LECTURE II

BLACK HOLE ENTROPY AND THE BLACK HOLE / QUBIT CORRESPONDENCE

BLACK HOLE ENTROPY

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SUMMARY:

STRING COMPACTIFICATIONS TO FOUR DIMENSIONS INCLUDE THE "STU MODEL" WHOSE LOW-ENERGY LIMIT IS $N=2$ SUPERGRAVITY + 3 VECTOR MULTIPLETS

BOSONIC SECTOR: 1 GRAVITON: $g_{\mu\nu}$

4 VECTORS: A_μ

3 COMPLEX SCALARS: S, T, U

ADMITS SYMMETRY $SL(2,Z)^3$ AND INTERCHANGE OF S, T & U. ("STRING TRIALITY") DUFF LIU RAHMFELD hep-th/9508094

ADMITS EXTREMAL BLACK HOLE SOLUTIONS WITH 4 ELECTRIC + 4 MAGNETIC CHARGES, ENTROPY:

[redacted]

NOTATION

$S = S_1 + i S_2 = a + i e^{-\eta}$ AXION-DILATON

$T = T_1 + i T_2 = b + i e^{-\sigma}$ KAHLER FORM

$U = U_1 + i U_2 = c + i e^{-\rho}$ COMPLEX STRUCTURE

2 FORM POTENTIAL: $B_{\mu\nu}(x) = - B_{\nu\mu}(x)$

3 FORM FIELD STRENGTH $H_{\mu\nu\rho}$:

$$H_{\mu\nu\rho} = 3\, \partial_{[\mu} B_{\nu\rho]} - \frac{1}{2} A_{[\mu} (\varepsilon_T \times \varepsilon_U) F_{\nu\rho]}$$

$$\varepsilon_T = \varepsilon_S = \varepsilon_U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

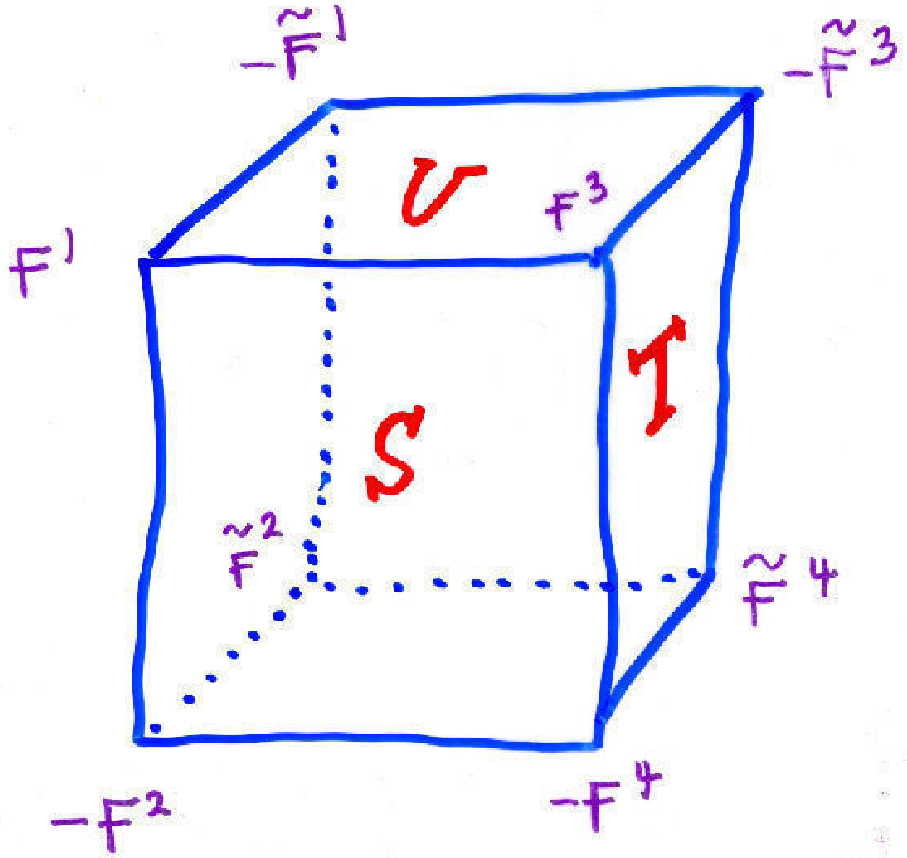
AXION $a(x)$:

$$\varepsilon^{\mu\nu\rho\sigma} \partial_\sigma a = \sqrt{g}\, e^{-\eta} H^{\mu\nu\rho}$$

4 MAXWELL FIELD STRENGTHS $F_{\mu\nu}$

DUAL $\tilde{F}_{\mu\nu}$

CUBE OF TRIALITY



T : KK electric $\leftrightarrow$ winding electric

U : CIRCLE 1 electric $\leftrightarrow$ CIRCLE 2 electric

S : KK electric $\leftrightarrow$ winding magnetic

EXPLICIT SOLUTION

PUT $q_1 = p_2 = q_3 = p_4$ FOR SIMPLICITY, OTHER CHARGES ZERO

$$ds^2 = -(\Delta_1 \Delta_2 \Delta_3 \Delta_4)^{-1} dt^2 + \Delta_1 \Delta_2 \Delta_3 \Delta_4 \, d\underline{x}^2$$

$$e^{-\eta} = \frac{\Delta_1 \Delta_3}{\Delta_2 \Delta_4} \qquad e^{-\sigma} = \frac{\Delta_1 \Delta_4}{\Delta_2 \Delta_3} \qquad e^{-\rho} = \frac{\Delta_1 \Delta_2}{\Delta_3 \Delta_4}$$

$$F^{1,3}{}_{0m} = \frac{Q \quad x^m_{1,3}}{r^3_{1,3} \, \Delta^4_{1,3}} \qquad \tilde{F}^{2,4}_{0m} = \frac{Q \quad x^m_{2,4}}{r^3_{2,4} \, \Delta^4_{2,4}}$$

$$\Delta_A = \left(1 + \frac{Q}{r_A}\right)^{1/2}$$

$$r_A = \sqrt{(x^1 - y^1_A)^2 + (x^2 - y^2_A)^2 + (x^3 - y^3_A)^2}$$

ENTROPY

DENOTE $\left\{\begin{array}{l}\text{ELECTRIC}\\\text{MAGNETIC}\end{array}\right.$ CHARGES

$$(p^0, q_0)\ (p^1, q_1)\ (p^2, q_2)\ (p^3, q_3)$$

ACCORDING TO BEHRDT ET AL hep-th/9608059

ENTROPY IS *

$$S = \pi \left[W(p, q) \right]^{1/2}$$

WHERE

$$W = -(p^0 q_0 + p^1 q_1 + p^2 q_2 + p^3 q_3)^2$$
$$+ 4(p^1 q_1 p^2 q_2 + p^3 q_3 p^1 q_1 + p^2 q_2 p^3 q_3)$$
$$- 4 p^0 q_1 q_2 q_3 + 4 q_0 p^1 p^2 p^3$$

WITH $W > 0$ FOR CONSISTENCY (BPS)

* NEED "ATTRACTOR" MECHANISM

FERRARA KALLOSH STROMINGER

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IDENTIFY

$$\begin{bmatrix} a_{000} \\ a_{001} \\ a_{010} \\ a_{011} \\ a_{100} \\ a_{101} \\ a_{110} \\ a_{111} \end{bmatrix} = \begin{bmatrix} -p^0 \\ -p^1 \\ -p^2 \\ -q_3 \\ p^3 \\ q_2 \\ q_1 \\ -q_0 \end{bmatrix}$$

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THEN

$$\boxed{S = \pi\sqrt{-\operatorname{Det} a}}$$

EXAMPLES

$p = (0\ \ -1\ \ 0\ \ 0)\quad q = (0\ \ 0\ \ 0\ \ 0)\quad S = 0$

$p = (1\ \ 0\ \ 0\ \ -1)\quad q = (0\ \ 0\ \ 0\ \ 0)\quad S = 0$

$p = (1\ \ 0\ \ 0\ \ -1)\quad q = (0\ \ -1\ \ 0\ \ 0)\quad S = 0$

* $p = (1\ \ 0\ \ 0\ \ -1)\quad q = (0\ \ -1\ \ -1\ \ 0)\quad S \neq 0$

* REISSNER-NORDSTROM B.H.

REBITS

QUANTUM ENTANGLEMENT

$a_{AA'A''}$ COMPLEX , NORMALIZED $\psi^2 = 1$

BLACK HOLE ENTROPY

$a_{AA'A''}$ REAL (INTEGERS)

BUT LEVAY 06 MAKES CONTACT WITH "REBITS"
(REAL QUANTUM BITS)

3 CLASSES

(1) $\mathrm{Det}\, a < 0$ NON-SEPARABLE GHZ

$$|\psi\rangle = \frac{1}{2}\left(-|000\rangle + |011\rangle + |101\rangle + |110\rangle\right)$$

(2) $\mathrm{Det}\, a = 0$ SEPARABLE e.g W

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|100\rangle + |010\rangle + |001\rangle\right)$$

(3) $\mathrm{Det}\, a > 0$ ALSO GHZ

sign flip

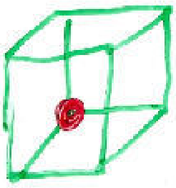
$$|\psi\rangle = \frac{1}{2}\left(|000\rangle + |011\rangle + |101\rangle + |110\rangle\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|111\rangle + |000\rangle\right)$$

BLACK HOLE CORRESPONDENCE

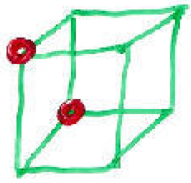
"SMALL" B.H. $\quad \mathrm{Det}\, a = 0$

$p = (0\ {-1}\ 0\ 0) \quad q = (0\ 0\ 0\ 0)$



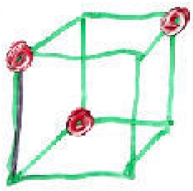
A–B–C

$p = (1\ 0\ 0\ {-1}) \quad q = (0\ 0\ 0\ 0)$



A–BC

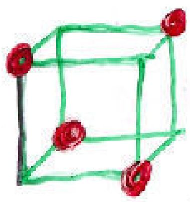
$p = (1\ 0\ 0\ {-1}) \quad q = (0\ {-1}\ 0\ 0)$



W[redacted]

"LARGE" B.H. $\quad \mathrm{Det}\, a < 0$

$p = (1\ 0\ 0\ {-1}) \quad q = (0\ {-1}\ {-1}\ 0)$



GHZ ?

NON-BPS $\quad \mathrm{Det}\, a > 0$

$p = ({-1}\ 0\ 0\ {-1}) \quad q = (0\ {-1}\ {-1}\ 0)$ ← sign flip

MJD+RAHMFELD 94

SIGN OF $\det a$

THERE ALSO EXIST SOLUTIONS IN THIS CLASS ("LARGE" BLACK HOLES) FOR WHICH

$$\det a > 0$$

REMARKABLY, THESE ARE <u>NON-BPS</u> SOLUTIONS

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e.g GHZ STATE

$$|000\rangle + |111\rangle$$

QUANTUM CORRECTIONS

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"SMALL" B.H. HAVE ZERO ENTROPY CLASSICALLY BUT MAY ACQUIRE IT THROUGH QUANTUM CORRECTIONS.

ONCE AGAIN, THERE IS A QUBIT DESCRIPTION e.g W-CLASS

$$S = \pi \sqrt{\frac{c_2}{3}\left(C_{AB} + C_{BC} + C_{CA}\right)}$$

c_2 = CONSTANT (8 FOR K3)

$C_{AB}^2 = \tau_{AB}$ "CONCURRENCE"