LECTURE III

a) E_7 AND THE TRIPARTITE ENTANGLEMENT OF SEVEN QUBITS

b) E_6 AND THE BIPARTITE ENTANGLEMENT OF THREE QUTRITS

POSSIBLE GENERALIZATIONS?

1) N=2 SUGRA + ℓ VECTOR MULTIPLETS

$SL(2,\mathbb{Z}) \times SO(\ell-1,2,\mathbb{Z})$

BH CHARGES : $(2,\ell+1)$ REPS

2) N=4 SUGRA + m VECTOR MULTIPLETS

$SL(2,\mathbb{Z}) \times SO(6,6+m,\mathbb{Z})$

BH CHARGES : $(2,12+m)$ REPS

3) N=8 SUGRA

$E_7(\mathbb{Z})$

BH CHARGES : 56 REP

IN ALL 3 CASES EXIST QUARTIC INVARIANT GIVING BH ENTROPY

N=8 EXAMPLE:

$$S = \pi\sqrt{|J_4|}$$

CARTAN $J_4 = -\mathrm{Tr}(xy)^2 + \frac{1}{4}(\mathrm{Tr}\,xy)^2 - 4(\mathrm{Pf}\,x + \mathrm{Pf}\,y)$

$28\ x^{ij}$; $28\ y_{ij}$

E_7 & TRIPARTITE ENTANGLEMENT OF 7 QUBITS

QUANTUM INFORMATION THEORETIC INTERPRETATION ?

CANNOT BE RANDOM ENTANGLEMENT OF n QUBITS WHICH GIVES $[SL(2)]^n$

CLUE:

$$E_7 \supset [SL(2)]^7$$

SUGGESTS 7 QUBIT INTERPRETATION

ALICE BOB CHARLIE DAISY EMMA FRED GEORGE

BUT OF A SPECIAL KIND

$$E_{7(7)} \supset SL(2)^7$$

$$E_7 \supset SL(2) \times SO(6,6)$$

$$56 \to (2,12) + (1,32)$$

$$\supset SL(2) \times SL(2) \times SL(2) \times SO(4,4)$$

$$\to (2,2,2,1)$$

$$+ (2,1,1,8_v) + (1,2,1,8_s) + (1,1,2,8_c)$$

S-T-U TRIALITY RELATED TO

$SO(4,4)$ $8_s - 8_c - 8_v$ TRIALITY

$$SO(4,4) \supset SO(2,2) \times SO(2,2)$$

$$\sim SL(2) \times SL(2) \times SL(2) \times SL(2)$$

$$E_7 \supset [SL(2)]^7$$

$$\begin{array}{llcccccccl}
 & & A & B & C & D & E & F & G & \\
56 & \rightarrow & (2, & 2, & 1, & 2, & 1, & 1, & 1) & a \\
 & + & (1, & 2, & 2, & 1, & 2, & 1, & 1) & b \\
 & + & (1, & 1, & 2, & 2, & 1, & 2, & 1) & c \\
 & + & (1, & 1, & 1, & 2, & 2, & 1, & 2) & d \\
 & + & (2, & 1, & 1, & 1, & 2, & 2, & 1) & e \\
 & + & (1, & 2, & 1, & 1, & 1, & 2, & 2) & f \\
 & + & (2, & 1, & 2, & 1, & 1, & 1, & 2) & g
\end{array}$$

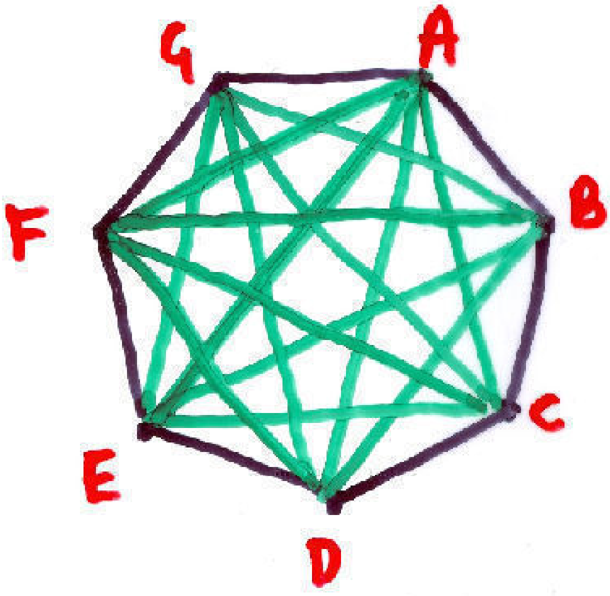
(2, 2, 2) UNDER 3 SL(2)

SINGLETS UNDER 4 SL(2)

S-T-U TRIALITY LINKED WITH

SO(4,4) TRIALITY $8_v - 8_s - 8_c$

TRIPARTITE ENTANGLEMENT OF SEVEN QUBITS



$\lvert\psi\rangle = a_{ABD}\lvert ABD\rangle$	ALICE
$+\, b_{BCE}\lvert BCE\rangle$	BOB
$+\, c_{CDF}\lvert CDF\rangle$	CHARLIE
$+\, d_{DEG}\lvert DEG\rangle$	DAISY
$+\, e_{EFA}\lvert EFA\rangle$	EMMA
$+\, f_{FGB}\lvert FGB\rangle$	FRED
$+\, g_{GAC}\lvert GAC\rangle$	GEORGE

QUTRIT INTERPRETATION

7 QUTRITS

$$|\psi\rangle = a_{ABCDEFG}\,|ABCDEFG\rangle \qquad A=0,1,2$$

$(3,3,3,3,3,3,3)$ UNDER $[SL(3)]^7$

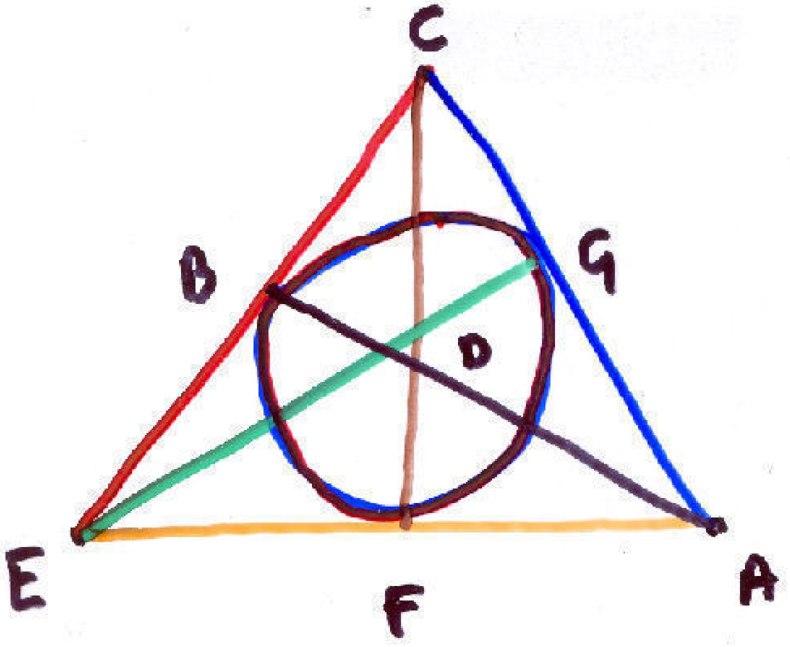
WHEN $SL(3) \to SL(2)$

$$3 \to 2+1$$

$(3,3,3,3,3,3,3)$ CONTAINS AS A

SUBSPACE
$$\begin{aligned}
&(2,2,1,2,1,1,1)\\
+&(1,2,2,1,2,1,1)\\
+&(1,1,1,2,2,1,2)\\
+&(2,1,1,1,2,2,1)\\
+&(1,2,1,1,1,2,2)\\
+&(2,1,2,1,1,1,2)\\
+&(1,1,2,2,1,2,1)
\end{aligned}$$

FANO PLANE



7 LINES

7 VERTICES

3 VERTICES ON EACH LINE

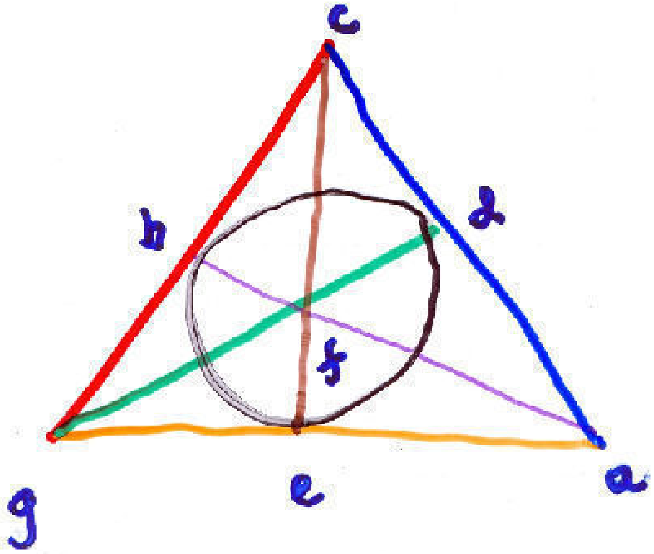
3 LINES ~~THROUGH~~ THROUGH EACH VERTEX

OCTONIONS

	A	B	C	D	E	F	G
A		D	G	-B	F	-E	-C
B	-D		E	A	-C	+G	F
C	-G	-E		F	B	-D	A
D	B	-A	-F		G	C	-E
E	-F	C	-B	-G		A	D
F	E	-G	D	-C	-A		B
G	C	F	-A	E	D	-B	

DUAL FANO PLANE

OBTAINED BY INTERCHANGING
7 LINES WITH 7 VERTICES



TRANSPOSE OF $E_7 \rightarrow SL(2)^7$ MATRIX

	A	B	C	D	E	F	G
a							
b							
c							
d							
e							
f							
g							

TRIPARTITE ENTANGLEMENT

$$\tau_3(ABCDEFG) = 4|J_4|$$

$$J_4 = -\mathrm{Tr}(xy)^2 + \frac{1}{4}(\mathrm{Tr}\, xy)^2$$ CARTAN

$$-4(\mathrm{Pf}\, x + \mathrm{Pf}\, y)$$

28 x

28 y

$$x^{ab} + i y_{ab} = -\frac{\sqrt{2}}{4} Z_{AB} (\Gamma^{AB})_{ab}$$

$$J_4 = \mathrm{Tr}(Z\bar{Z})^2 - \frac{1}{4}(\mathrm{Tr}\, Z\bar{Z})^2$$

$$+4(\mathrm{Pf}\, Z + \mathrm{Pf}\, \bar{Z})$$

CREMMER -JULIA 79

= HYPERDETERMINANT

IN CANONICAL BASIS

KALLOSH - LINDE 06

ENTANGLEMENT MEASURE

3-TANGLE

$$\tau_3(ABCDEFG) = 4\,|J_4|$$

$0 \leqq \tau_3 \leqq 1$ FOR NORMALIZED $|\psi\rangle$

FOR $N=8$, AS FOR $N=2$, LARGE BLACK HOLES CORRESPOND TO THE TWO CLASSES OF GHZ-TYPE (ENTANGLED) STATES AND SMALL BLACK HOLES TO SEPERABLE OR W[redacted] CLASSES.

N.B. J_4 REDUCES TO CAYLEY IN CANONICAL BASIS KALLOSH & LINDE

I_4

IS THE SINGLET IN $56 \times 56 \times 56 \times 56$

$$
\begin{aligned}
I \sim\; & a^4 + b^4 + c^4 + d^4 + e^4 + f^4 + g^4 \\
& + 2\big[a^2b^2 + b^2c^2 + c^2d^2 + d^2e^2 + e^2f^2 + f^2g^2 + g^2a^2 \\
& \quad + a^2c^2 + b^2d^2 + c^2e^2 + d^2f^2 + e^2g^2 + f^2a^2 + g^2b^2 \\
& \quad + a^2d^2 + b^2e^2 + c^2f^2 + d^2g^2 + e^2a^2 + f^2b^2 + g^2c^2 \big] \\
& + 8\big[bcdf + cdeg + defa + efgb + fgac + gabd + abce \big]
\end{aligned}
$$

WHERE

$abce \sim$

$$\varepsilon^{A_1B_2}\,\varepsilon^{C_2D_3}\,\varepsilon^{D_1F_3}\,\varepsilon^{C_3E_4}\,\varepsilon^{E_2A_4}\,\varepsilon^{B_1F_4}$$

$$a_{A_1B_1D_1}\; b_{B_2C_2E_2}\; c_{C_3D_3F_3}\; e_{E_4F_4A_4}$$

$\Rightarrow$ 7 CAYLEY + CROSS TERMS

FANO PLANE IS THE

$n=2$ CASE OF PROJECTIVE

PLANES OF ORDER n

$n+1$	n^2+n+1	
1	1	
2	3	
3	7	← FANO
4	13	
5	21	
⋮	⋮	

$(n+1)$-PARTITE ENTANGLEMENT

OF (n^2+n+1) QUBITS ? ?

BUT NO ANALOGUE OF E_7 (AS FAR AS I KNOW)

D = 5 SUPERGRAVITY

N=8 BLACK HOLE HAS 27 of $E_{6(6)}$ (ELECTRIC) CHARGES AND BLACK STRING 27' (MAGNETIC)

QUTRIT (3 STATE SYSTEM)

$$E_6(C) \supset SL(3,C)_A \times SL(3,C)_B \times SL(3,C)_C$$

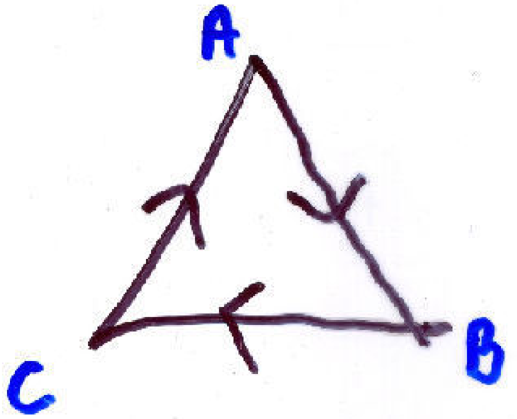
$$27 \rightarrow (3,3,1) + (3',1,3) + (1,3',3')$$

SUGGESTS BIPARTITE ENTANGLEMENT OF 3 QUTRITS (ALICE, BOB & CHARLIE)

$$|\Psi\rangle = a_{AB}|AB\rangle + b^{B}{}_{C}|BC\rangle + c^{CA}|CA\rangle$$

$A = 0, 1, 2$

ENTANGLEMENT DIAGRAM



ENTANGLEMENT MEASURE

$$\tau(ABC) = 4|J_3|$$

J_3 = CARTAN CUBIC INVARIANT

$$\sim a^3 + b^3 + c^3 + 6abc$$

c.f. BLACK HOLE ENTROPY

$$S = \pi |J_3|$$

MAGIC N=2 SUPERGRAVITIES

$D=4$

$\mathbb{R}$	14	$Sp(6,2)$
$\mathbb{C}$	20	$SU(3,3)$
$\mathbb{H}$	32	$SO^*(12)$
$\mathbb{O}$	56	$E_{7(-25)}$

ALSO ADMIT QUBIT ANALOGY SINCE

$$E_7(\mathbb{C}) \supset E_{7(-25)}$$

$D=5$

$\mathbb{R}$	6	$SL(3,\mathbb{R})$
$\mathbb{C}$	9	$SL(3,\mathbb{C})$
$\mathbb{H}$	15	$SU^*(6)$
$\mathbb{O}$	27	$E_{6(-26)}$

ALSO ADMIT QUTRIT ANALOGY SINCE

$$E_6(\mathbb{C}) \supset E_{6(-26)}$$

CONCLUSION:

STRINGS GET ENTANGLED!