BLACK HOLES, QUBITS AND THE FANO PLANE

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M. J. DUFF

DUFF hep-th/0601134

0602160

KALLOSH & LINDE 0602061

LEVAY 0603136

0610314

0708.2799 [hep-th]

DUFF & FERRARA

quant-ph/0609227

hep-th/0612036

0704.0507 [hep-th]

BLACK HOLES & QUBITS

TWO DIFFERENT AREAS OF THEORETICAL PHYSICS SHARING THE SAME MATHEMATICS:

1) BLACK HOLE ENTROPY IN STRING THEORY

2) QUBIT ENTANGLEMENT IN QUANTUM INFORMATION THEORY:

CAYLEY'S HYPERDETERMINANT : $[SL(2)]^3$

$\Rightarrow$ BOTH TRIPARTITE ENTANGLEMENT MEASURE AND $N=2$ BLACK HOLE ENTROPY !

CARTAN'S QUARTIC INVARIANT : E_7

$\Rightarrow$ BOTH $N=8$ BLACK HOLE ENTROPY AND MEASURE OF TRIPARTITE ENTANGLEMENT OF SEVEN QUBITS :

$\Rightarrow$ THE FANO PLANE & OCTONIONS

LECTURES

I QUBIT ENTANGLEMENT AND CAYLEY'S HYPERDETERMINANT

II BLACK HOLE ENTROPY AND THE QUBIT CORRESPONDENCE

III E_7 AND THE TRIPARTITE ENTANGLEMENT OF 7 QUBITS

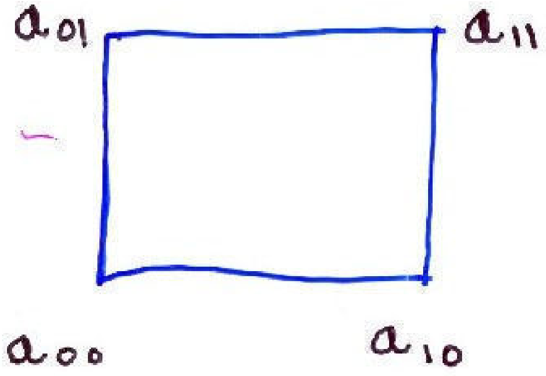
E_6 AND THE BIPARTITE ENTANGLEMENT OF 3 QUTRITS

IV CAYLEY'S HYPERDETERMINANT AND HIDDEN SYMMETRIES OF THE NAMBU-GOTO STRING

LECTURE I

QUBIT ENTANGLEMENT AND CAYLEY'S HYPERDETERMINANT

DETERMINANT



$$\det a = \frac{1}{2} \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'} a_{BB'}$$

$\varepsilon^{01} = 1$

$\varepsilon^{10} = -1$

$$= a_{00} a_{11} - a_{01} a_{10}$$

VANISHES IFF

$$a_{AA'} p^{A} = 0$$

HAS NONTRIVIAL SOLUTION FOR p^{A}

$\det a$ INARIANT UNDER $SL(2) \times SL(2)$

AND UNDER INTERCHANGE OF A & A'

SL(2)

$$\det a = \frac{1}{2} \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'} a_{BB'}$$

INVARIANT UNDER

$$a_{AA'} \to \Omega_A{}^B a_{BA'}$$

PROVIDED

$$\varepsilon^{AB} \Omega_A{}^C \Omega_B{}^D = \varepsilon^{CD}$$

Ω'S ARE 2x2 MATRICES WITH UNIT DETERMINANT

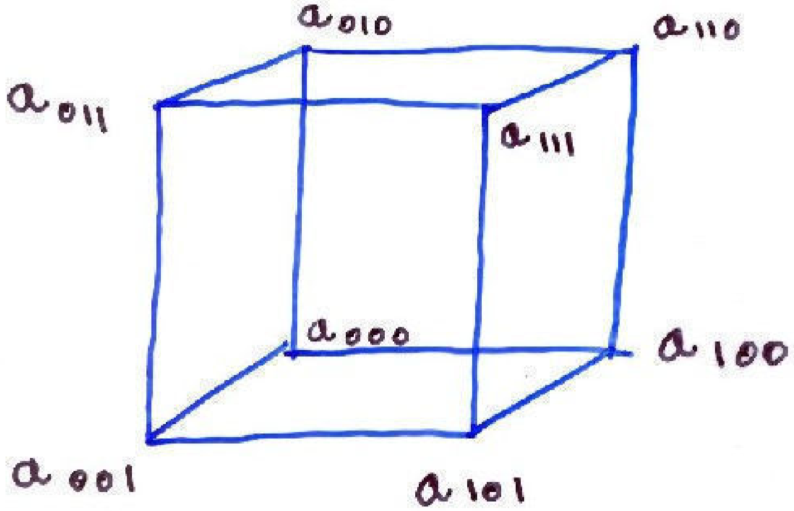
$$\Omega = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad ad - bc = 1$$

HENCE SL(2, C) abcd COMPLEX
SL(2, R) REAL
SL(2, Z) INTEGERS

* SAME FOR PRIMED INDICES SO SYMMETRY IS SL(2) x SL(2)

HYPERDETERMINANT

CAYLEY 1845



"HYPERMATRIX"

$a_{AA'A''}$

$$\mathrm{Det}\, a = -\frac{1}{2} \varepsilon^{AB} \varepsilon^{A'B'} \varepsilon^{CD} \varepsilon^{C'D'} \varepsilon^{A''D''} \varepsilon^{B''C''}$$

$$\times \; a_{AA'A''} \, a_{BB'B''} \, a_{CC'C''} \, a_{DD'D''}$$

$$= a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2$$

$$-2\big(a_{000} a_{001} a_{110} a_{111} + a_{000} a_{010} a_{101} a_{111}$$

$$+ a_{000} a_{100} a_{011} a_{111} + a_{001} a_{010} a_{101} a_{110}$$

$$+ a_{001} a_{100} a_{011} a_{110} + a_{010} a_{100} a_{011} a_{101}\big)$$

$$+4\big(a_{000} a_{011} a_{101} a_{110} + a_{001} a_{010} a_{100} a_{111}\big)$$

Det a VANISHES IFF

$$a_{AA'A''}\, p^{A}\, q^{A'} = 0$$

$$a_{AA'A''}\, p^{A}\, r^{A''} = 0$$

$$a_{AA'A''}\, q^{A'}\, r^{A''} = 0$$

HAS NONTRIVIAL SOLUTION FOR

p^{A}, $q^{A'}$ & $r^{A''}$.

Det a INVARIANT UNDER

$$SL(2) \times SL(2) \times SL(2)$$

AND UNDER INTERCHANGE OF

A, A' & A''.

GELFAND, KAPRANOV & ZELEVINSKY
" MULTIDIMENSIONAL ~~HYPER~~ DETERMINANTS "
BIRKHAUSER 1994

OR SEE FOR EXAMPLE
V. FERNANDO (OXFORD, STRUCTURAL BIOLOGY)
" SINGULAR 2 x 2 x 2 ARRAYS "

QUBITS

2 LEVEL QUANTUM SYSTEM

(e.g SPIN UP $\uparrow$, SPIN DOWN $\downarrow$)

BASIS STATES $|0\rangle = |\uparrow\rangle$

$|1\rangle = |\downarrow\rangle$

1 QUBIT (ALICE)

$$|\Psi\rangle = a_0|0\rangle + a_1|1\rangle \quad a_A \text{ COMPLEX}$$

2 QUBITS (ALICE & BOB)

$$\begin{aligned} |\Psi\rangle &= a_{00}|00\rangle + a_{01}|01\rangle \\ &\quad + a_{10}|10\rangle + a_{11}|11\rangle \\ &= a_{AB}|AB\rangle = a_{AA'}|AA'\rangle \end{aligned}$$

A MEASURE OF THE ALICE-BOB ENTANGLEMENT IS THE "2-TANGLE"

$$\tau_{AB} = 4|\det a|^2$$

EXAMPLES

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

IS SEPARABLE $\tau_{AB} = 0$

ALICE MEASURES SPIN UP WITH PROB 1 AND BOB CAN MEASURE EITHER UP OR DOWN WITH PROB $\frac{1}{2}$

BUT $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

IS ENTANGLED $\tau_{AB} = 1$

ALICE MEASURES SPIN UP OR DOWN WITH PROB $\frac{1}{2}$, STATE COLLAPSES TO EITHER $|00\rangle$ OR $|11\rangle$

BOB'S MEASUREMENT FIXED BY ALICE'S. $\Rightarrow$ EPR "PARADOX"

RECAP

2 QUBITS (ALICE & BOB)

$$\psi = a_{AA'} |AA'\rangle$$

(2,2) UNDER $[SL(2)]^2$

$$= a_{00}|00\rangle + a_{01}|01\rangle$$

$$+ a_{10}|10\rangle + a_{11}|11\rangle$$

ARE ENTANGLED IF

$$\det a = a_{00}a_{11} - a_{01}a_{10} \neq 0$$

THE "2-TANGLE" $\det a$ IS INVARIANT UNDER

$$SL(2,C) \times SL(2,C)$$

AND UNDER INTERCHANGE

$$A \leftrightarrow A'$$

3 QUBITS (ALICE, BOB & CHARLIE)

$$|\Psi\rangle = a_{ABC}|ABC\rangle$$

$$= a_{000}|000\rangle + a_{001}|001\rangle$$

$$+ a_{010}|010\rangle + a_{011}|011\rangle$$

$$+ a_{100}|100\rangle + a_{101}|101\rangle$$

$$+ a_{110}|110\rangle + a_{111}|111\rangle$$

MEASURE OF ALICE-BOB-CHARLIE ENTANGLEMENT IS THE "3-TANGLE"

$$\tau_{ABC} = \tau_{A(BC)} - \tau_{AB} - \tau_{AC}$$

$$= \tau_{B(CA)} - \tau_{BC} - \tau_{BA}$$

$$= \tau_{C(AB)} - \tau_{CA} - \tau_{CB}$$

COFFMAN, KUNDU & WOOTERS quant-ph/9907047

SUDBERY quant-ph/0001116

HYPERDETERMINANT RELATION

$$\tau_{ABC} = 4|\mathrm{Det}\, a|$$

MIYAKE & WADATI quant-ph/0212146

LEVAY (TWISTORS) quant-ph/0403060

ENTANGLEMENT MEASURES

$$\rho_A = Tr_{BC} |\psi\rangle\langle\psi| \quad etc$$

$$\tau_{A(BC)} = 4 \det \rho_A$$

$\tau_{A(BC)} = C^2_{A(BC)}$

A-TANGLE BETWEEN ALICE & BOB-CHARLIE SYSTEM

$$\tau_{ABC} = \tau_{A(BC)} - \tau_{AB} - \tau_{AC}$$

$\tau_{AB} = C^2_{AB}$ IS 2-TANGLE BETWEEN ALICE & BOB WITHIN THE A-B-C SYSTEM.

EQUIVALENTLY

$$\tau_{ABC} = \tau_{B(CA)} - \tau_{BC} - \tau_{AB}$$

$$\tau_{ABC} = \tau_{C(AB)} - \tau_{CA} - \tau_{BC}$$

DUR, VIDAL & CIRAC quant-ph/000515

CLASSIFICATION OF 3 QUBIT STATES

	$\tau_{A(BC)}$	$\tau_{B(AC)}$	$\tau_{C(AB)}$	τ_{ABC}
A-B-C	×	×	×	×
A-BC	×	✓	✓	×
B-CA	✓	×	✓	×
C-AB	✓	✓	×	×
W	✓	✓	✓	×
GHZ	✓	✓	✓	✓

A-B-C COMPLETELY SEPARABLE

A-BC, B-CA, C-AB: BIPARTITE ENTANGLEMENT

W W $|001\rangle + |010\rangle + |100\rangle$

GHZ GREENBERGER-HORNE-ZEILINGER

$$|000\rangle + |111\rangle$$

GENUINE TRIPARTITE ENTANGLEMENT

SLOCC

IN QUANTUM INFORMATION THEORY

$$SL(2,\mathbb{C})^3$$

IS THE GROUP OF

STOCHASTIC LOCAL OPERATIONS

AND

CLASSICAL COMMUNICATION

RECAP

3 QUBITS (ALICE, BOB & CHARLIE)

$$\psi = a_{AA'A''} \, |AA'A''\rangle$$

$(2,2,2)$ UNDER $[SL(2)]^3$

ARE ENTANGLED (TRIPARTITE) IF

$$\text{Det } a \neq 0$$

WHERE Det IS CAYLEY'S HYPER-DETERMINANT.

THE "3-TANGLE" Det a IS INVARIANT UNDER

$$SL(2,C) \times SL(2,C) \times SL(2,C)$$

AND UNDER "TRIALITY"

$$A \leftrightarrow A' \leftrightarrow A''$$

SEE ALSO: BRODY, GUSTAVSSON & HUGHSTON quant-ph/0612117