Black Holes Galore
in D>4

Roberto Emparan

ICREA & U. Barcelona

Collaborations w/ R.Myers, H.Reall, H.Elvang, P.Figueras, A.Virmani, T.Harmark, N.Obers, V.Niarchos, M.J.Rodríguez (2000-2007)

Higher-dimensional gravity

Motivations

As applications:

- String / M theory
- Large Extra Dimensions & TeV gravity
- AdS/CFT
- Mathematics: Lorentzian geometry

But, not least, also of intrinsic interest:

- ***D as a* tunable parameter for gravity and black holes**

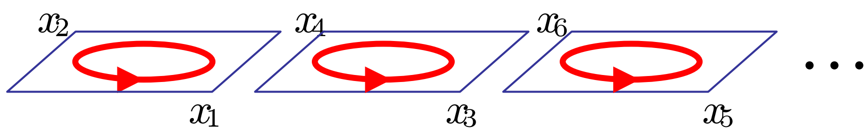
  What properties of black holes are

  - 'intrinsic' ? → *Laws of bh mechanics…*
  - D-dependent? → *Uniqueness, topology, shape, stability…*

What can a black hole (i.e. *spacetime*) do?

FAQ's

- Why is D>4 richer?
  - More degrees of freedom
  - Rotation:
    - more rotation planes



    - gravitational attraction ⇔ centrifugal repulsion

$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

  - ∃ extended black objects: black p-branes

- Why is D>4 harder?
  - More degrees of freedom
  - Axial symmetries: U(1)'s at asymptotic infinity appear only every 2 more dimensions -- not enough to reduce to 2D σ-model if D>5

Phases of black holes

- Find all stationary solutions that
  - are non-singular on and outside event horizons
  - satisfy Einstein's equations → $R_{\mu\nu} = 0$
  - with specified boundary conditions → *Asymp Flat*
- What does the phase diagram look like?
- Which solutions maximize the total horizon area (ie entropy)?
- *N.B.*: I (almost) won't discuss stability today

Phases of 4D black holes

- Static: Schwarzschild

$$ds^2 = -\left(1 - \frac{\mu}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{\mu}{r}} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\mu = 2GM$$

- Stationary: Kerr

$$ds^2 = -dt^2 + \frac{\mu r}{\Sigma}\left(dt + a \sin^2\theta d\phi\right)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta d\phi^2$$

$$\Sigma = r^2 + a^2 \cos^2\theta\,, \qquad \Delta = r^2 - \mu r + a^2\,, \qquad a = \frac{J}{M}$$

Horizon: $\Delta = 0 \Rightarrow M \geqslant a$: Upper bound on J for given M

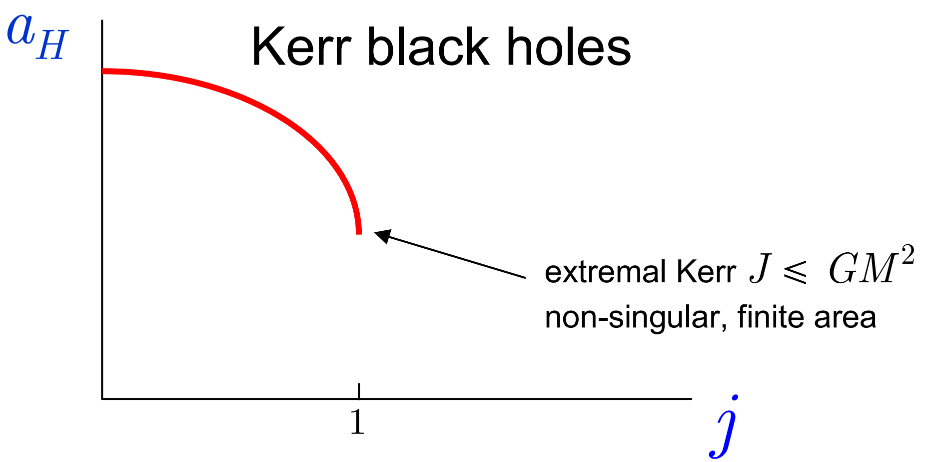
$$J \leqslant GM^2$$

Phase plots

- To compare solutions we need to fix a common scale
- *Classical* GR doesn't have any intrinsic scale

  → We'll fix the mass M

  equivalently factor it out to get dimensionless quantities



$$a_H \equiv \frac{\mathcal{A}_H}{(GM)^2}$$

$$j \equiv \frac{J}{GM^2}$$

- Uniqueness theorem: *End of the story!*

  multi-bhs are unlikely

Black holes in *D*>4

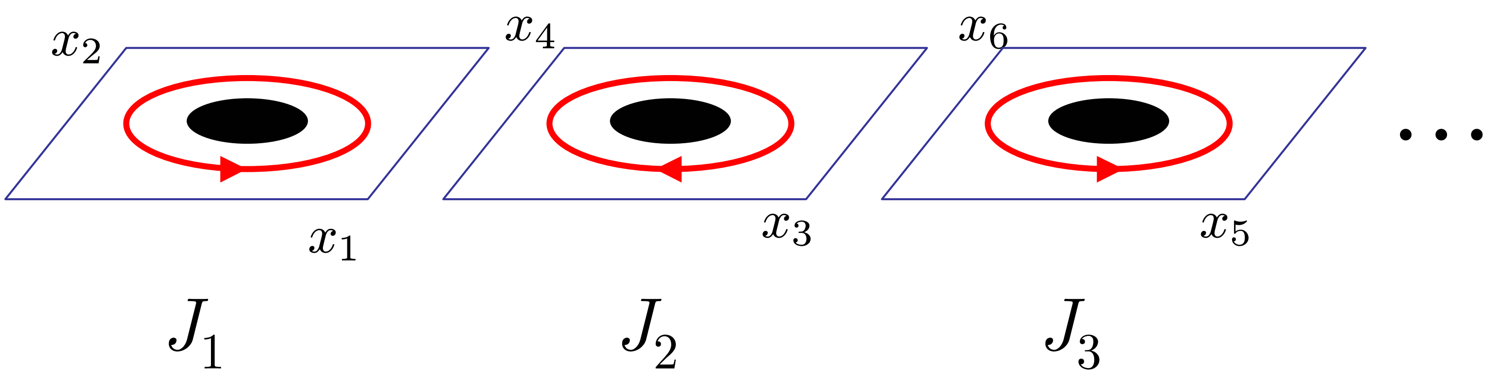
- Schwarzschild is easy: *Tangherlini 1963*

$$ds^2 = -\left(1 - \frac{\mu}{r^{D-3}}\right) dt^2 + \frac{dr^2}{1 - \mu/r^{D-3}} + r^2 d\Omega_{(D-2)}$$

$$\mu \propto M$$

- Unique static black hole *Gibbons+Ida+Shiromizu*
- Dynamically linearly stable *Ishibashi+Kodama*

- Kerr is harder. But *Myers+Perry (1986)* found rotating black hole solutions with angular momentum in an arbitrary number of planes



- They all have spherical topology S^{D-2}

- Consider a **single spin**:

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2+a^2)\sin^2\theta\, d\phi^2$$
$$+ r^2\cos^2\theta\, d\Omega^2_{(D-4)}$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}\,,$$

$$\mu \propto M$$

$$a \propto \frac{J}{M}$$

$$\frac{\Delta}{r^2} - 1 = \underbrace{-\frac{\mu}{r^{D-3}}}_{\text{gravitational}} + \underbrace{\frac{a^2}{r^2}}_{\text{centrifugal}}$$

- Consider a **single spin**:

Horizon: $\Delta=0$ $$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

$$\boldsymbol{D=5:} \qquad r_h^2 + a^2 - \mu = 0 \quad \Rightarrow \quad r_h = \sqrt{\mu - a^2}$$

$\Rightarrow \quad a^2 \leqslant \mu \quad \Rightarrow$ upper bound on J for given M

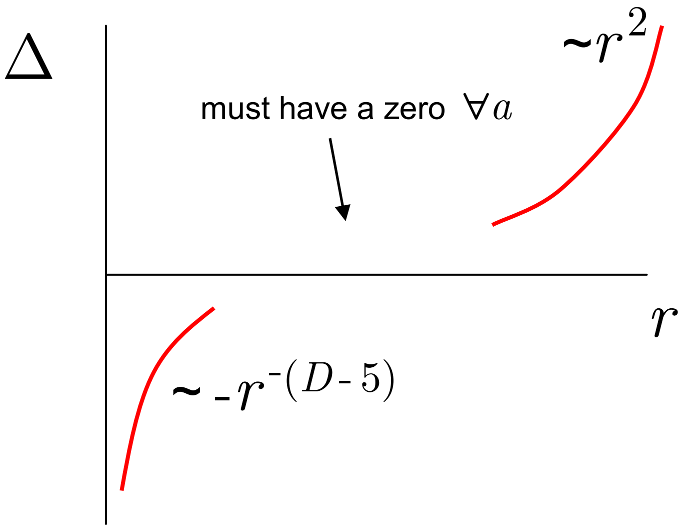
- similar to 4D
- but extremal limit $a^2 = \mu \;\Rightarrow\; r_h=0$

this is *singular, zero-area*

$D \geqslant 6$:

Horizon: $\Delta = 0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$



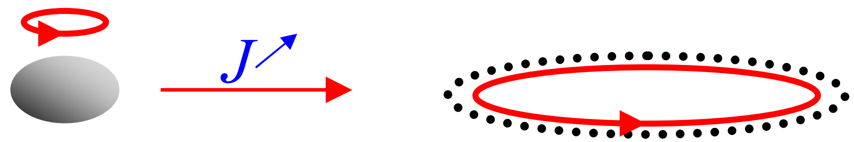
For fixed μ there is an outer event horizon for *any* value of a

$\Rightarrow$ No upper bound on J for given M

$\Rightarrow \exists$ ***ultra-spinning black holes***

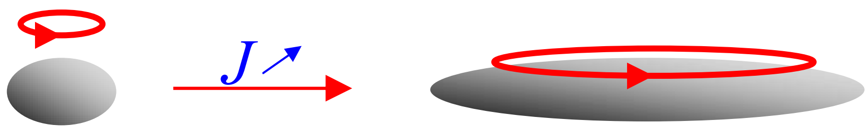
- **Single spin MP black holes:**

$D = 5$

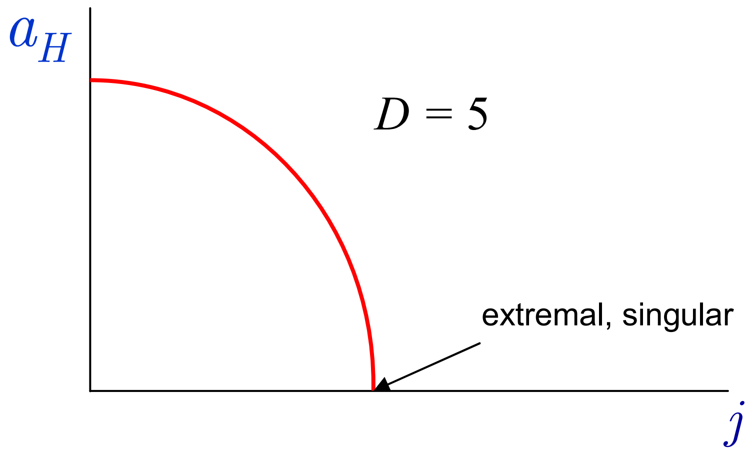


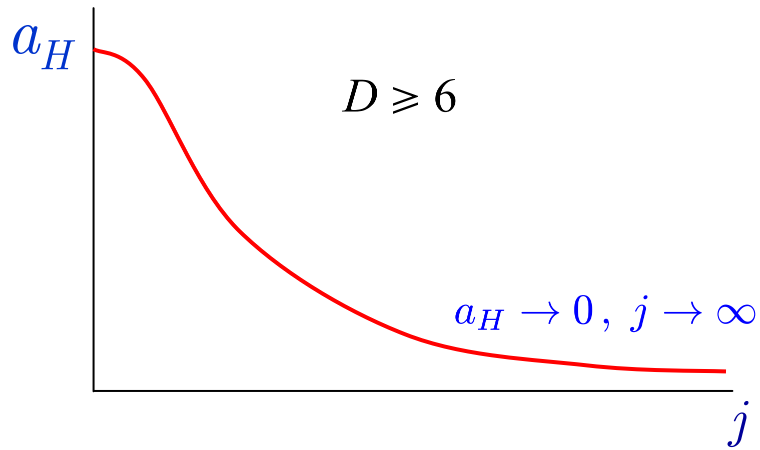
upper (extremal) limit on J is singular

$D \geqslant 6$



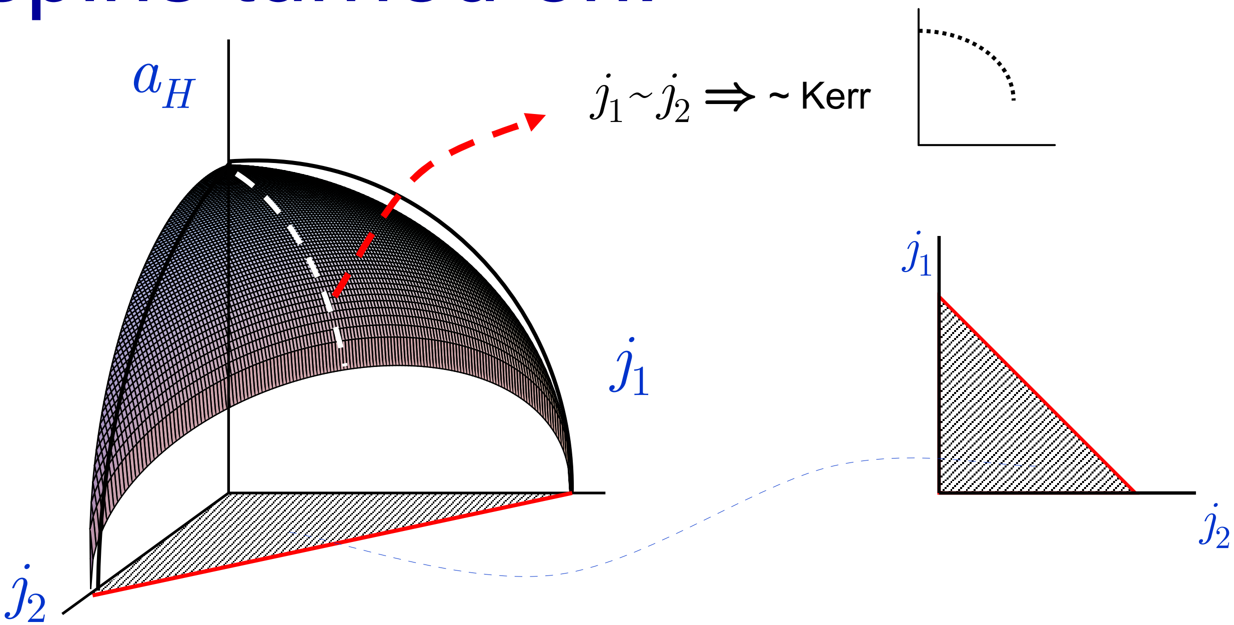
no upper limit on J for fixed M



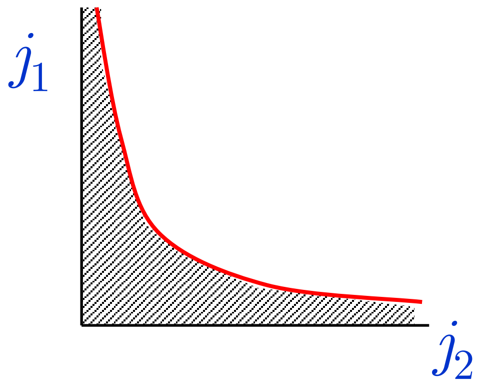


Several spins turned on:

$D{=}5$



$D{=}6$



$D \geqslant 6$

- If all $j_1 \sim j_2 \sim \ldots \sim j_{\lfloor (D\text{-}1)/2 \rfloor} \Rightarrow$ ~ Kerr
- $\exists$ ultra-spinning regimes if one (two) j_i are much smaller than the rest

Is this all there is in D>4?

Not at all

Combine black branes & rotation:

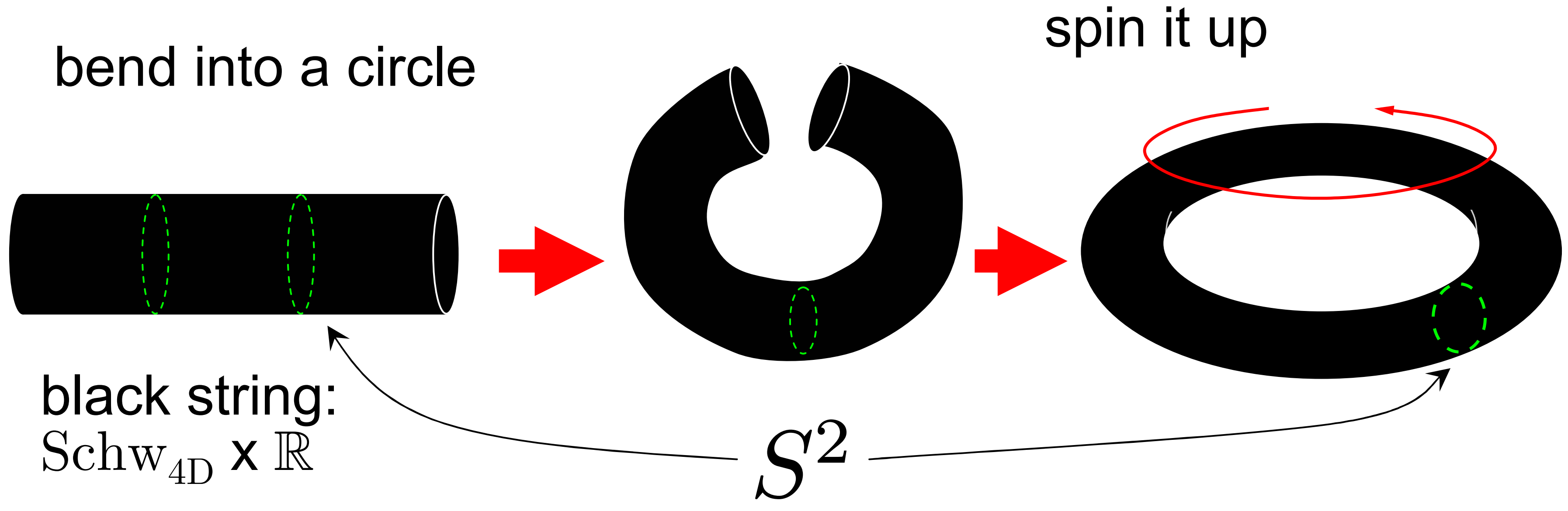
$\Rightarrow$ Black Rings + other blackfolds in $D \geqslant 5$

$\Rightarrow$ *Pinched* black holes in $D \geqslant 6$

D=5

End may be in sight

The forging of the ring



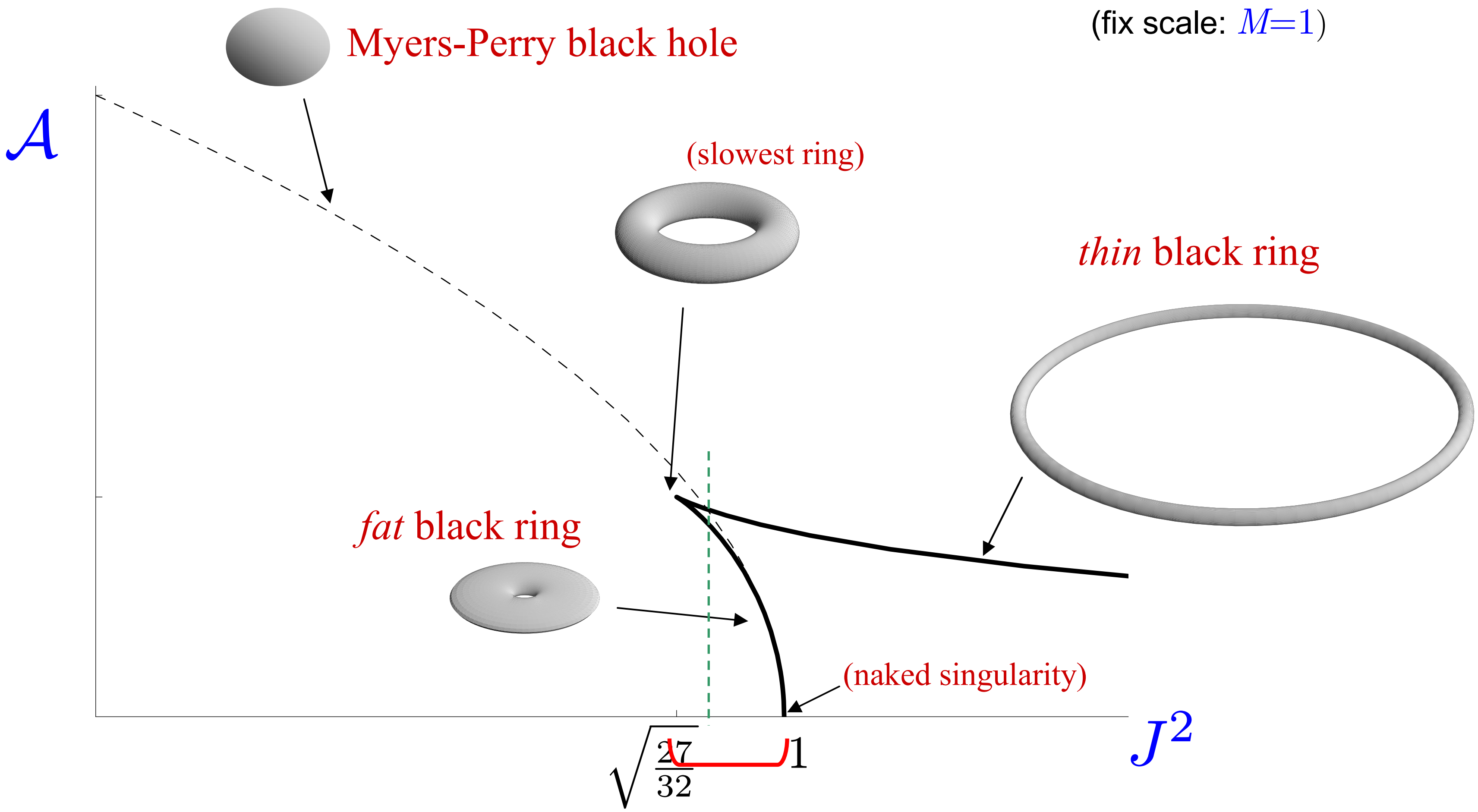
Horizon topology $S^1 \times S^2$

Exact solution available -- and fairly simple

RE+Reall 2001

5D: one-black hole phases

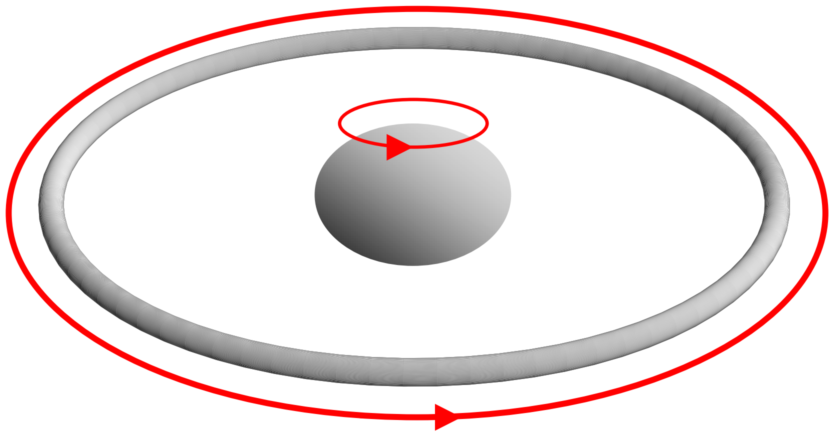
(fix scale: $M=1$)



3 different black holes with the same value of M,J

Multi-black holes

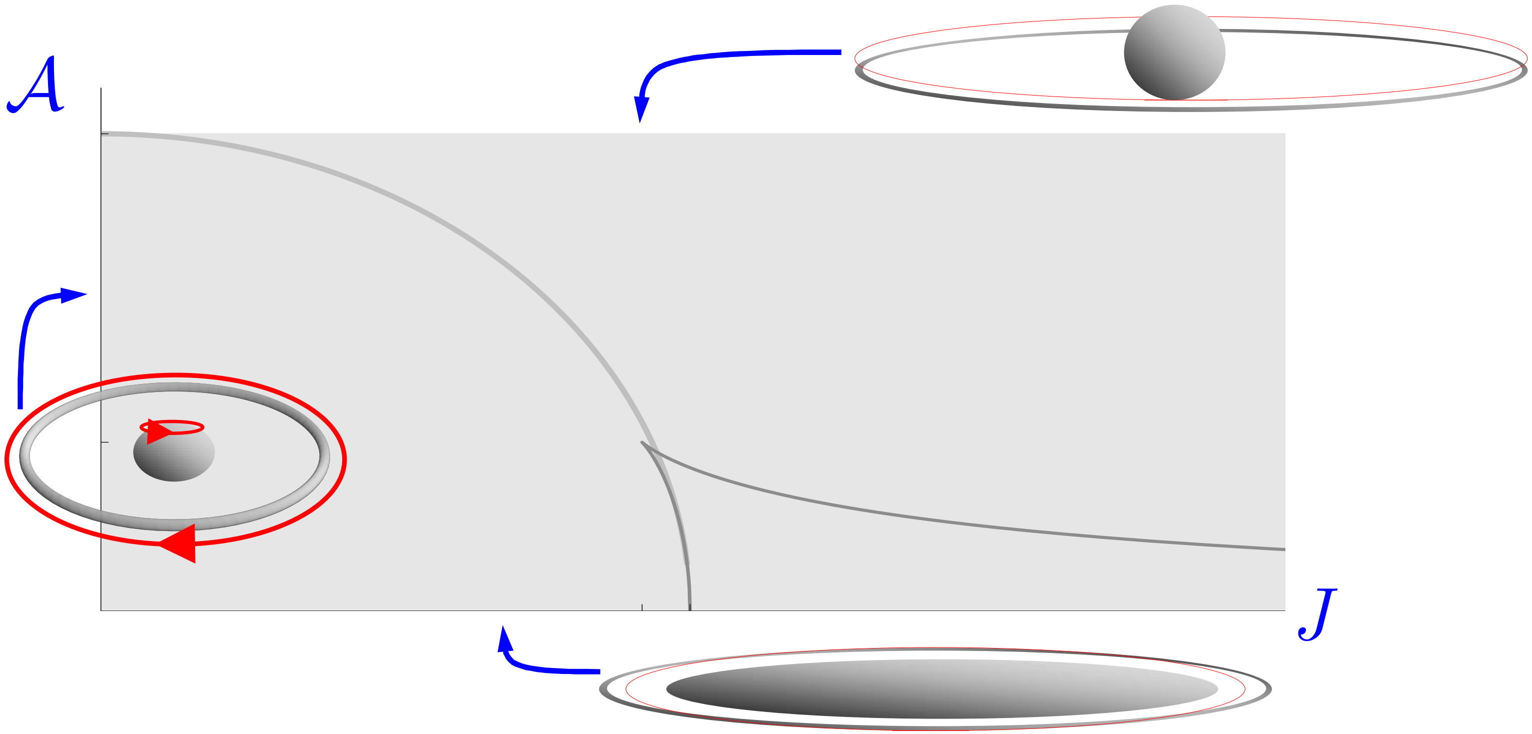
- *Black Saturn:*



- Exact solutions available *Elvang+Figueras*
- Co- & counter-rotating, rotational dragging…

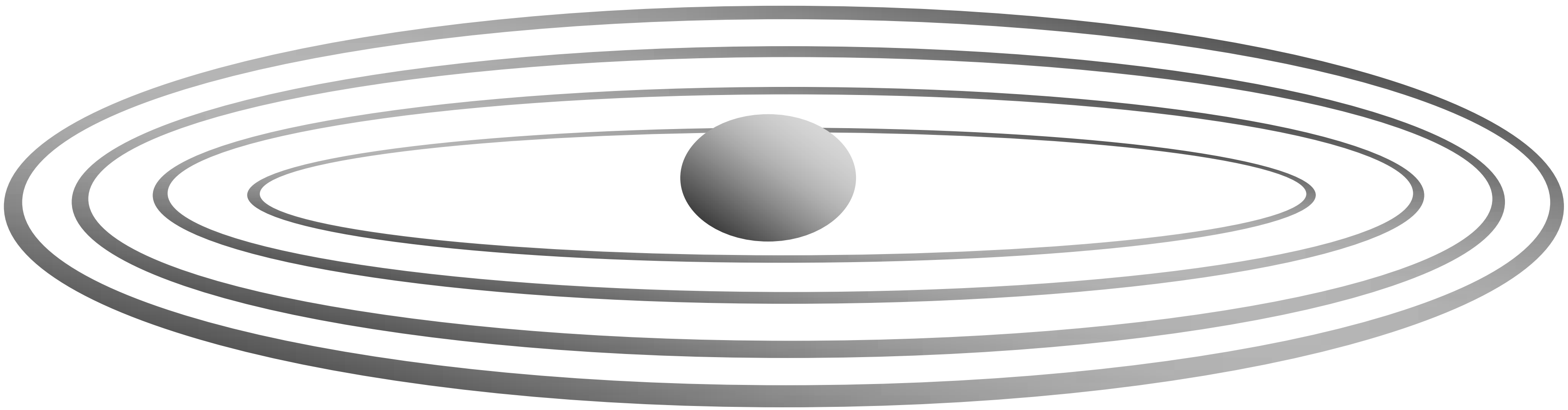
Filling the phase diagram

- Black Saturns cover a semi-infinite strip

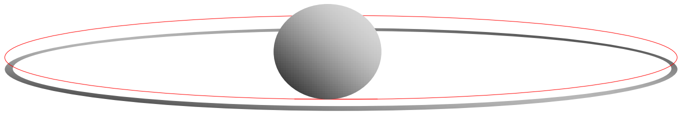


Multi-rings are also possible

- Di-rings explicitly constructed *Iguchi+Mishima* *Evslin+Krishnan*
- Systematic method available (but messy)



Thermodynamical equilibrium

 is not in thermo-equil

$$T_r \gg T_h \,, \quad \Omega_r \neq \Omega_h$$

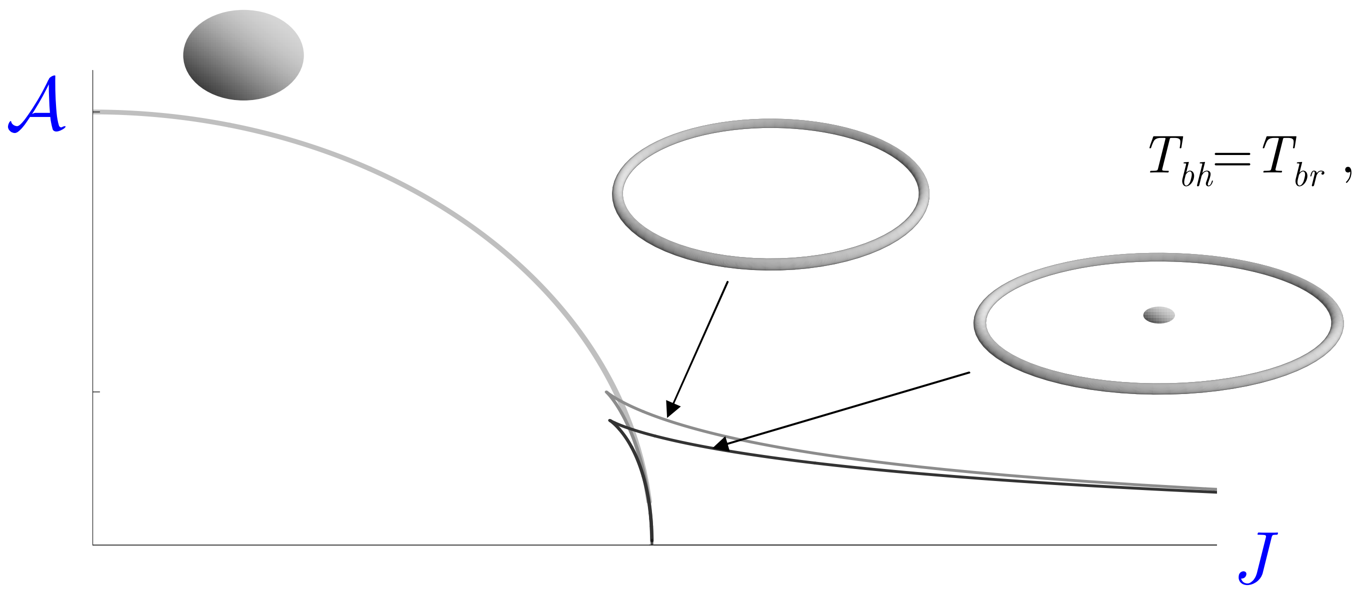
- Beware: bh thermodynamics makes sense only with Hawking radiation
- Radiation is in equilibrium only if

$$T_i = T_j \,, \; \Omega_i = \Omega_j$$

$\Rightarrow$ continuous degeneracies removed

- Multi-rings unlikely

5D phases in thermal equilibrium



$$T_{bh}=T_{br}\ ,\quad \Omega_{bh}=\Omega_{br}$$

is there anything else?

Towards a complete classification of 5D black holes

- Topology: S^3, S^1x S^2 *Galloway+Schoen*
- Rigidity: stationarity → one axial $U(1)$, but not (yet?) necessarily two *Hollands et al*
- If $\mathbb{R}_t$ x $U(1)_\phi$x $U(1)_\psi$ then
  - complete integrability *Pomeransky*
  - "uniqueness" *Hollands+Yazadjiev*
- Essentially all bh solutions *may* have been found:

  MP, black rings, multi-bhs (saturns & multi-rings)

  (including also with two spins: *Pomeransky+Sen'kov, Elvang+Rodríguez*)

  (bubbly black holes?)

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in D>4 (II)

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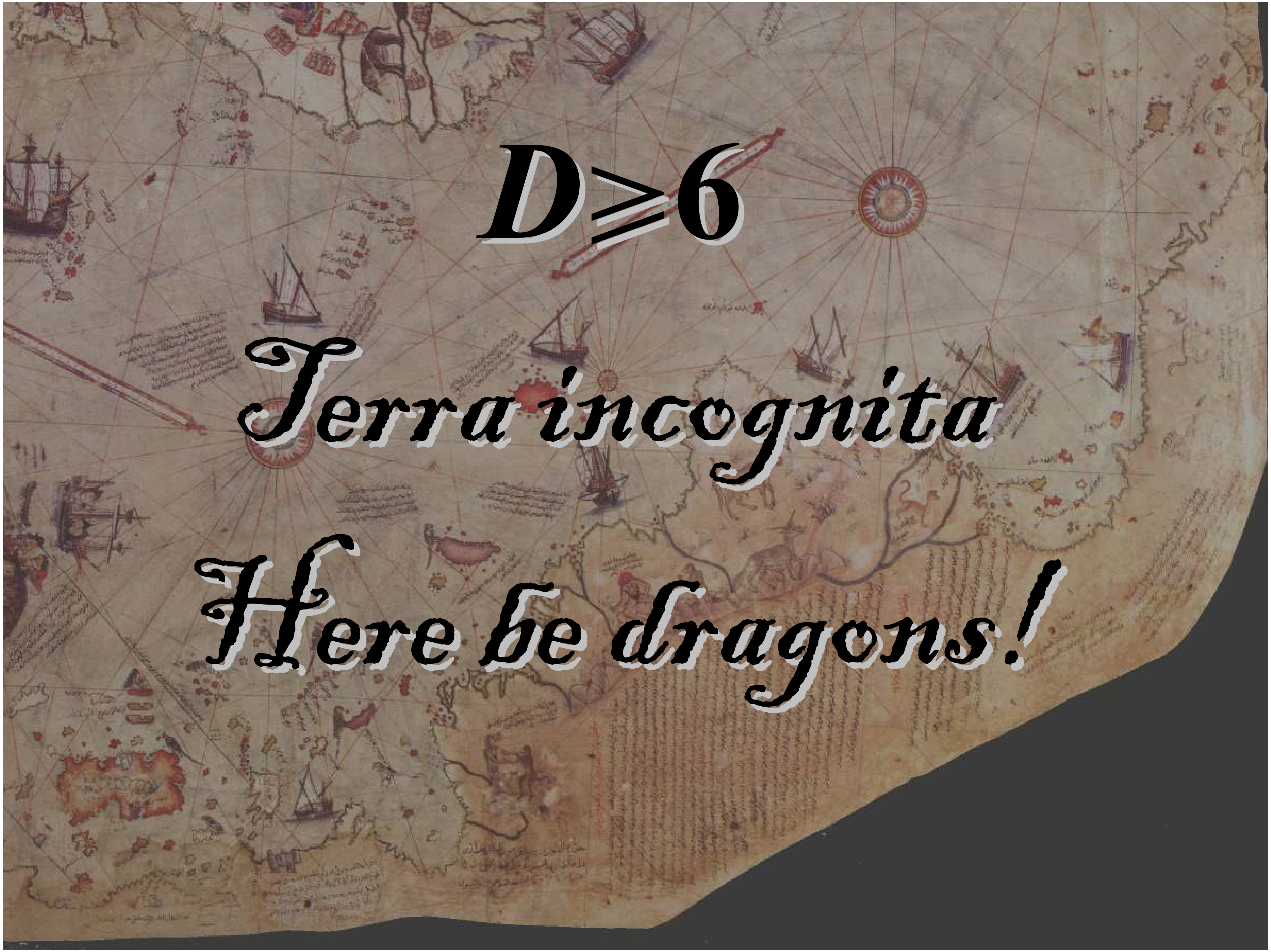
D=5

End may be in sight



D≥6

Terra incognita

Here be dragons!

Difficulties:

Explicit construction techniques

- Newman-Penrose formalism: unwieldy in $D>4$
- In $D=4,\ 5$: $\mathbb{R} \times U(1)$, $\mathbb{R} \times U(1) \times U(1)$

  $\Rightarrow$ 2D σ-model

  But in $D\geqslant 6$: $\mathbb{R} \times U(1)^{\lfloor (D-1)/2 \rfloor}$ not enough:

  $$1+\lfloor (D\text{-}1)/2 \rfloor < D\text{-}2$$

- Kerr-Schild class: $g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu$

  MP black holes are K-S, but black rings are not

Difficulties:
Horizon topology

- Hawking's 4D theorem relies on Gauss-Bonnet thm:

$$\int_{\mathcal{H}} R^{(2)} > 0 \Rightarrow \mathcal{H} = S^2$$

- *Galloway+Schoen*: +ve Yamabe $R^{(D\text{-}2)} > 0$ → S^3, S^1x S^2
- $D{=}6$: *Helfgott et al*: S^4, S^2x S^2, S^1x S^3, $(\mathbb{H}^2/\Gamma)_g$ x S^2

  So far: S^4 *(exact soln),* S^1x S^3 *(approximate soln)*

- $D{>}6$: essentially unknown

  $S^{D\text{-}2}$ *(exact soln),* S^1x $S^{D\text{-}3}$ *(approximate soln),*

  $\mathbf{T}^p$ x $S^{D\text{-}p\text{-}2}$ *(in progress)*

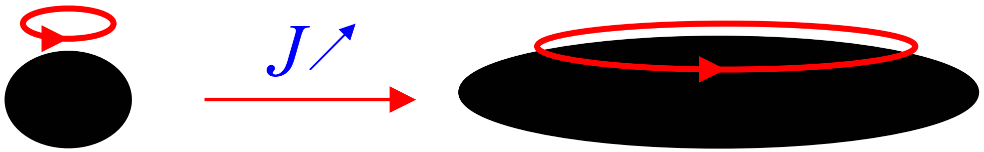
So, *very limited* success in extending 4D approaches

$\Rightarrow$ Need new ideas

- more qualitative & less rigorous methods (physics-guided)
- may guide later numerical attacks

Pinched (*lumpy*) black holes in $D \geqslant 6$

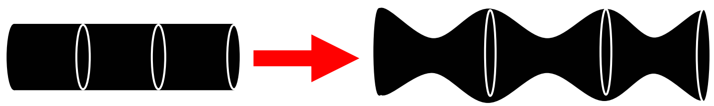
Ultraspinning regime in $D \geqslant 6$



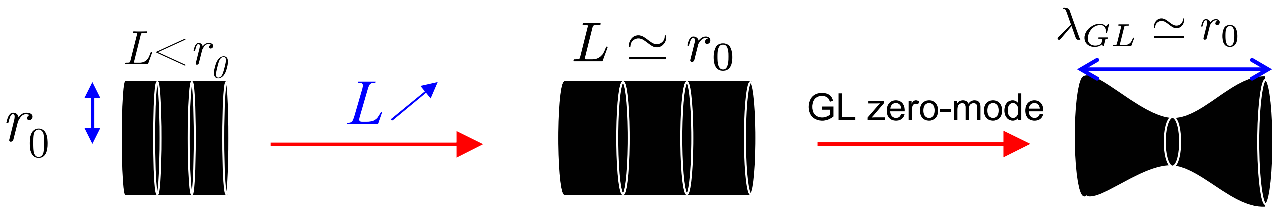
$\Rightarrow$ ***black membrane*** along rotation plane

Black strings and branes exhibit

- Gregory-Laflamme instability



- GL zero-mode → branching into non-uniform horizons



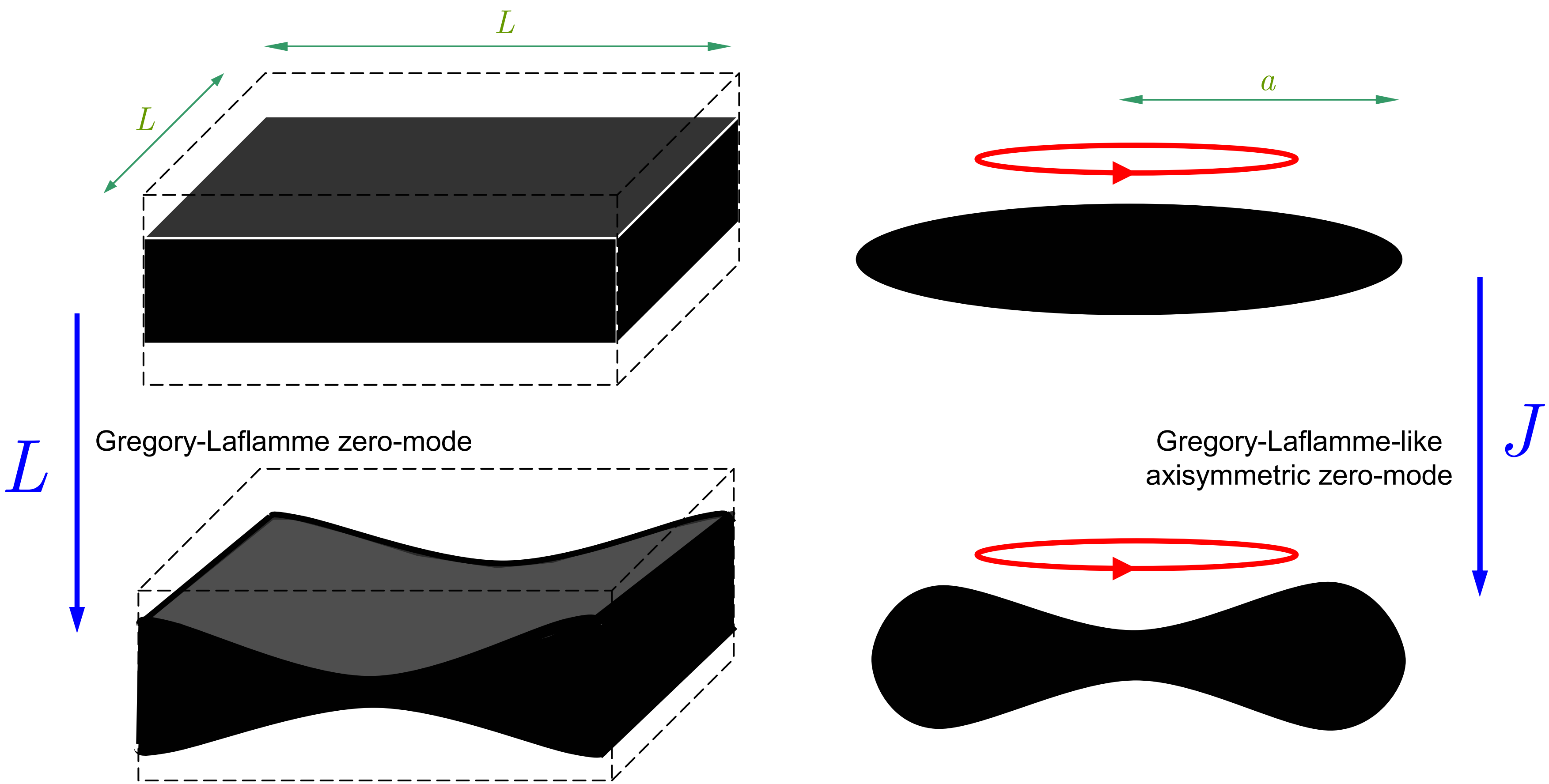
branch of static
lumpy black strings

Gubser, Wiseman

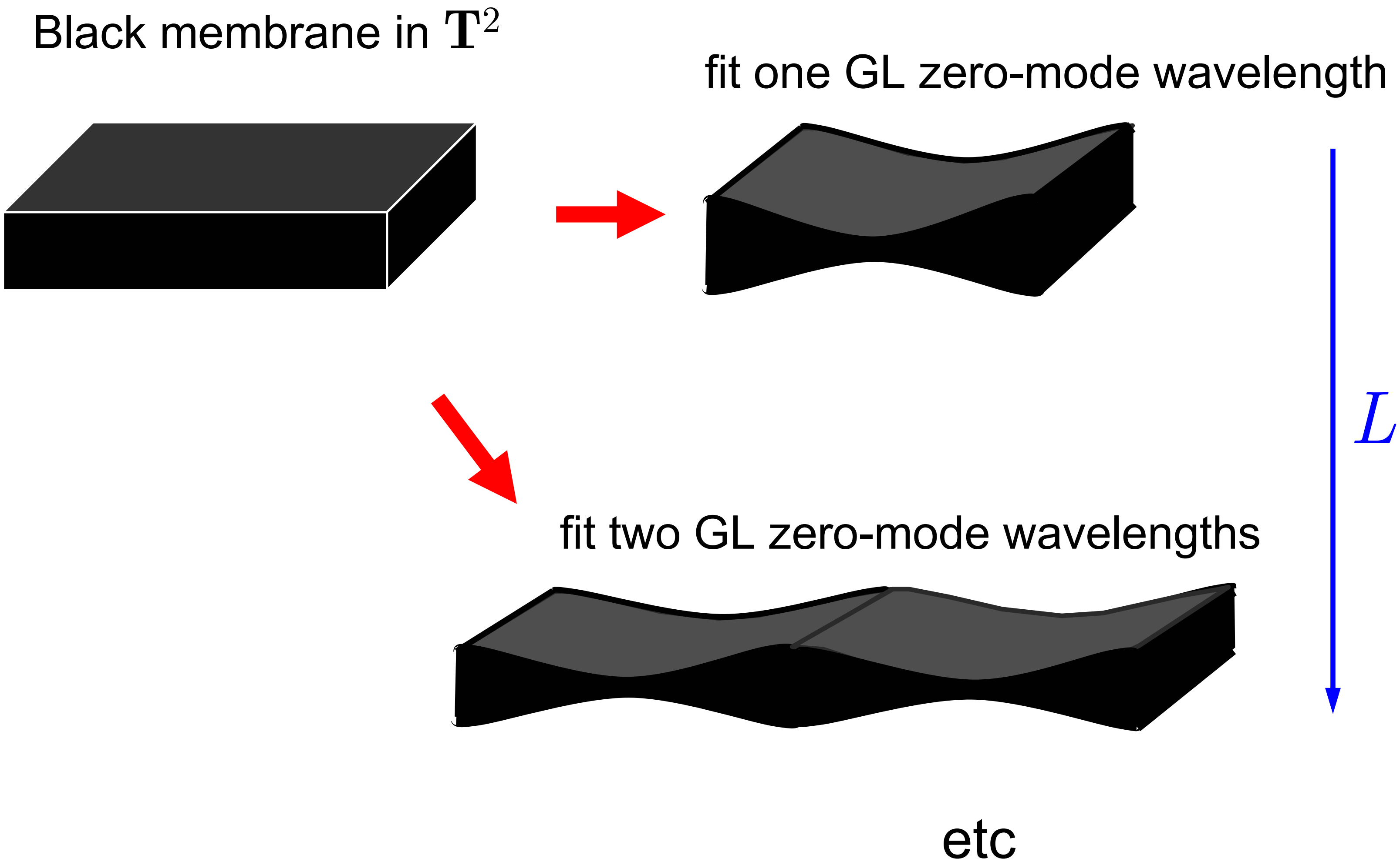
Pinched (*lumpy*) black holes in $D \geqslant 6$

Ultra-spinning = membrane-like

RE+Myers

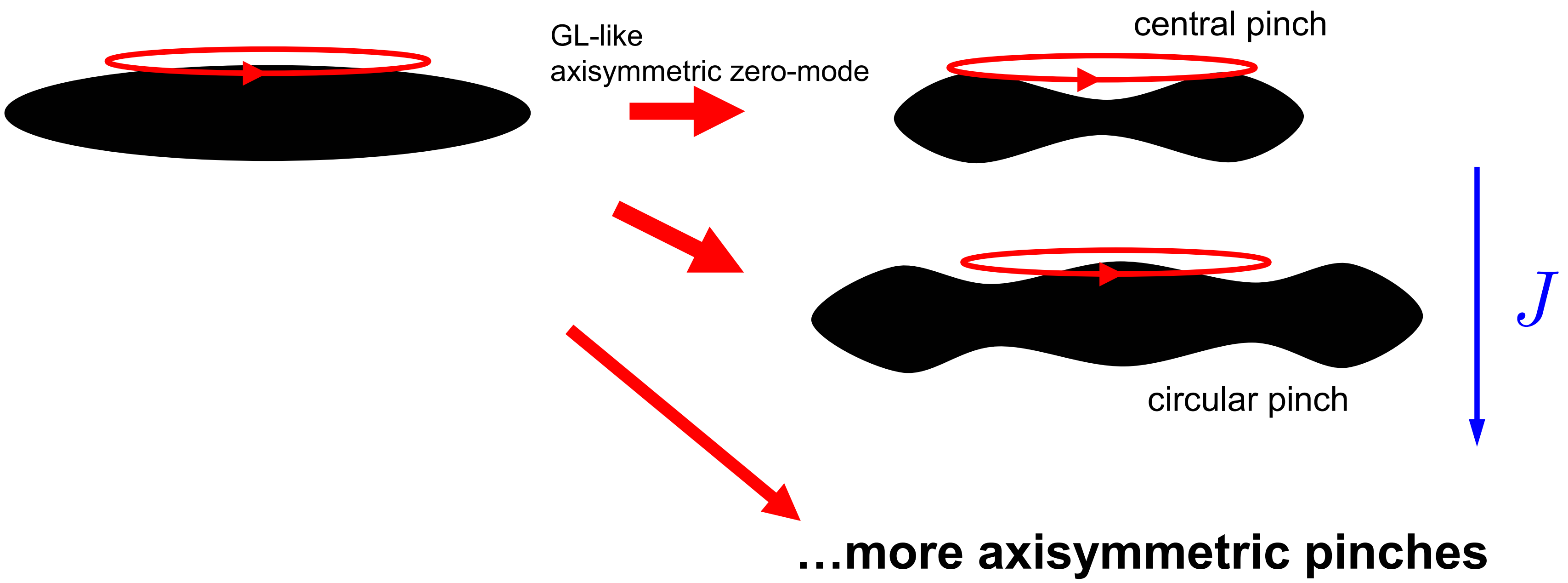


Replicas: multiple pinches



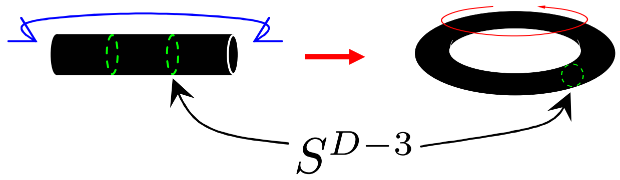
(fixed mass)

Multiply pinched black holes from axisymmetric zero-modes:



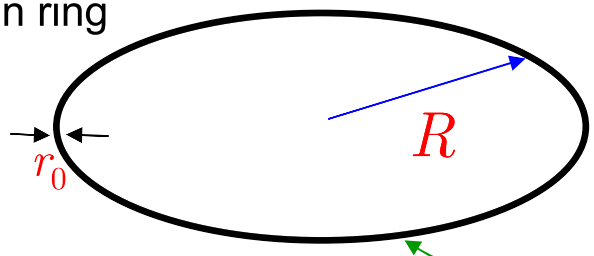
- Not yet found --- presumably numerically or approximately
- They're necessary to complete the phase diagram
  $\Rightarrow$ connect to black rings
- *Pinched plasma balls* recently found by *Lahiri+Minwalla*:
  dual to (large) pinched black holes in AdS

Thin black rings in $D>5$

- Heuristic:  seems plausible

- Thin black rings ≃ circular boosted black strings
- Equilibrium can be analyzed w/in linearized gravity:
  - balance: tension ⇔ centrifugal repulsion
  - gravitational self-attraction is subdominant
- Approximate construction via matched asymptotic expansion *T. Harmark's talk*

Matched asymptotic expansion



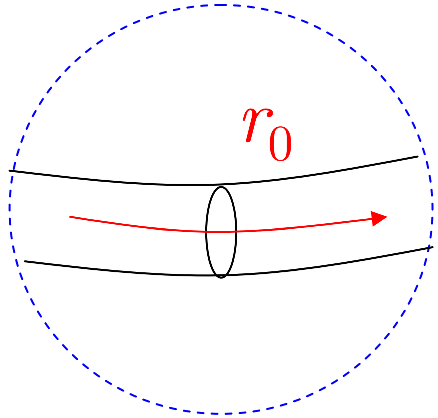
1- linearized soln around flat space

$$\frac{r_0}{r} \ll 1$$

2- perturbations of a boosted black string

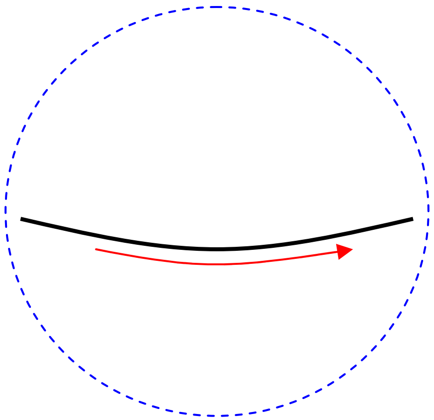
$$\frac{r}{R} \ll 1$$

need bdry conditions to
fix integration constants

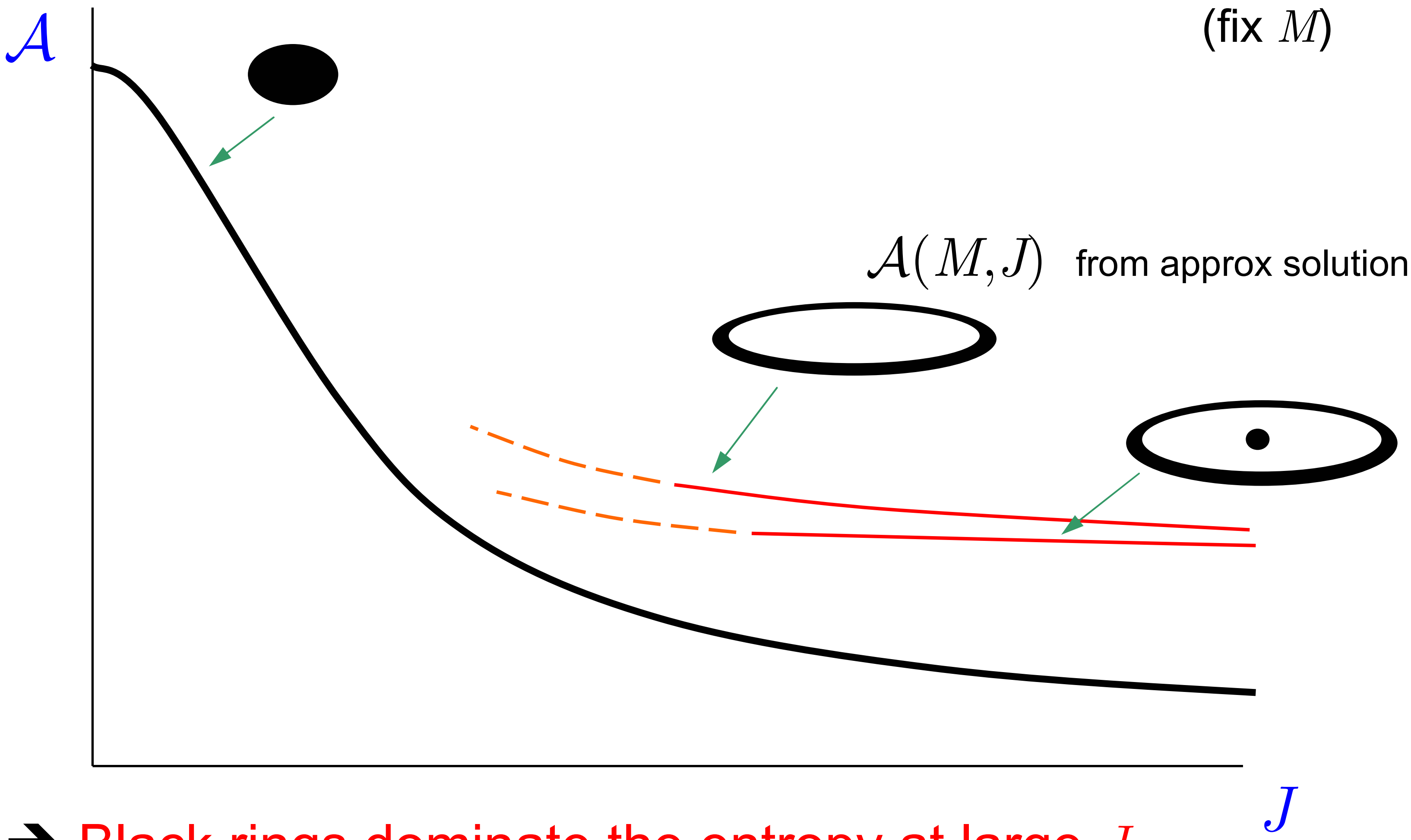


Match in overlap zone

$$r_0 \ll r \ll R$$



$D \geqslant 6$ phase diagram

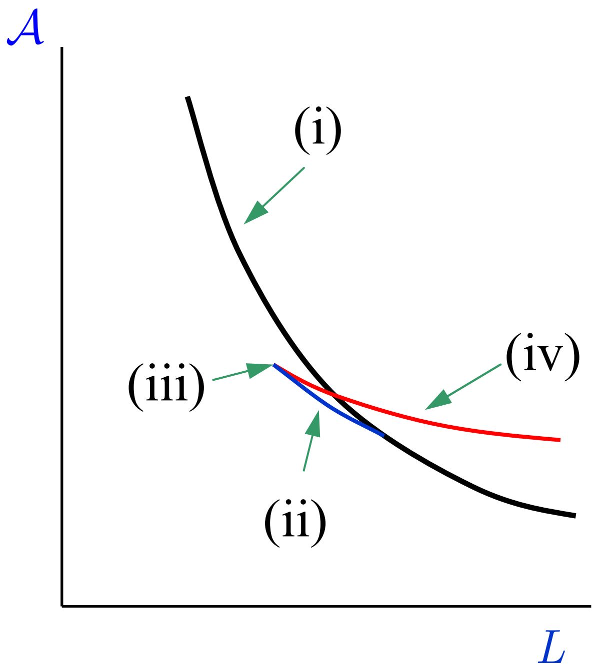


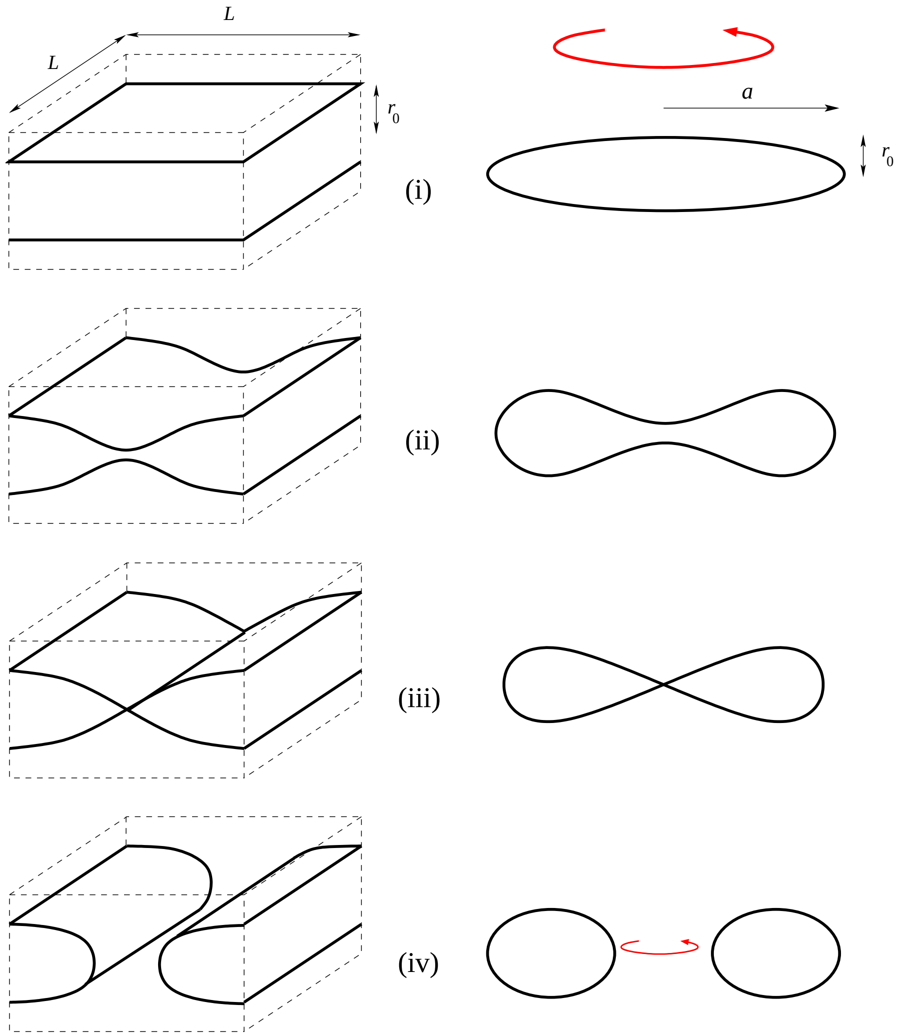
➔ Black rings dominate the entropy at large J

Black membrane ⇔ Rot Black Hole

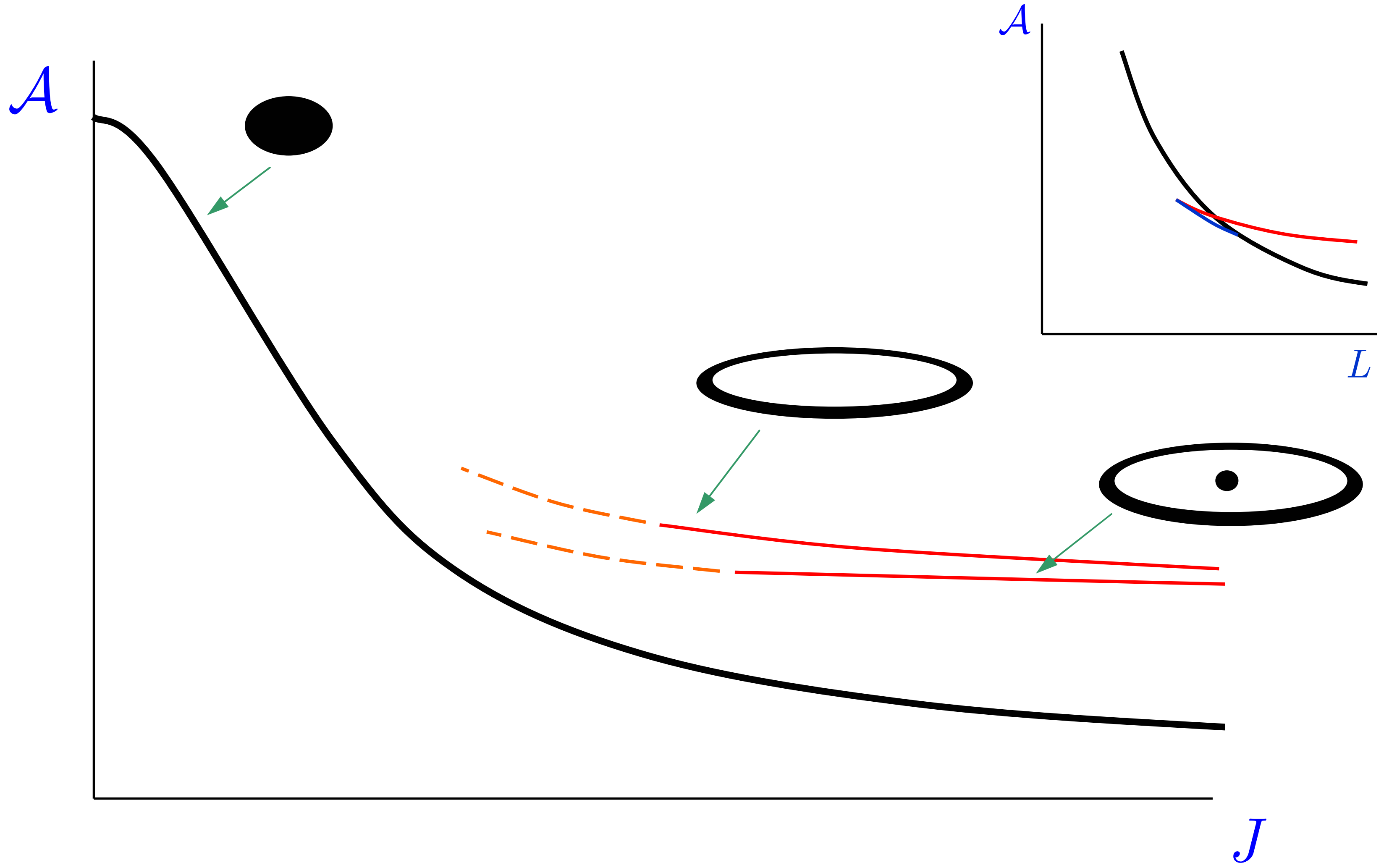
Black membrane in $\mathbf{T}^2$

(fixed mass)

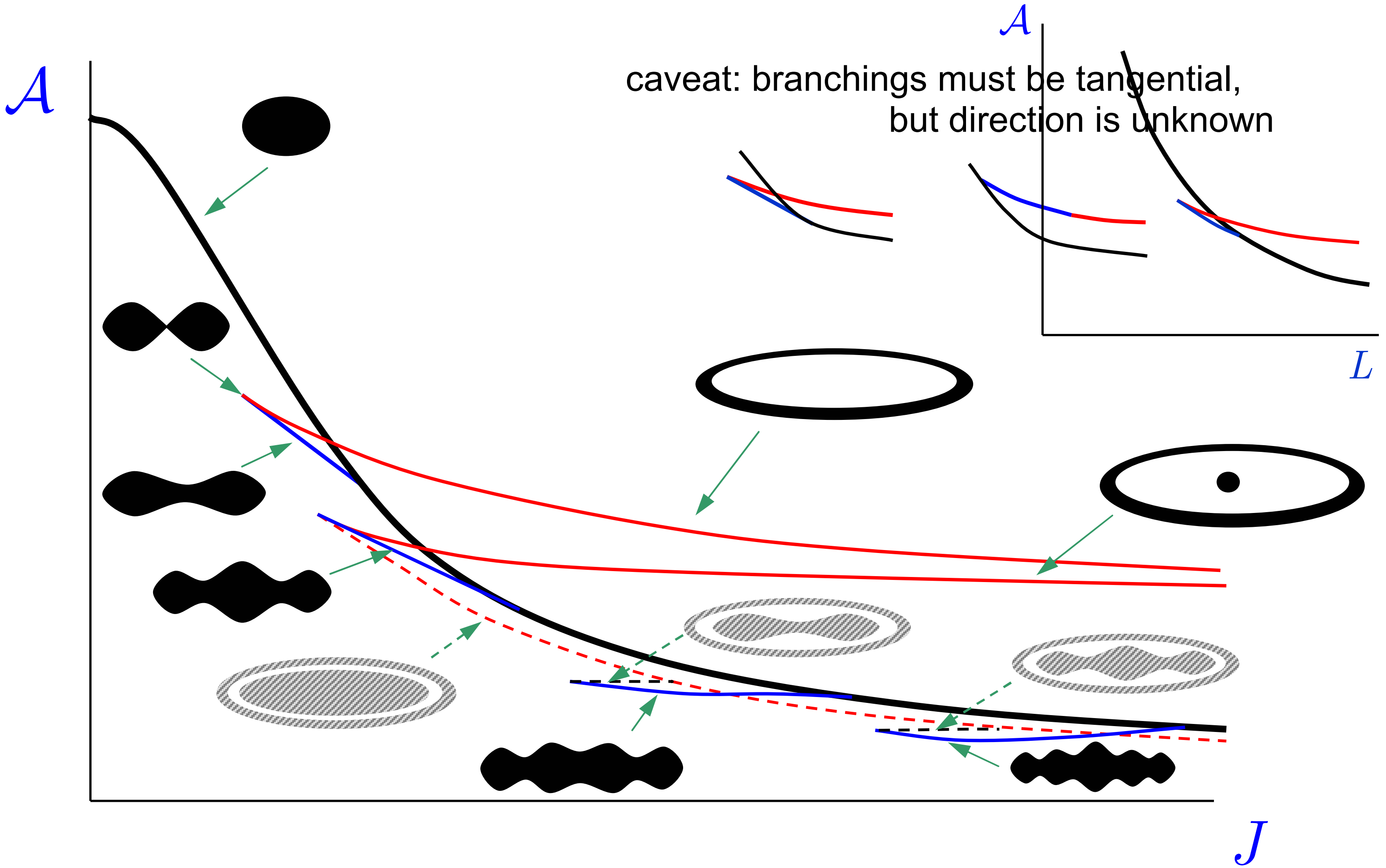




$D \geqslant 6$ phase diagram: a proposal



$D \geqslant 6$ phase diagram: a proposal



less tight arguments, but still plausible
arguments for this phase structure can be made in detail

Possible extensions

- Curving thin black branes:

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

Law of brane dynamics $K_{\mu\nu}{}^{\rho} T^{\mu\nu} = F^{\rho}$
(*Carter*)

extrinsic curvature

eg, for tori $\frac{T_{11}}{R_1} + \frac{T_{22}}{R_2} + \cdots = 0$ easily satisfied!

  – *dynamical* constraints on possible topologies

- Branes in gravitational potentials (AdS, dS, black saturns, etc)
- Charged and susy black holes

Conclusions: *More is different*

Vacuum gravity $R_{\mu\nu} = 0$ in

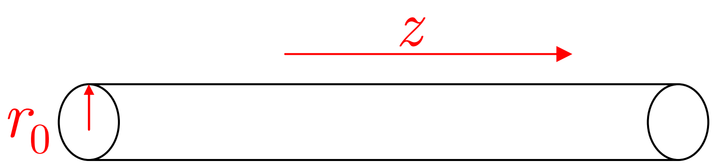
- $D=3$ has no black holes
  - GM is dimensionless ➔ can't construct a length scale

    (Λ, or h, provide length scale)
- $D=4$ has **one** black hole
  - but no 3D bh ➔ no 4D black strings → no 4D black rings
- $D=5$ has **three** black holes (two topologies); black strings → black rings, infinitely many multi-bhs…
- $D\geqslant 6$ seem to have **infinitely many** black holes (many topologies, lumpy horizons…); black branes→ rings, toroids…, infinitely many multi-bhs…

we've just begun

Curving a black string

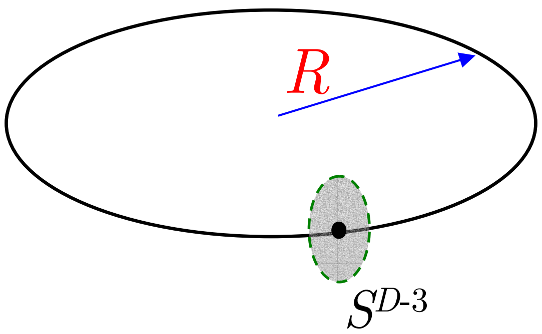
Boosted black string:



ADM stress tensor:

$$
\begin{aligned}
T_{tt} &\propto r_0^{D-4}((D-4)\cosh^2\sigma + 1)\\
T_{tz} &\propto r_0^{D-4}(D-4)\cosh\sigma\sinh\sigma\\
T_{zz} &\propto r_0^{D-4}((D-4)\sinh^2\sigma - 1)
\end{aligned}
$$

→ Thin boosted black string along a circle



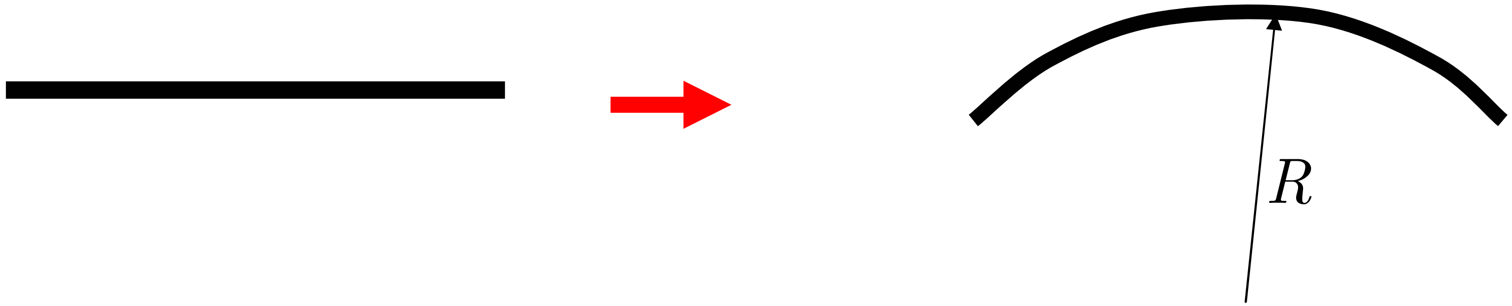
equivalent delta-source $T^{(\delta)}_{\mu\nu} = T^{ADM}_{\mu\nu}\,\delta(r)$

$$
\begin{aligned}
M &= 2\pi R \int_{S^{D-3}} T^{(\delta)}_{tt}\\
J &= 2\pi R^2 \int_{S^{D-3}} T^{(\delta)}_{tz}\\
\mathcal{A} &= 2\pi R \int_{S^{D-3}} \sqrt{g}_{hor}
\end{aligned}
\quad \Rightarrow \mathcal{A}(M,J,R)
$$

What fixes σ ? or: how is R fixed in terms of M, J ?

Curving a black string: equilibrium condition

Overlap zone analysis: $r_0 \ll r \ll R$ (still thin ring)



Compute $1/R$ corrections to linearized black string

→ Solution is singular unless $T_{zz}=0$

equivalently from

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\Rightarrow \sinh^2\sigma = \frac{1}{D-4}$$

$$MR = \frac{D-2}{\sqrt{D-3}} J$$

← equilibrium conditions