LECTURE IV

HIDDEN SYMMETRIES OF THE NAMBU-GOTO STRING

3 APPLICATIONS

1) $SL(2,C)^3$

3 QUBIT QUANTUM ENTANGLEMENT

MIYAKE & WADATI quant-ph/02121146

arxiv.org

2) $SL(2,R)^3$

NAMBU-GOTO STRING

DUFF hep-th/0602160

3) $SL(2,Z)^3$ **S-T-U MODEL**

BLACK HOLE ENTROPY

DUFF hep-th/0601134

KALLOSH & LINDE hep-th/0602061

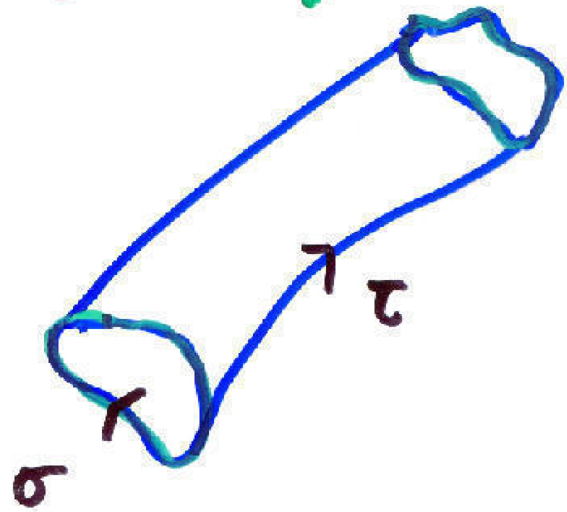
LEVAY hep-th/0603136

DUFF & FERRARA quant-ph/0609227

$\hookrightarrow E_7(C)$ 7 QUBITS!

4D BOSONIC STRING

(Nambu, Goto 1970)



WORLDSHEET COORDINATES (τ, σ)

SPACETIME COORDINATES $X^\mu(\tau, \sigma)$

$\mu = 0\ 1\ 2\ 3$

DERIVATIVES $\dot{X} = \frac{\partial X}{\partial \tau}$, $X' = \frac{\partial X}{\partial \sigma}$

INDUCED METRIC

$$\gamma_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu} \quad i = 1, 2$$

$\eta_{\mu\nu}$: diag$(-1, 1, 1, 1)$

AREA SWEPT OUT

$$\int d\sigma\, d\tau \sqrt{-\det \gamma} = \int d\sigma\, d\tau \sqrt{\dot{X}^2 X'^2 - (\dot{X} \cdot X')^2}$$

NAMBU-GOTO STRING

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ORGANIZE 8 VARIABLES OF 4D STRING $(\dot{X}^{\mu}, X'^{\mu})$ $\mu = 0,1,2,3$ INTO HYPERMATRIX $a_{AA'A''}$.

CAVEAT: REQUIRE $(2,2)$ SIGNATURE (BOSONIC SECTOR OF CRITICAL $N=2$ STRING)

ACTION:

$$I_{NG} = -\frac{T}{2}\int d\tau\, d\sigma \sqrt{-\det\gamma}$$

γ IS THE WORLDSHEET METRIC

$$\gamma_{A''B''} = \partial_{A''} X^{\mu}\, \partial_{B''} X^{\nu}\, \eta_{\mu\nu}$$

$$\eta_{\mu\nu} = \mathrm{diag}\,(-1,-1,+1,+1)$$

HYPERMATRIX

ORGANIZE 8 VARIABLES OF 4D STRING $(\dot{x}^\mu, x'^\mu)$ $\mu = 0123$ INTO HYPERMATRIX $a_{AA'A''}$

$$a_{000} = -\dot{x}^0 + \dot{x}^2$$

$$a_{010} = \dot{x}^1 - \dot{x}^3$$

$$a_{100} = -\dot{x}^1 - \dot{x}^3$$

$$a_{110} = -x'^0 + x'^3$$

$$a_{011} = x'^1 - x'^3$$

$$a_{101} = -x'^1 - x'^3$$

$$a_{111} = -x'^0 - x'^2$$

WORLDSHEET METRIC

$$\gamma_{A''B''} = \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'A''} a_{BB'B''}$$

$$\boxed{\det \gamma = - \mathrm{Det}\, a}$$

TWO-COMPONENT NOTATION

$$X^{AA'} = \frac{1}{\sqrt{2}} \begin{pmatrix} -X^0 + X^2 & X^1 - X^3 \\ -X^1 - X^3 & -X^0 - X^2 \end{pmatrix}$$

DEFINE

$$a_{AA'A''} = \begin{pmatrix} \dot{X}^{AA'} \\ X'^{AA'} \end{pmatrix}$$

$$\dot{X} = \partial X / \partial \tau \qquad X' = \partial X / \partial \sigma$$

HENCE

$$\gamma_{A''B''} = \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'A''} a_{BB'B''}$$

SO

$$\boxed{\det \gamma = - \operatorname{Det} a}$$

ALICE-BOB-CHARLIE ANALOGY: 3 WORLDSHEETS

ALICE $\quad \alpha_{AB} = \varepsilon^{A'B'} \varepsilon^{A''B''} a_{AA'A''} a_{BB'B''}$

BOB $\quad \beta_{A'B'} = \varepsilon^{A''B''} \varepsilon^{AB} a_{AA'A''} a_{BB'B''}$

CHARLIE $\quad \gamma_{A''B''} = \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'A''} a_{BB'B''}$

$$\det \alpha = \det \beta = \det \gamma = - \operatorname{Det} a$$

ALICE - BOB - CHARLIE ANALOGY

THREE DIFFERENT WORLDSHEETS WITH THREE DIFFERENT METRICS

ALICE

$$\alpha_{AB} = \varepsilon^{A'B'} \varepsilon^{A''B''} a_{AA'A''} a_{BB'B''}$$

BOB

$$\beta_{A'B'} = \varepsilon^{A''B''} \varepsilon^{AB} a_{AA'A''} a_{BB'B''}$$

CHARLIE

$$\gamma_{A''B''} = \varepsilon^{AB} \varepsilon^{A'B'} a_{AA'A''} a_{BB'B''}$$

BUT ALL EQUIVALENT

$$\boxed{\det \alpha = \det \beta = \det \gamma = - \mathrm{Det}\, a}$$

SYMMETRIES

1) $SL(2,R)^3$

ACTING ON A & A' : JUST SPACETIME

$$O(2,2) \sim SL(2,R) \times SL(2,R)$$

ACTING ON A'' : WORLDSHEET $SL(2,R)$

$$\begin{pmatrix} \dot{X}^\mu \\ X'^\mu \end{pmatrix} \to \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} \dot{X}^\mu \\ X'^\mu \end{pmatrix}$$

$$ad - bc = 1$$

2) INTERCHANGE A A' A''

INTERCHANGING A & A' : JUST $X^1 \leftrightarrow -X^1$

BUT A & A'' : NEW DISCRETE SYMMETRY OF N-G STRING !

e.g

$$\begin{bmatrix} \dot{X}^0 \\ \dot{X}^1 \\ \dot{X}^2 \\ \dot{X}^3 \\ X'^0 \\ X'^1 \\ X'^2 \\ X'^3 \end{bmatrix} \to \frac{1}{2} \begin{bmatrix} \dot{X}^0 - X'^1 - \dot{X}^2 + X'^3 \\ -X'^0 - \dot{X}^1 + X'^2 + \dot{X}^3 \\ -\dot{X}^0 - X'^1 + \dot{X}^2 + X'^3 \\ X'^0 - \dot{X}^1 - X'^2 + \dot{X}^3 \\ \dot{X}^0 + \dot{X}^1 + \dot{X}^2 + \dot{X}^3 \\ \dot{X}^0 - \dot{X}^1 + \dot{X}^2 - \dot{X}^3 \\ X'^0 - \dot{X}^1 + X'^2 - \dot{X}^3 \\ \dot{X}^0 + X'^1 + \dot{X}^2 + X'^3 \end{bmatrix}$$

RECAP

NAMBU-GOTO STRING (FIRST WRITTEN DOWN IN 1970)

IS DESCRIBED IN (2,2) SIGNATURE

BY CAYLEY'S HYPERDETERMINANT

AND HENCE HAS SYMMETRY

$SL(2,R) \times SL(2,R) \times SL(2,R)$

AND $A \leftrightarrow A' \leftrightarrow A''$ TRIALITY

$\Rightarrow$

THIS REVEALS HITHERTO

UNNOTICED DISCRETE SYMMETRY

* (2,2) PHYSICAL SIGNIFICANCE NOT OBVIOUS

BUT FORMS BOSONIC SECTOR OF

N=2 CRITICAL SUPERSTRING

E_7 GENERALIZATION?

$$I = -\frac{T}{2}\int d\tau\, d\sigma \sqrt{I_4}$$

$$E_{7(7)} \supset SO(6,6) \times SL(2)$$

$$\supset SO(2,2) \times SL(2)$$

STRING IN 28 DIMENSIONS

NOTE NO CONFLICT WITH

COLEMAN-MANDULA

E_7 ACTS ON $\dot{X}$ AND X' NOT X

c.f. "GENERALIZED SPACETIMES"

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