rightarrow W$ determines the shape of the brane
- Maximally coisotropic brane on T^4 and 4d hyperkähler manifold $\mathcal{K}$ with $V \equiv V(z - \bar{z}, w + \bar{w})$

  In both cases:

$$F_{zw} = i$$

$$W = \frac{i}{2}(zV_z + wV_w - \bar{z}V_{\bar{z}} - \bar{w}V_{\bar{w}})$$

In general: possible corrections from higher derivatives on V

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  In general: possible corrections from higher derivatives on V
- 5-brane on T^6 or $T^2 \times \mathcal{K}$

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  - Carries holomorphic flux: $F_{\alpha\bar{\beta}} = -iW_{\alpha\bar{\beta}}\,, \quad F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0$

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- Application: space filling brane in flat space

$$D'X^{\alpha} = F^{\alpha}_{\ \beta} DX^{\beta}$$

$\longrightarrow$ study stability by computing β-function

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N=(2,2) supersymmetry	$\longrightarrow$	Kähler manifold
One-loop conformal invariance	$\longrightarrow$	Ricci flat
Four-loop conformal invariance	$\longrightarrow$	R^4 term

Conformal invariance

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N=(2,2) supersymmetry	→	Kähler manifold
One-loop conformal invariance	→	Ricci flat
Four-loop conformal invariance	→	R^4 term

- Four-loop calculation is greatly simplified by using $N = (2,2)$ superspace techniques [Grisaru, van de Ven, Zanon '86]

Deformed stability condition from β-function

- Open strings on flat space-filling B brane coupled to boundary potential W

[Nevens, Sevrin, Troost, AW '06]

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- At one loop: $\beta(W) \propto g^{\alpha\bar{\beta}}(\text{arcth}\, F)_{\alpha\bar{\beta}} \longrightarrow$ BI action

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- Two loops: no contribution to the β-function $\rightarrow$ no two-derivative corrections
- Three loops: four-derivative correction to the BI action
- All results in complete agreement with renormalization group equations

[Nevens, Sevrin, Troost, AW '06]

Result: BI + derivative corrections

Abelian effective action up to 4 derivatives [Wyllard '01]

$$\mathcal{S} = -\tau_9 \int d^{10}x \sqrt{-h_+}\Big[1 + \frac{1}{96}\Big(\frac{1}{2}h_+^{\mu\nu}h_+^{\rho\sigma}S_{\nu\rho}S_{\sigma\mu} - h_+^{\rho_2\mu_1}h_+^{\mu_2\rho_1}h_+^{\sigma_2\nu_1}h_+^{\nu_2\sigma_1}S_{\mu_1\mu_2\nu_1\nu_2}S_{\rho_1\rho_2\sigma_1\sigma_2}\Big)\Big]$$

With stability condition [Koerber '04]

$$g^{\alpha\bar\beta}\left(\operatorname{arcth} F\right)_{\alpha\bar\beta} + \tfrac{1}{96}S_{ab\alpha\bar\beta}S_{cd\gamma\bar\delta}\, h_+^{bc}h_+^{da}\left(h_+^{\alpha\bar\delta}h_+^{\gamma\bar\beta} - h_-^{\alpha\bar\delta}h_-^{\gamma\bar\beta}\right) = 0$$

where $S_{abcd} = \partial_a\partial_b F_{cd} + 2h_+^{ef}\partial_a F_{[c|e}\,\partial_b F_{|d]f}\,, \quad h^{\pm}_{\alpha\bar\beta} = \eta_{\alpha\bar\beta} \pm F_{\alpha\bar\beta}$

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Duality transformations

- Closed strings
    - In the presence of an isometry: chiral field dualized to twisted chiral field
    - Explicit transformation by gauging the isometry and passing through a first order action

Duality transformations

- Closed strings
  - In the presence of an isometry: chiral field dualized to twisted chiral field
  - Explicit transformation by gauging the isometry and passing through a first order action
- Open strings
  - Analogous to closed string case, but boundary conditions of first order action should be treated with care
  - Example 1: space filling B brane on $d = 2n$-dimensional Kähler manifold parameterized by n chiral fields is dual to n-dimensional Lagrangian A brane on dual Kähler manifold parameterized by n twisted chiral fields
  - Example 2: One of the two isometries of maximal coisotropic brane on hyperkähler manifold can be dualized $\rightarrow$ 3-brane on generalized Kähler manifold

Outline

1. Motivation
   - Nonlinear σ-models in superspace
   - D-brane effective actions
2. Boundary superspace
   - $N = 1$
   - $N = 2$
3. Type A branes
   - Formalism
   - Coisotropic branes
4. Type B branes
   - Formalism
   - Derivative corrections
5. Duality transformations
6. Remarks

Conclusion and outlook

Conclusions

- $N = 2$ superspace description of A and B branes on Kähler manifolds
- Explicit examples of coisotropic branes
- Duality transformations
- Application: D-brane effective action

Conclusion and outlook

Conclusions

- $N = 2$ superspace description of A and B branes on Kähler manifolds
- Explicit examples of coisotropic branes
- Duality transformations
- Application: D-brane effective action

Outlook

- Geometries parameterized by chiral + twisted chiral (+ semi-chiral) $\rightarrow$ branes on generalized complex geometries
- Conformal invariance $\rightarrow$ stability conditions [Marino, Minasian, Moore, Strominger '00; Kapustin, Li '03; Koerber '05]