Is a point-like ultraviolet finite theory of quantum gravity possible?

Valencia, Oct. 5, 2007

Zvi Bern, UCLA

Lecture 2

Lecture 1: Scattering amplitudes in quantum field theories. On-shell methods, unitarity and twistors.

Lecture 2: Ultraviolet properties of quantum gravity theories.

Based on following :

ZB, N.E.J. Bjerrum-Bohr, D. Dunbar, hep-th/0501137
ZB, L. Dixon , R. Roiban, hep-th/0611086
ZB, J.J. Carrasco, L. Dixon, H. Johansson, D. Kosower, R. Roiban, hep-th/0702112
ZB, J.J. Carrasco, D. Forde, H. Ita and H. Johansson, arXiv:0707.1035 [hep-th]

Outline

Will present concrete evidence for perturbative UV finiteness of $N = 8$ supergravity.

- Review of conventional wisdom on UV divergences in quantum gravity –dimensionful coupling.
- Surprising one-loop cancellations: "no triangle hypothesis".
- Additional observations and hints of UV finiteness.
- Calculational method – reduce gravity to gauge theory:
  - (a) Kawai-Lewellen-Tye tree-level relations.
  - (b) Unitarity method.
- All-loop arguments for UV finiteness of $N = 8$ supergravity.
- Explicit three-loop calculation and "superfinitness".
- Origin of cancellation -- high energy behavior.

$N = 8$ Supergravity

The most supersymmetry allowed for maximum particle spin of 2 is $N = 8$. Eight times the susy of $N = 1$ theory of Ferrara, Freedman and van Nieuwenhuizen

We consider the $N = 8$ theory of Cremmer and Julia.

256 massless states

$N=8:$	1	8	28	56	70	56	28	8	1
helicity :	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
	h^-	ψ_i^-	v_{ij}^-	χ_{ijk}^-	s_{ijkl}	χ_{ijk}^+	v_{ij}^+	ψ_i^+	h^+

Reasons to focus on this theory:

- With more susy suspect better UV properties.
- High symmetry implies technical simplicity.

Finiteness of $N = 8$ Supergravity?

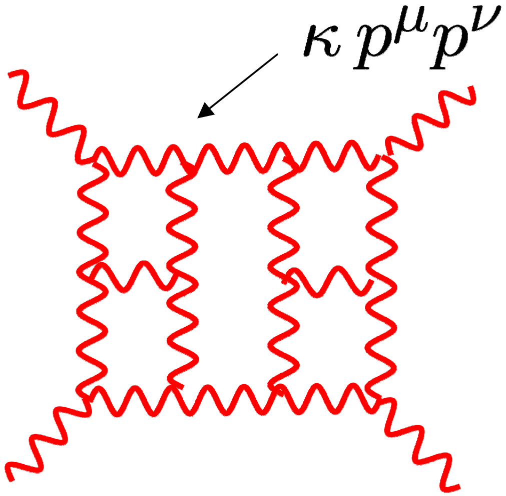
We are interested in UV finiteness of $N = 8$ supergravity because it would imply a new symmetry or non-trivial dynamical mechanism.

The discovery of either would have a fundamental impact on our understanding of gravity.

- **Here we only focus on order-by-order UV finiteness.**
- **Non-perturbative issues and viable models of Nature are not the goal for now.**

Power Counting at High Loop Orders

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV

Much more sophisticated power counting in supersymmetric theories but this is the basic idea.

Quantum Gravity at High Loop Orders

A key unsolved question is whether a finite point-like quantum gravity theory is possible.

- Gravity is non-renormalizable by power counting.

$$\kappa = \sqrt{32\pi G_N} \longleftarrow \text{Dimensionful coupling}$$

- Every loop gains $G_N \sim 1/M_{\rm Pl}^2$ mass dimension –2.

  At each loop order potential counterterm gains extra

$$R^{\mu}_{\nu\sigma\rho} \sim \; g^{\mu\kappa}\partial_\rho\partial_\nu g_{\kappa\sigma} \quad \text{or } D^2$$

- As loop order increases potential counterterms must have either more R's or more derivatives

Divergences in Gravity

One loop: Vanish on shell

$$R^2,\ R_{\mu\nu}^2,\ R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$$ vanishes by Gauss-Bonnet theorem

Pure gravity 1-loop finite (but not with matter) ‘t Hooft, Veltman (1974)

Two loop: Pure gravity counterterm has non-zero coefficient:

$$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu}R^{\mu\nu}{}_{\sigma\tau}R^{\sigma\tau}{}_{\lambda\rho}$$

Any supergravity: Goroff, Sagnotti (1986); van de Ven (1992)

R^3 is ***not*** a valid supersymmetric counterterm.

Produces a helicity amplitude $(-,+,+,+)$ forbidden by susy.

Grisaru (1977); Tomboulis (1977)

The first divergence in *any* supergravity theory can be no earlier than three loops.

R^4 **Bel-Robinson tensor expected counterterm**

Deser, Kay, Stelle (1977); Kaku, Townsend, van Nieuwenhuizen (1977), Ferrara, Zumino (1978)

Opinions from the 80's

If certain patterns that emerge should persist in the higher orders of perturbation theory, then … $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops.

Green, Schwarz, Brink, (1982)

Unfortunately, in the absence of further mechanisms for cancellation, the analogous $N = 8$ $D = 4$ supergravity theory would seem set to diverge at the three-loop order.

Howe, Stelle (1984)

There are no miracles… It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions. … The final word on these issues may have to await further explicit calculations.

Marcus, Sagnotti (1985)

The idea that *all* supergravity theories diverge at 3 loops has been accepted for over 20 years

Where are the $N = 8$ Divergences?

Depends on who you ask and when you ask.

Howe and Lindstrom (1981)
Green, Schwarz and Brink (1982)
Howe and Stelle (1989)
Marcus and Sagnotti (1985)

3 loops: Conventional superspace power counting.

5 loops: Partial analysis of unitarity cuts. ZB, Dixon, Dunbar, Perelstein, and Rozowsky (1998)

If harmonic superspace with $N = 6$ susy manifest exists
Howe and Stelle (2003)

6 loops: If harmonic superspace with $N = 7$ susy manifest exists
Howe and Stelle (2003)

7 loops: If a superspace with $N = 8$ susy manifest were to exist.
Grisaru and Siegel (1982)

8 loops: Explicit identification of potential susy invariant counterterm with full non-linear susy. Kallosh; Howe and Lindstrom (1981)

9 loops: Assume Berkovits' superstring non-renormalization theorems can be naively carried over to $N = 8$ supergravity.
Paper actually argues for finiteness. Green, Vanhove, Russo (2006)

Note: none of these are based on demonstrating a divergence. They are based on arguing susy protection runs out after some point. Really just dimensional analysis arguments.

Reasons to Reexamine This

1) The number of *established* counterterms in *any* supergravity theory is zero.

2) Discovery of remarkable cancellations at 1 loop – the “no-triangle hypothesis”. ZB, Dixon, Perelstein, Rozowsky
ZB, Bjerrum-Bohr and Dunbar; Bjerrum-Bohr, Dunbar, Ita, Perkins and Risager

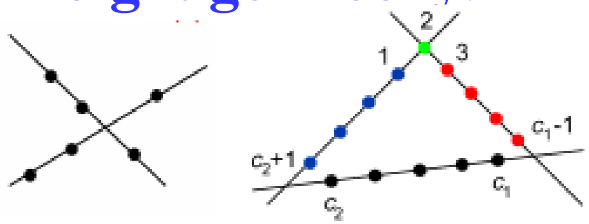
3) *Every* explicit loop calculation to date finds $N = 8$ supergravity has identical power counting as in $N = 4$ super-Yang-Mills theory, which is UV finite. Green, Schwarz and Brink; ZB, Dixon, Dunbar, Perelstein, Rozowsky; Bjerrum-Bohr, Dunbar, Ita, PerkinsRisager; ZB, Carrasco, Dixon, Johanson, Kosower, Roiban.

4) Very interesting hint from string dualities. Chalmers; Green, Vanhove, Russo
   – Dualities restrict form of effective action. May prevent divergences from appearing in $D = 4$ supergravity.
   – Difficulties with decoupling of towers of massive states.

See Vanhove’s talk

5) Gravity twistor-space structure similar to gauge theory. Derivative of delta function support

Witten; ZB, Bjerrum-Bohr, Dunbar



Gravity Feynman Rules

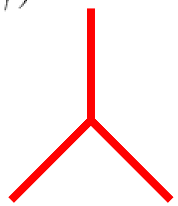
$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

Propagator in de Donder gauge:

$$P_{\mu\nu;\alpha\beta}(k) = \frac{1}{2}\left[\eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{D-2}\eta_{\mu\alpha}\eta_{\nu\beta}\right]\frac{i}{k^2 + i\epsilon}$$

Three vertex:

$$\begin{aligned}
&G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1, k_2, k_3) = \\
&\quad \mathrm{sym}[-\tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\
&\qquad + P_6(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\
&\qquad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\
&\qquad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1 \cdot k_2 \eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$



About 100 terms in three vertex

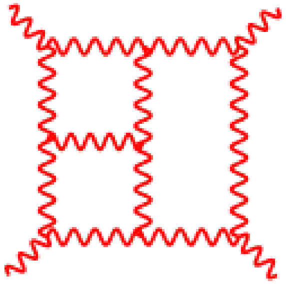
An infinite number of other messy vertices.

Naive conclusion: Gravity is a nasty mess.

Feynman Diagrams for Gravity

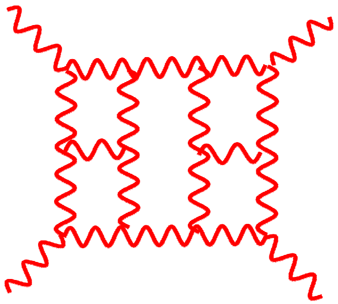
Suppose we want to put an end to the speculations by explicitly calculating to see what is true and what is false:

Suppose we wanted to check superspace claims with Feynman diagrams:



If we attack this directly get $\sim 10^{20}$ terms in diagram. There is a reason why this hasn't been evaluated.

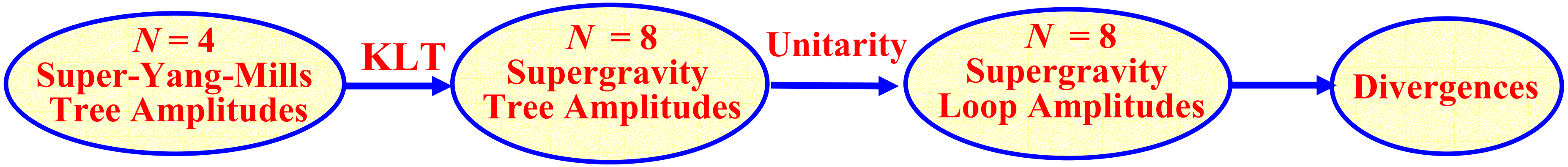
In 1998 we suggested that five loops is where the divergence is:



This single diagram has $\sim 10^{30}$ terms prior to evaluating any integrals. More terms than atoms in your brain!

Basic Strategy

ZB, Dixon, Dunbar, Perelstein and Rozowsky (1998)



- Kawai-Lewellen-Tye relations: sum of products of gauge theory tree amplitudes gives gravity tree amplitudes.
- Unitarity method: efficient formalism for perturbatively quantizing gauge and gravity theories. Loop amplitudes from tree amplitudes.

ZB, Dixon, Dunbar, Kosower (1994)

Key features of this approach:

- **Gravity calculations mapped into much simpler gauge theory calculations.**
- **Only on-shell states appear.**

KLT Relations

$$M_4^{\text{tree}}(1,2,3,4) = s_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = s_{12}s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5) + s_{13}s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5)$$

Gravity amplitude

Color stripped gauge theory amplitude

where we have stripped all coupling constants

$$\mathcal{A}_4^{\text{tree}} = g^2 \sum_{\text{non-cyclic}} \text{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4})\, A_4^{\text{tree}}(1,2,3,4)$$

Full gauge theory amplitude

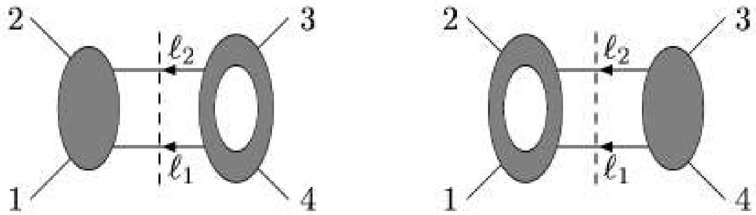
Holds for any external states.
See review: gr-qc/0206071

Progress in gauge theory can be imported into gravity theories

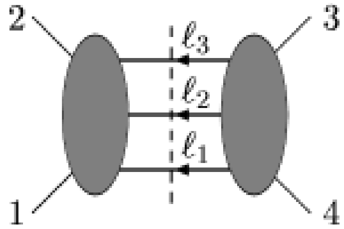
Bern, Dixon, Dunbar and Kosower

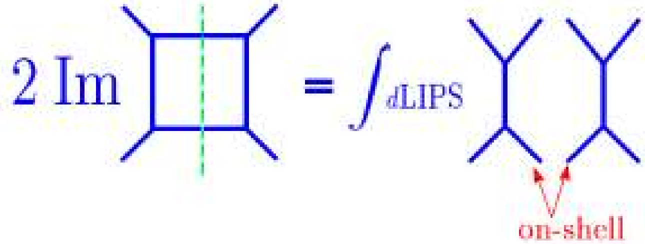
Onwards to Loops: Unitarity Method

Two-particle cut:



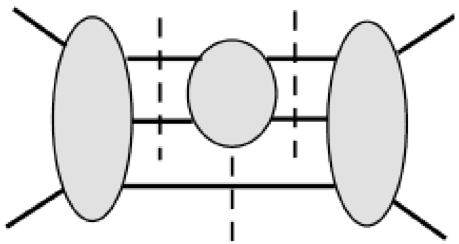
Three- particle cut:





Generalized unitarity:

Bern, Dixon and Kosower

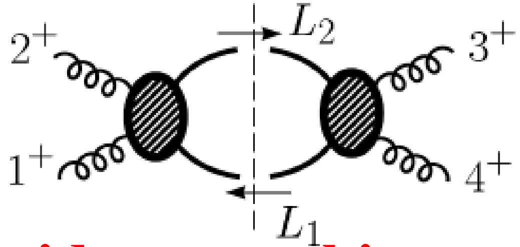


Apply decomposition of cut amplitudes in terms of product of tree amplitudes.

Onwards to Loops

KLT only valid at tree level.

To answer questions of divergences in quantum gravity we need loops.



Unitarity method provides a machinery for turning tree amplitudes into loop amplitudes.

Apply KLT to unitarity cuts:

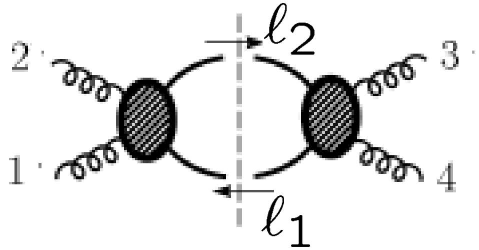
$$\sum_{\text{states}} M^{\text{tree}}_{\text{gravity}} \times M^{\text{tree}}_{\text{gravity}} \sim \left(\sum_{\text{states}} A^{\text{tree}}_{\text{gauge}} \times A^{\text{tree}}_{\text{gauge}} \right) \times \left(\sum_{\text{states}} A^{\text{tree}}_{\text{gauge}} \times A^{\text{tree}}_{\text{gauge}} \right)$$

Unitarity cuts in gravity theories can be reexpressed as sums of products of unitarity cuts in gauge theory.

Allows advances in gauge theory to be carried over to gravity.

$N = 8$ Supergravity from $N = 4$ Super-Yang-Mills

Using unitarity and KLT we express cuts of $N = 8$ supergravity amplitudes in terms of $N = 4$ amplitudes.



$$\sum_{N=8\ \text{states}} M_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\times M_4^{\text{tree}}(-\ell_2,3,4,\ell_1)$$

$$= s^2 \sum_{N=4\ \text{states}}\Big(A_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\times A_4^{\text{tree}}(\ell_2,3,4,-\ell_1)\Big)\times\sum_{N=4\ \text{states}}\Big(A_4^{\text{tree}}(\ell_2,1,2,-\ell_1)\times A_4^{\text{tree}}(\ell_1,3,4,-\ell_2)\Big)$$

Key formula for $N = 4$ Yang-Mills two-particle cuts:

$$\sum_{N=4\ \text{states}} A_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\times A_4^{\text{tree}}(-\ell_2,3,4,\ell_1) = -\frac{st\,A_4^{\text{tree}}(1,2,3,4)}{(\ell_1-k_1)^2(\ell_2-k_3)^2}$$

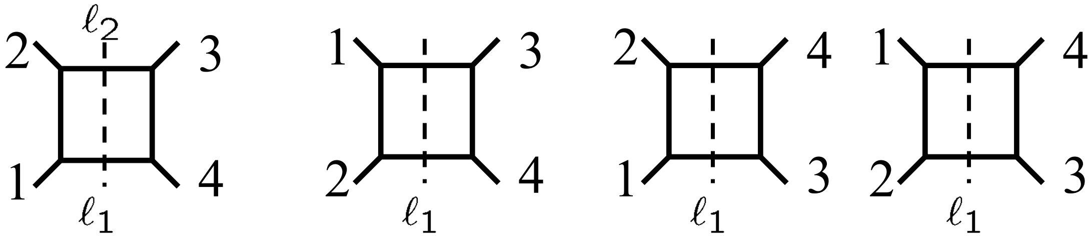
Key formula for $N = 8$ supergravity two-particle cuts:

Note recursive structure!

$$\sum_{N=8\ \text{states}} M_4^{\text{tree}}(-\ell_1,1,2,\ell_2)\times M_4^{\text{tree}}(-\ell_2,\ell_3,-\ell_4,\ell_1)$$

$$= istu\,M_4^{\text{tree}}(1,2,3,4)\left[\frac{1}{(\ell_1-k_1)^2}+\frac{1}{(\ell_1-k_2)^2}\right]\left[\frac{1}{(\ell_2-k_3)^2}+\frac{1}{(\ell_2-k_4)^2}\right]$$

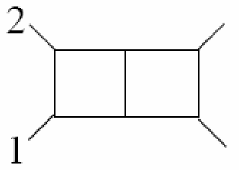
Generates all contributions with s-channel cuts.

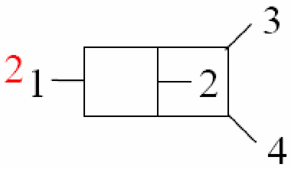


Two-Loop $N = 8$ Amplitude

Z.B., Dixon, Dunbar, Perelstein and Rozowsky

From two- and three-particle cuts we get the $N = 8$ amplitude:

$(s\,K)^2$ 

$+$ $(s\,K)^2$ 

$+$ perms

where $K = st A_4^{\rm tree}$ ⟵ **Yang-Mills tree**

$$(sK)^2 = stu M_4^{tree}$$

gravity tree

First divergence is in $D = 7$

$$\mathcal{M}_4^{\text{2-loop},\ D=7-2\epsilon}\Big|_{\rm pole} = \frac{1}{2\epsilon\,(4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2)\,stu M_4^{\rm tree}$$

Note: theory diverges at one loop in $D = 8$

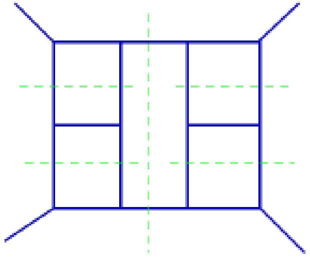
Counterterms are derivatives acting on R^4

$$t_8 t_8 R^4 \equiv t_8^{\mu_1\mu_2\cdots\mu_8}\, t_8^{\nu_1\nu_2\cdots\nu_8}\, R_{\mu_1\mu_2\nu_1\nu_2}\, R_{\mu_3\mu_4\nu_3\nu_4}\, R_{\mu_5\mu_6\nu_5\nu_6}\, R_{\mu_7\mu_8\nu_7\nu_8}$$

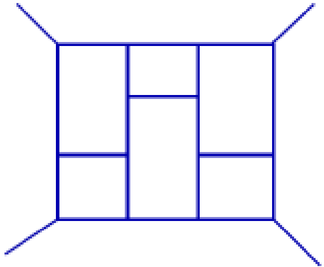
For D=5, 6 the amplitude is finite contrary to traditional superspace power counting. First indication of better behavior.

Iterated Two-Particle Cuts to All Loop Orders

ZB, Dixon, Dunbar, Perelstein, Rozowsky (1998)



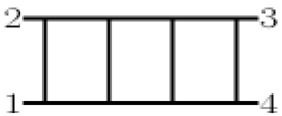
constructible from iterated 2 particle cuts

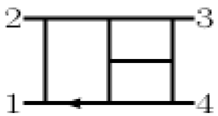


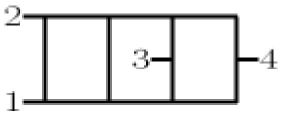
not constructible from iterated 2 particle cuts

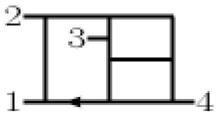
Rung rule for iterated two-particle cuts

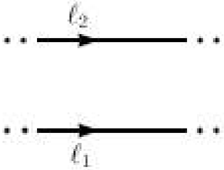
N = 4 super-Yang-Mills

s^2


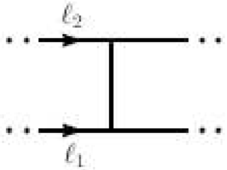
$s(\ell + k_4)^2$


s^2


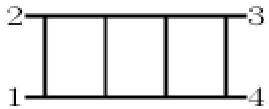
$s(\ell + k_4)^2$


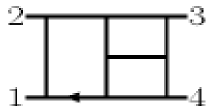


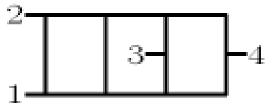
$\longrightarrow i\,(\ell_1 + \ell_2)^2 \times$

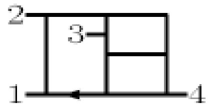


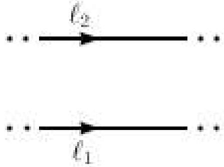
N = 8 supergravity

s^4


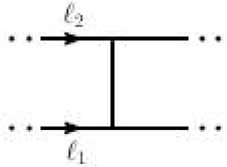
$s^2(\ell + k_4)^4$


s^4


$s^2(\ell + k_4)^4$




$\longrightarrow i\,(\ell_1 + \ell_2)^4 \times$

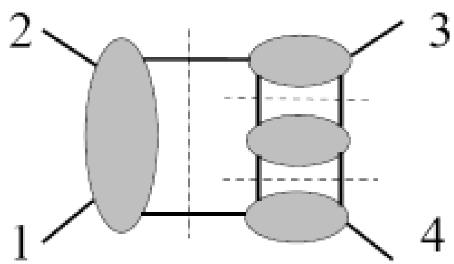


Power Counting To All Loop Orders

Z.B., Dixon, Dunbar, Perelstein and Rozowsky

From '98 paper:

- Assumed rung-rule contributions give the generic UV behavior.
- Assumed no cancellations with other uncalculated terms.



- No evidence was found that more than 12 powers of loop momenta come out of the integrals.
- This is precisely the number of loop momenta extracted from the integrals at two loops.

Elementary power counting for 12 loop momenta coming out of the integral gives finiteness condition:

$$\boxed{D < \frac{10}{L} + 2} \qquad (L > 1)$$

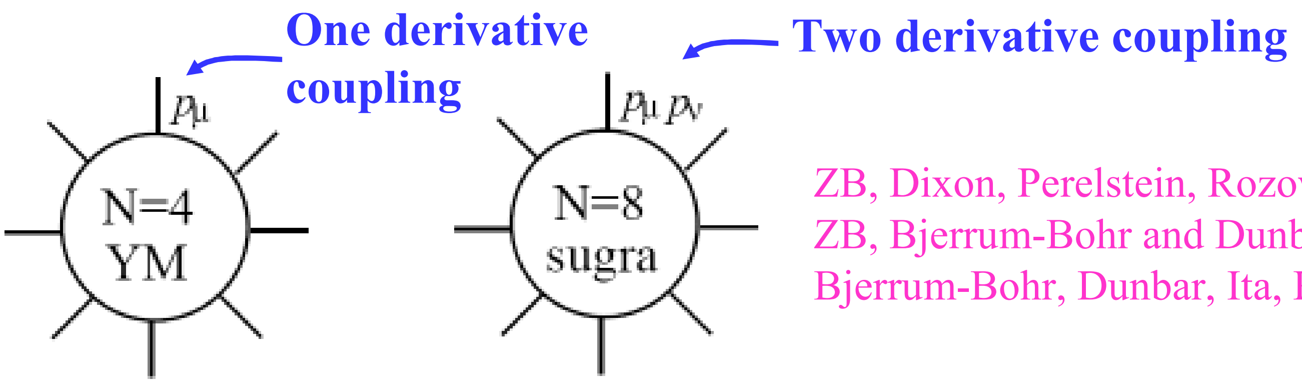
In $D = 4$ finite for $L < 5$.
L is number of loops.

$D^4 R^4$ counterterm was expected in $D = 4$, for $L = 5$

Cancellations at One Loop

Crucial hint of additional cancellation comes from one loop.

Surprising cancellations not explained by any known susy mechanism are found beyond four points



ZB, Dixon, Perelstein, Rozowsky (1998);
ZB, Bjerrum-Bohr and Dunbar (2006);
Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager (2006)

Two derivative coupling means $N = 8$ integrals should have a worse power counting relative to $N = 4$ super-Yang-Mills theory for the same integral type.

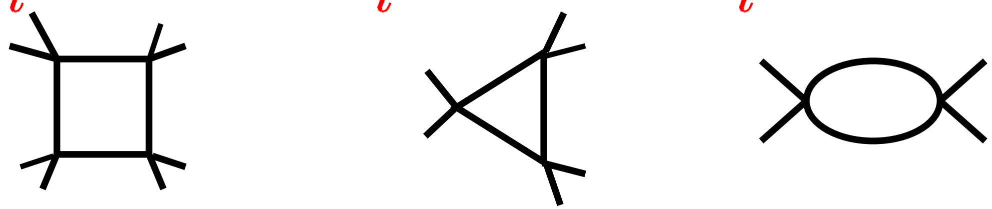
However, we have strong evidence that the UV behavior of both theories is the same at one loop.

No-Triangle Hypothesis

ZB, Bjerrum-Bohr and Dunbar (2006)
Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager (2006)

One-loop $D = 4$ theorem: Any one loop amplitude is a linear combination of scalar box, triangle and bubble integrals with rational coefficients:

$$A_n^{\text{1-loop}} = \sum_i d_i I_4^{(i)} + \sum_i c_i I_3^{(i)} + \sum_i b_i I_2^{(i)}$$

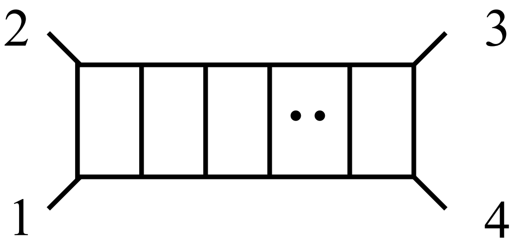


$$\int \frac{d^4p}{(p^2)^4} \qquad \int \frac{d^4p}{(p^2)^3} \qquad \int \frac{d^4p}{(p^2)^2}$$

- **In $N = 4$ Yang-Mills *only box* integrals appear. No triangle integrals and no bubble integrals.**
- **The "no-triangle hypothesis" is the statement that same holds in $N = 8$ supergravity. Explict calculations plus factorization arguments give strong support.**

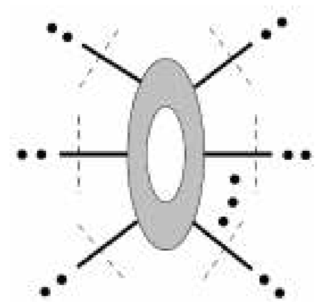
L-Loop Observation

ZB, Dixon, Roiban

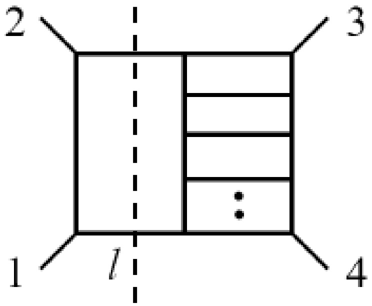


$$[(k_1 + k_2)^2]^{2(L-2)}$$

numerator factor



From 2 particle cut:



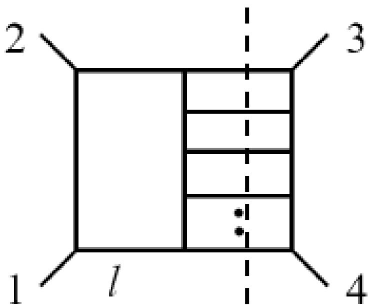
1 in $\mathcal{N} = 4$ YM

$$[(l + k_4)^2]^{2(L-2)}$$

numerator factor

Using generalized unitarity and no-triangle hypothesis *all* one-loop subamplitudes should have power counting of $\mathcal{N} = 4$ Yang-Mills

From *L*-particle cut:

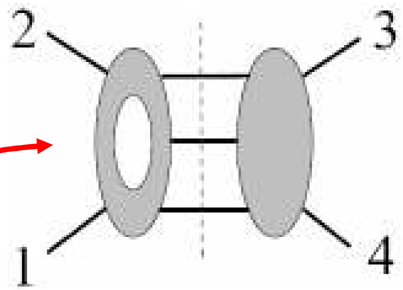


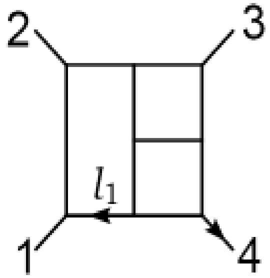
Above numerator violates no-triangle hypothesis. Too many powers of loop momentum.

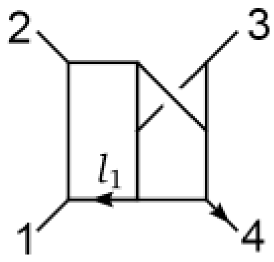
There must be additional cancellation with other contributions!

$\mathcal{N} = 8$ All Orders Cancellations

5-point
1-loop
known
explicitly

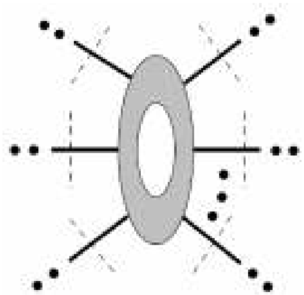






$$[(l_1 + k_4)^2]^2$$

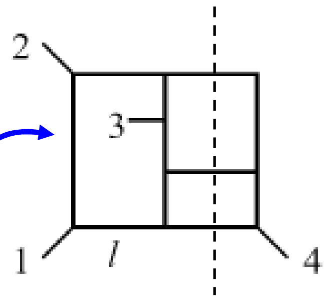
must have cancellations between
planar and non-planar



Using generalized unitarity and no-triangle hypothesis
any one-loop subamplitude should have power counting of
$\mathcal{N} = 4$ Yang-Mills

But contributions with bad overall power counting yet no violation of no-triangle hypothesis might be possible.

One-loop
hexagon
OK



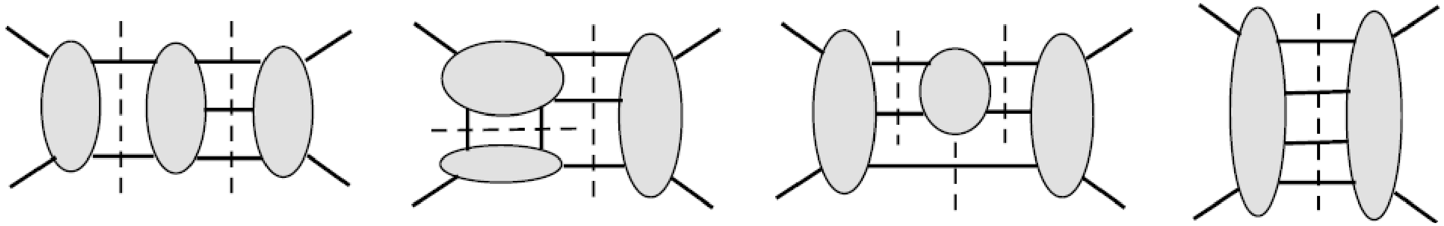
$$(l + k_4)^4$$

Total contribution is
worse than for $\mathcal{N} = 4$
Yang-Mills.

Full Three-Loop Calculation

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban

Besides iterated two-particle cuts need following cuts:



For first cut have:

reduces everything to product of tree amplitudes

$$\sum_{N=8\ \text{states}} M_4^{\text{tree}}(1,2,l_3,l_1) \times M_5^{\text{tree}}(-l_1,-l_3,q_3,q_2,q_1) \times M_5^{\text{tree}}(3,4,-q_1,-q_2,-q_3)$$

Use KLT

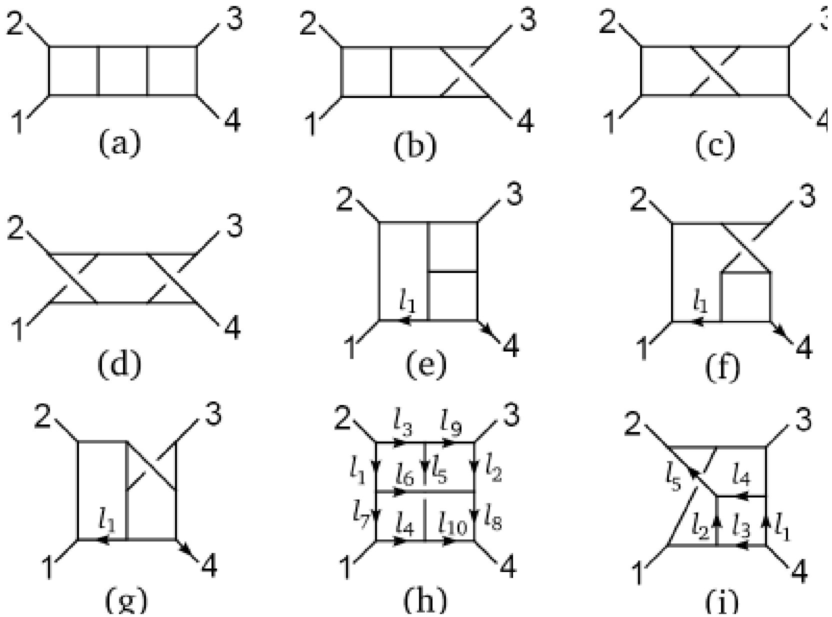
$$M_4^{\text{tree}}(1,2,l_3,l_1) = -is_{12}A_4^{\text{tree}}(1,2,l_3,l_1)A_4^{\text{tree}}(2,1,l_3,l_1)$$

$$M_5^{\text{tree}}(-l_1,-l_3,q_3,q_2,q_1) = i\, s_{l_1q_1} s_{l_3q_3} A_5^{\text{tree}}(-l_1,-l_3,q_3,q_2,q_1)\, A_5^{\text{tree}}(-l_1,q_1,q_3,-l_3,q_2) + \{l_1 \leftrightarrow l_3\}\,,$$

supergravity **super-Yang-Mills**

$N = 8$ supergravity cuts are sums of products of $N = 4$ super-Yang-Mills cuts

Complete three loop result



ZB, Carrasco, Dixon, Johansson, Kosower, Roiban; hep-th/0702112

All obtainable from rung rule, except (h), (i) which are new.

$$l_{i,j}^2 = (l_i + l_j)^2$$

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$sl_{1,2}^2 + tl_{3,4}^2 - sl_5^2 - tl_6^2 - st$	$(sl_{1,2}^2 + tl_{3,4}^2 - st)^2 - s^2(2(l_{1,2}^2 - t) + l_5^2)l_5^2 - t^2(2(l_{3,4}^2 - s) + l_6^2)l_6^2$ $- s^2(2l_1^2 l_8^2 + 2l_2^2 l_7^2 + l_1^2 l_7^2 + l_2^2 l_8^2) - t^2(2l_3^2 l_{10}^2 + 2l_4^2 l_9^2 + l_3^2 l_9^2 + l_4^2 l_{10}^2) + 2st\,l_5^2 l_6^2$
(i)	$sl_{1,2}^2 - tl_{3,4}^2 - \frac{1}{3}(s-t)l_5^2$	$(sl_{1,2}^2 - tl_{3,4}^2)^2 - (s^2 l_{1,2}^2 + t^2 l_{3,4}^2 + \frac{1}{3}stu)l_5^2$

$$M_4^{(3)} = \left(\frac{\kappa}{2}\right)^8 stu M_4^{\rm tree} \sum_{S_3} \Big[I^{(\rm a)} + I^{(\rm b)} + \tfrac{1}{2} I^{(\rm c)} + \tfrac{1}{4} I^{(\rm d)}$$
$$+ 2I^{(\rm e)} + 2I^{(\rm f)} + 4I^{(\rm g)} + \tfrac{1}{2} I^{(\rm h)} + 2I^{(\rm i)} \Big]$$

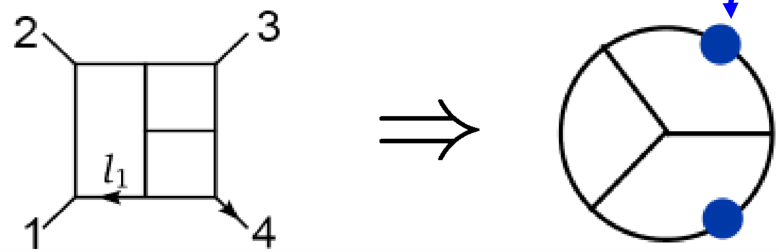
Cancellation of Leading Behavior

To check leading UV behavior we can expand in external momenta keeping only leading term.

Get vacuum type diagrams:

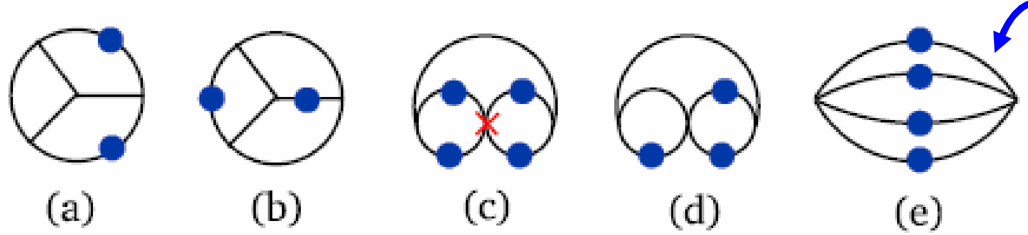
Doubled propagator

$$(l_1 + k_4)^4$$



Violates NTH

Does not violate NTH but bad power counting



After combining contributions:

The leading UV behavior cancels!!

Finiteness Conditions

Through $L = 3$ loops the correct finiteness condition is ($L > 1$):

"superfinite" in $D = 4$

$$\boxed{D < \frac{6}{L} + 4}$$

same as $N = 4$ super-Yang-Mills

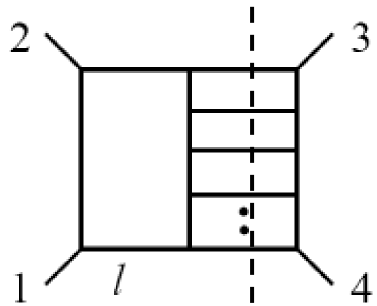
***not* the weaker result from iterated two-particle cuts:**

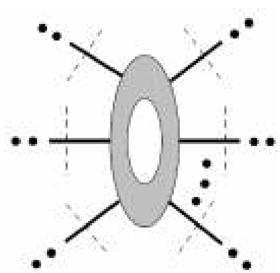
finite in $D = 4$ for $L = 3,4$

$$D < \frac{10}{L} + 2$$

(old prediction)

Beyond $L = 3$, as already explained, from special cuts we have strong evidence that the cancellations continue.





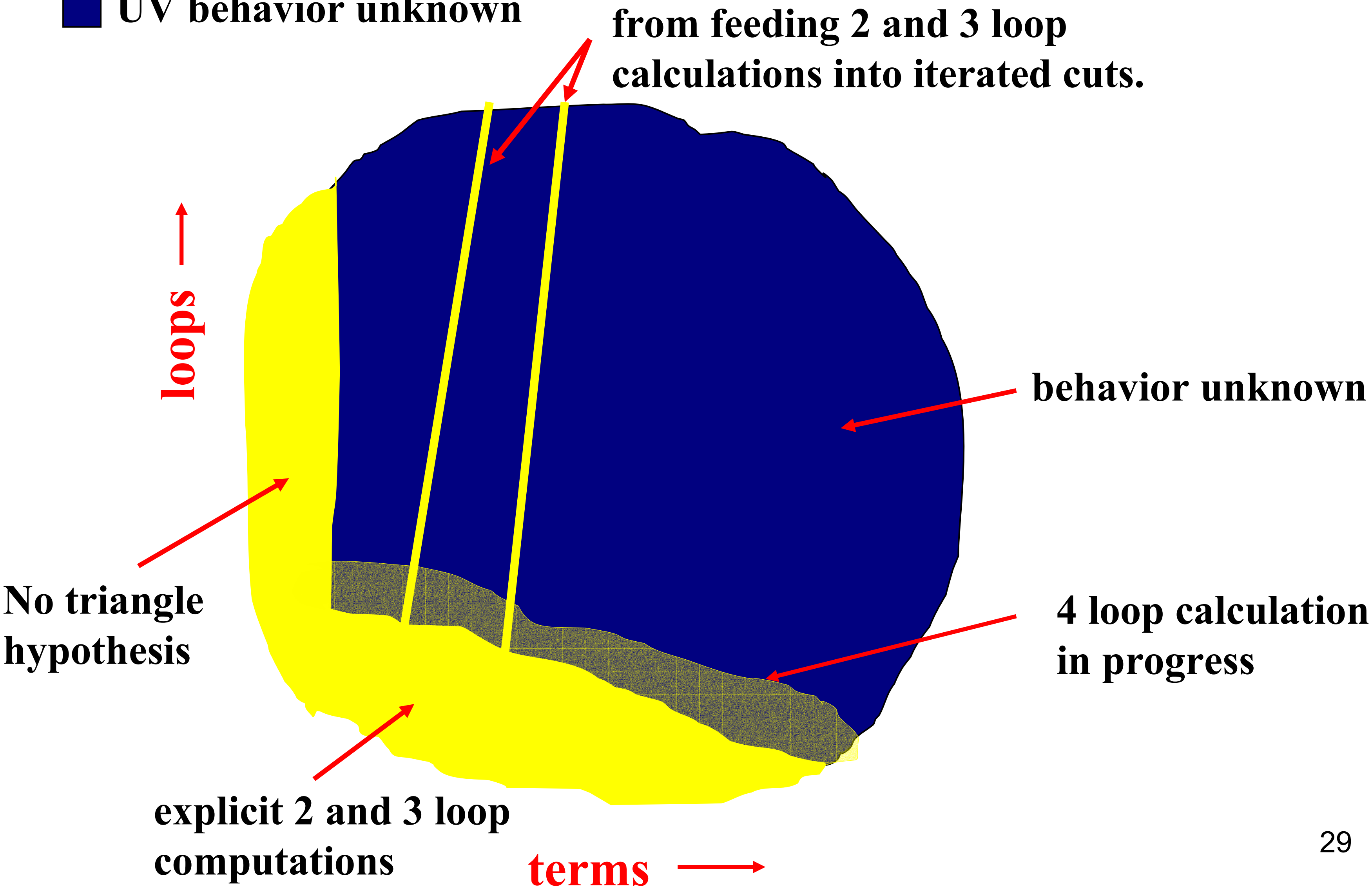
All one-loop subdiagrams should have same UV power-counting as $N = 4$ super-Yang-Mills theory.

No known susy argument explains these cancellations

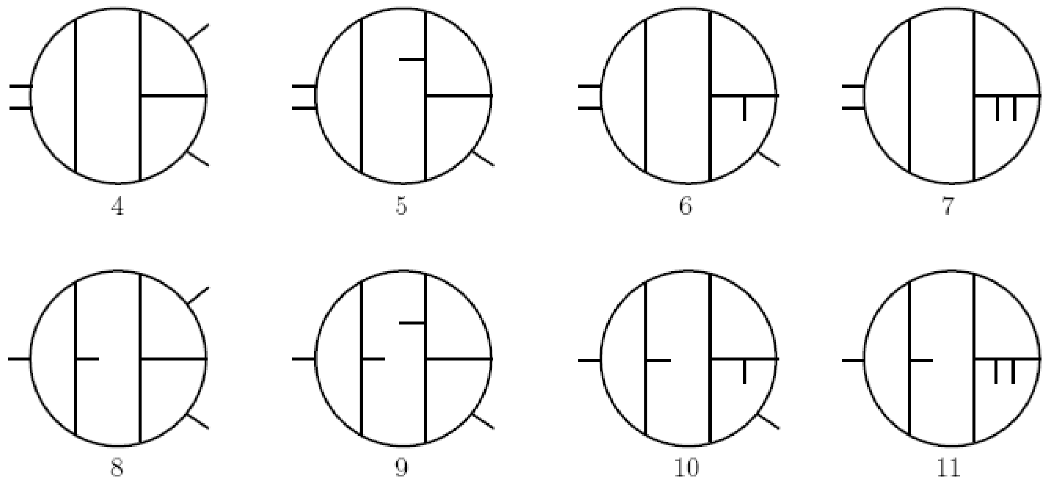
Schematic Illustration of Status

Same power count as $\mathcal{N}$=4 super-Yang-Mills

UV behavior unknown



Four-Loop Calculation in Progress



ZB, Carrasco, Dixon, Johansson, Kosower, Roiban

50 distinct planar and non-planar diagrammatic topologies

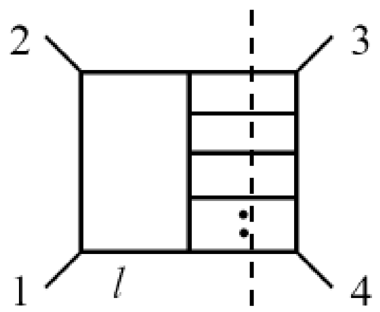
Four-loops will teach us a lot:

1. **Direct challenge to potential superspace explanations.**
2. **Study of cancellations may lead to all orders proof.**
3. **Need 16 not 14 powers of loop momenta to come out of integrals to get power counting of $\mathcal{N} = 4$ sYM**

Origin of Cancellations?

There does not appear to be a supersymmetry explanation for observed cancellations, especially ones due to no-triangle hypothesis.

If it is *not* supersymmetry what might it be?



Tree Cancellations in Pure Gravity

Unitarity method implies all loop cancellations come from tree amplitudes. Can we find tree cancellations?

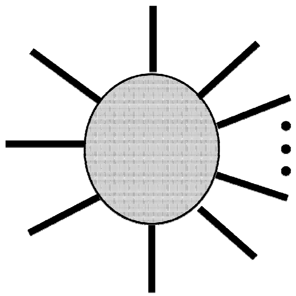
You don't need to look far: proof of BCFW tree-level on-shell recursion relations in gravity relies on the existence such cancellations!

Susy not required

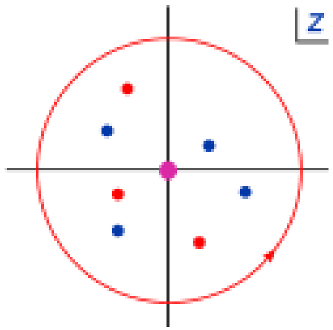
Britto, Cachazo, Feng and Witten;
Bedford, Brandhuber, Spence and Travaglini
Cachazo and Svrcek; Benincasa, Boucher-Veronneau and Cachazo

Consider the shifted tree amplitude:

$$k_1^\mu \to k_1^\mu + \frac{z}{2}\langle k_1^- | \gamma^\mu | k_2^- \rangle, \qquad k_2^\mu \to k_2^\mu - \frac{z}{2}\langle k_1^- | \gamma^\mu | k_2^- \rangle,$$



How does $M(z)$ behave as $z \to \infty$?

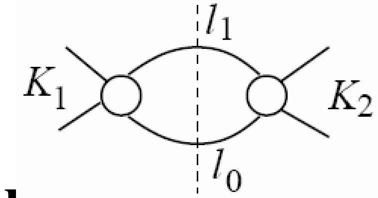


Proof of BCFW recursion requires $M(z) \to 0$

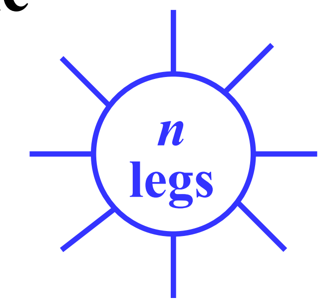
Loop Cancellations in Pure Gravity

ZB, Carrasco, Forde, Ita, Johansson

Powerful new one-loop integration method due to Forde makes it much easier to track the cancellations. Allows us to link one-loop cancellations to tree-level cancellations.



Observation: Most of the one-loop cancellations observed in $N = 8$ supergravity leading to "no-triangle hypothesis" are already present in non-supersymmetric gravity. Susy cancellations are on top of these.



$$(l^\mu)^{2n} \longrightarrow (l^\mu)^{n+4} \times (l^\mu)^{-8}$$

Maximum powers of Loop momenta

Cancellation generic to Einstein gravity

Cancellation from $N = 8$ susy

Proposal: This continues to higher loops, so that most of the observed $N = 8$ multi-loop cancellations are *not* due to susy but in fact are generic to gravity theories!

What needs to be done?

- $\mathcal{N} = 8$ **four-loop computation.** Can we demonstrate that four-loop $\mathcal{N} = 8$ amplitude has the same UV power counting as $\mathcal{N} = 4$ super-Yang-Mills? In progress.

- Can we construct a proof of perturbative UV finiteness of $\mathcal{N} = 8$? Perhaps possible using unitarity method – formalism is recursive.

- Investigate higher-loop non-susy gravity power counting to study cancellations. (It does diverge.) Goroff and Sagnotti; van de Ven

- Twistor structure of gravity loop amplitudes? ZB, Bjerrum-Bohr, Dunbar

- Link to a twistor string description of $\mathcal{N} = 8$? Abou-Zeid, Hull, Mason

- Can we find other examples with less susy that may be finite?

Guess: $\mathcal{N} = 6$ supergravity theories will be perturbatively finite.

Summary

- **Gravity ~ (gauge theory) x (gauge theory) at tree level.**
- **Unitarity method gives us means of applying this to loop calculations. Extremely Efficient way to calculate.**
- ***N* = 8 supergravity has cancellations which no known supersymmetry argument explains.**
  - **One-loop "no-triangle hypothesis" – one-loop cancellations.**
  - **No-triangle hypothesis implies cancellations strong enough for finiteness to *all* loop orders, in a class of terms. No known susy explanation.**
  - **At four points three loops, *established* that cancellations are complete and *N* = 8 supergravity has the same power counting as *N* = 4 Yang-Mills.**

***N* = 8 supergravity may well be the first example of a point-like perturbatively UV finite theory of gravity.**

Proof remains a challenge.