Higher Spins: a Key Corner of Field and String Theory

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Some related reviews:

- *N. Bouatta, G. Compere, A.S., hep-th/0609068*
- *D. Francia and A.S., hep-th/0601199*
- *A.S., Sezgin, Sundell, hep-th/0501156*
- *X. Bekaert, S. Cnockaert, C. Iazeolla, M.A. Vasiliev, hep-th/0503128*
- *D. Sorokin, hep-th/0405069*
- *A.S., P. Sundell, D. Sorokin, M.A. Vasiliev, Phys. Reports, 2008 (?)*

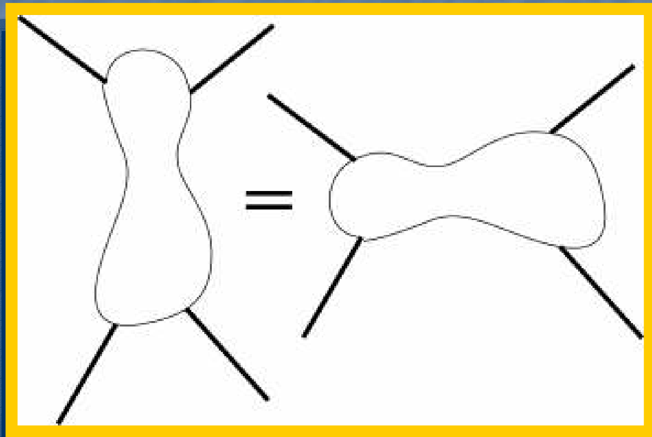
EU-RTN Workshop, Valencia, October 2007

Some Motivations for HS

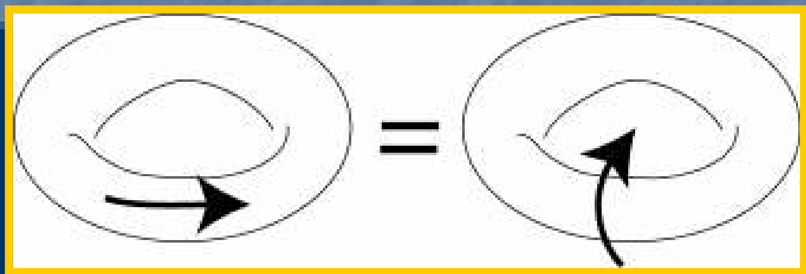
- *Key (old) problem in (classical) Field Theory:*
  - *Only s=0,1/2,1,3/2,2*
- *Key role in String Theory:*
  - *(Non) Planar duality of tree amplitudes*
  - *Modular invariance and soft U.V.*
  - *Open-closed duality*
  - *…………………*

For instance ….

$$\sum_n \frac{R_n(t)}{\alpha(s)-n} = \sum_n \frac{R_n(s)}{\alpha(t)-n}$$



- *(Non-) planar duality rests on infinitely many poles*
- *[Actual t (or s) dependence implies a growing sequence of spins]*
- *Similarly for modular invariance:*



What do we know ?

- *Flat-space formulation* *(with a number of recent surprises which I will try to illustrate)*
- *Extension to (A) dS backgrounds*
- *Inconsistency of more general backgrounds for individual HS fields*
- *Two well-defined frameworks with infinitely many interacting HS fields :*
    1. *STRING THEORY: broken HS symmetries, same scale in masses and interactions*
    2. *VASILIEV' EQUATIONS: unbroken HS symmetries, same scale in s=2 C.C. and interactions*

       [BACKGROUND INDEPENDENT , non Lagrangian]

What is "Spin" here ?

- *D=4 :*
  - *Up to dualities, all cases exhausted by fully symmetric (spinor)tensors:*

$$\Phi_{\mu_1 \ldots \mu_s} \ , \qquad \psi_{\mu_1 \ldots \mu_s}$$

- *D > 4 :*
  - *Arbitrary Young tableaux : "spin" somehow number of columns. Less developed, many general lessons can be drawn from previous special set of fields. Key contributions in the 80's by J.M.F. Labastida.*

See: - X. Bekaert and N. Boulanger, hp-th/0606198

- A. Campoleoni, D. Francia, J. Mourad and AS, to appear

Summary, I

- *Fierz-Pauli conditions: s=2 in detail*
- *Bose fields: Singh-Hagen and Fronsdal formulations*
- *Removal of trace constraints via non-local terms*
- *Non-local bosonic formulation and Higher-Spin Geometry*

[NO FERMI FIELDS FOR BREVITY]

Fierz-Pauli conditions: Bose

(Fierz, Pauli, 1939)

- Spin-s boson of mass m :

$$\begin{aligned}(\Box - M^2)\Phi_{\mu_1\ldots\mu_s} &= 0\\ \partial^{\mu_1}\Phi_{\mu_1\ldots\mu_s} &= 0\\ \Phi^{\mu_1}{}_{\mu_1\ldots\mu_s} &= 0\end{aligned}$$

- Correct degrees of freedom ($M^2>0$) :

$$\Phi \sim e^{-iMt} \quad \Rightarrow \quad \Phi_{0\mu_2\ldots\mu_s} = 0$$

Combine with trace: Only traceless spatial components

Massive case: low spins

- *Spin 1:* *(Singh, Hagen, 1974)*

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \Phi_\nu)^2 - \frac{1}{2}(\partial \cdot \Phi)^2 - \frac{M^2}{2}(\Phi_\mu)^2$$

- *Gives Proca equation:*

$$\Box \Phi_\mu - \partial_\mu(\partial \cdot \Phi) - M^2 \Phi_\mu = 0$$

- *For s=2 try (Φ traceless):*

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \Phi_{\nu\rho})^2 + \frac{\alpha}{2}(\partial \cdot \Phi_\nu)^2 - \frac{M^2}{2}(\Phi_{\mu\nu})^2$$

- *Does not give the correct Fierz-Pauli conditions*

Massive case: spin 2

$$\Box\Phi_{\mu\nu}-\frac{\alpha}{2}\left(\partial_\mu\partial\cdot\Phi_\nu+\partial_\nu\partial\cdot\Phi_\mu-\frac{2}{D}\,\eta_{\mu\nu}\,\partial\cdot\partial\cdot\Phi\right)-M^2\Phi_{\mu\nu}=0$$

- *Can still take the divergence:*

$$\left(1-\frac{\alpha}{2}\right)\Box\partial\cdot\Phi_\nu+\alpha\left(-\frac{1}{2}+\frac{1}{D}\right)\partial_\nu\partial\cdot\partial\cdot\Phi-M^2\partial\cdot\Phi_\nu=0$$

- *Take* $\alpha=2$ *(eliminate* $\Box$ *)*
- *If we could also impose:* $\partial\cdot\partial\cdot\Phi=0 \;\Rightarrow\; \partial\cdot\Phi_\nu=0$

- *Add a Lagrange multiplier (with its kinetic and mass terms):*

$$\Delta\mathcal{L}=\varphi\partial\cdot\partial\cdot\Phi+c_1(\partial_\mu\varphi)^2+c_2\varphi^2$$

Massive case: spin 2

$$\Delta\mathcal{L} = \varphi\, \partial\cdot\partial\cdot\Phi + c_1(\partial_\mu \varphi)^2 + c_2 \varphi^2$$

- *2x2 homogeneous system:*

$$\left[(2-D) - DM^2\right] \partial\cdot\partial\cdot\Phi + (D-1)\,\Box^2 \varphi = 0$$
$$\partial\cdot\partial\cdot\Phi + 2(c_2 - c_1\Box)\varphi = 0$$

- *Determinant is algebraic if :*

$$c_1 = \frac{(D-1)}{2(D-2)}$$
$$c_2 = \frac{M^2 D(D-1)}{2(D-2)^2}$$

$$\varphi = 0\ , \qquad \partial\cdot\partial\cdot\Phi = 0$$
$$\partial\cdot\Phi_\nu = 0\ , \qquad \Box\Phi_{\mu\nu} - M^2\Phi_{\mu\nu} = 0$$

Massless case: spin 2

- *Not surprisingly:* gauge symmetry as $M \to 0$ *(Fronsdal, 1978)*

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \Phi_{\nu\rho})^2 + (\partial\cdot\Phi_\nu)^2 + \varphi\,\partial\cdot\partial\cdot\Phi + \frac{D-1}{2(D-2)}(\partial_\mu\varphi)^2$$

- *The equations become:*

$$\Box\Phi_{\mu\nu} - \partial_\mu \partial\cdot\Phi_\nu - \partial_\nu \partial\cdot\Phi_\mu + \frac{2}{D}\,\eta_{\mu\nu}\,\partial\cdot\partial\cdot\Phi + \partial_\mu\partial_\nu\varphi = 0$$

$$\frac{D-1}{D-2}\Box\varphi - \partial\cdot\partial\cdot\Phi = 0$$

- *Gauge invariant under:*

$$\delta\Phi_{\mu\nu} = \partial_\mu\Lambda_\nu + \partial_\nu\Lambda_\mu - \frac{2}{D}\eta_{\mu\nu}\partial\cdot\Lambda$$

$$\delta\varphi = 2\frac{D-2}{D}\partial\cdot\Lambda$$

Massless case: spin 2

- *In terms of a traceful spin 2:*

$$\varphi_{\mu\nu} = \Phi_{\mu\nu} + \frac{1}{D-2}\,\eta_{\mu\nu}\,\varphi$$

- *"Fronsdal " eq:*

$$\mathcal{F}_{\mu\nu} \equiv \Box\varphi_{\mu\nu} - (\partial_\mu \partial\cdot\varphi_\nu + \partial_\nu \partial\cdot\varphi_\mu) + \partial_\mu\partial_\nu\varphi' = 0$$

$$\delta\varphi_{\mu\nu} = \partial_\mu\Lambda_\nu + \partial_\nu\Lambda_\mu$$

- *"Fronsdal " action:*

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi_{\nu\rho})^2 + (\partial\cdot\varphi_\mu)^2 + \frac{1}{2}\left(\partial_\mu\varphi'\right)^2 + \varphi'\partial\cdot\partial\cdot\varphi$$

- *This gives:*

$$\mathcal{F}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\mathcal{F}' = 0 \quad \Rightarrow \quad \mathcal{F}_{\mu\nu} = 0$$

Fronsdal equation, spin s

(Fronsdal, 1978)

Gauge invariance for massless symmetric tensors:

$$\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{\mu_1}\Lambda_{\mu_2\ldots\mu_s} + \ldots + \partial_{\mu_s}\Lambda_{\mu_1\ldots\mu_{s-1}}$$

$$F_{\mu_1\ldots\mu_s} \equiv \Box\varphi_{\mu_1\ldots\mu_s} - (\partial_{\mu_1}\partial.\varphi_{\mu_2\ldots\mu_s} + \ldots) + (\partial_{\mu_1}\partial_{\mu_2}\varphi'_{\mu_3\ldots\mu_s} + \ldots) = 0$$

(Originally from massive Singh-Hagen equations)

(Singh and Hagen, 1974)

Unusual constraints:

$$\Lambda' = 0, \quad \varphi'' = 0$$

Kaluza-Klein masses

- *Can extend the K-K construction to spin-s case (Stueckelberg gauge symmetries)*

$$\varphi_{D+1}^{(s)} \longrightarrow \varphi_D^{(s)}, \varphi_D^{(s-1)}, \varphi_D^{(s-2)}, \varphi_D^{(s-3)}$$
$$\Lambda_{D+1}^{(s-1)} \longrightarrow \Lambda_D^{(s-1)}, \Lambda_D^{(s-2)}$$

- *[(s-3)- parameter missing due to trace condition]*
- *[(s-4)-field missing due to double trace condition]*

- *Gauge fixing the Stueckelberg symmetries one is left with:*

$$\varphi_{D+1}^{(s)} \longrightarrow \varphi_D^{(s)}, \varphi_D^{(s-3)}$$

In terms of traceless tensors → Singh-Hagen fields

Bianchi identity

Why the unusual constraints ?

1. *Gauge variation of F :*

$$\delta F_{\mu_1 \dots \mu_s} = 3\,(\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\Lambda'_{\mu_4 \dots \mu_s} + \dots)$$

2. *Gauge invariance of the Lagrangian :*
   - *As in the spin-2 case, F not integrable*
   - *Bianchi identity:*

$$\partial \cdot F_{\mu_2 \dots \mu_s} - \frac{1}{2}\left(\partial_{\mu_2} F'_{\mu_3 \dots \mu_s} + \dots\right) = -\frac{3}{2}\left(\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\varphi''_{\mu_5 \dots \mu_s} + \dots\right)$$

Constrained gauge invariance

$$\delta L = \delta\varphi_{\mu_1\ldots\mu_s}\left[F_{\mu_1\ldots\mu_s} - \frac{1}{2}\left(\eta_{\mu_1\mu_2}F'_{\mu_3\ldots\mu_s} + \ldots\right)\right]$$

If in the variation of L one inserts

$$\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{\mu_1}\Lambda_{\mu_2\ldots\mu_s} + \ldots$$

$$\delta L = -s\Lambda_{\mu_2\ldots\mu_s}\left[\underbrace{\partial\cdot F_{\mu_2\ldots\mu_s} - \frac{1}{2}\left(\partial_{\mu_2}F'_{\mu_3\ldots\mu_s} + \ldots\right)}_{\text{Bianchi identity}:\varphi''} - \underbrace{\frac{1}{2}\left(\eta_{\mu_2\mu_3}\partial\cdot F'_{\mu_4\ldots\mu_s} + ..\right)}_{\Lambda'}\right]$$

Are the constraints really necessary?

(Francia, AS, 2002)

The spin-3 case

$$\delta F_{\mu\nu\rho} = 3\,\partial_\mu\partial_\nu\partial_\rho\Lambda' \rightarrow \delta\left(\frac{\partial_\mu\partial_\nu\partial_\rho}{\Box^2}\,\partial\cdot F'\right) = 3\,\partial_\mu\partial_\nu\partial_\rho\Lambda'$$

A fully gauge invariant (non-local) equation:

$$F_{\mu\nu\rho} = 0 \quad \Rightarrow \quad F_{\mu\nu\rho} - \frac{\partial_\mu\partial_\nu\partial_\rho}{\Box^2}\,\partial\cdot F' = 0$$

Reduces to local Fronsdal form upon partial gauge fixing

Spin 3: other non-local eqs

Other equivalent forms:

$$F_{\mu\nu\rho} - \frac{1}{3\Box}\left(\partial_\mu \partial_\nu F'_\rho + \partial_\nu \partial_\rho F'_\mu + \partial_\rho \partial_\mu F'_\nu\right) = 0$$

$$F_{\mu\nu\rho} - \frac{1}{3\Box}\left(\partial_\mu \partial\cdot F_{\nu\rho} + \partial_\nu \partial\cdot F_{\rho\mu} + \partial_\rho \partial\cdot F_{\mu\nu}\right) = 0$$

Lesson: full gauge invariance with non-local terms

Spin 3: non-local action

One can simply arrive at a non-local action (from a proper "Einstein" tensor)

$$L = -\frac{1}{2}\left(\partial_\mu \varphi_{\alpha\beta\gamma}\right)^2 + \frac{3}{2}\left(\partial \cdot \varphi_{\beta\gamma}\right)^2 - \frac{1}{2}\left(\partial \cdot \varphi'\right)^2 + \frac{3}{2}\left(\partial_\mu \varphi'_\alpha\right)^2$$

$$+3\varphi'_\alpha \, \partial \cdot \partial \cdot \varphi_\alpha + 3\partial \cdot \partial \cdot \partial \cdot \varphi \frac{1}{\Box} \partial \cdot \varphi' - \partial \cdot \partial \cdot \partial \cdot \varphi \frac{1}{\Box^2} \partial \cdot \partial \cdot \partial \cdot \varphi$$

fully invariant under

$$\delta \varphi_{\alpha\beta\gamma} = \partial_\alpha \Lambda_{\beta\gamma} + \partial_\beta \Lambda_{\gamma\alpha} + \partial_\gamma \Lambda_{\alpha\beta}$$

Implicit Notation

- *For all spins, one can eliminate all indices*
- *Need only some unfamiliar combinatoric rules*

(Francia, AS, 2002)

$$(\partial^{\,p}\,\varphi)^{\,\prime} \;=\; \Box\,\partial^{\,p-2}\,\varphi \;+\; 2\,\partial^{\,p-1}\,\partial\cdot\varphi \;+\; \partial^{\,p}\,\varphi^{\,\prime}$$

$$\partial^{\,p}\,\partial^{\,q} \;=\; \binom{p+q}{p}\;\partial^{\,p+q}$$

$$\partial\cdot(\partial^{\,p}\,\varphi) \;=\; \Box\,\partial^{\,p-1}\,\varphi \;+\; \partial^{\,p}\,\partial\cdot\varphi$$

$$\partial\cdot\eta^{\,k} \;=\; \partial\,\eta^{\,k-1}\,, \qquad \eta\eta^{k-1} = k\eta^{k}$$

$$\left(\eta^{k}\,T_{(s)}\right)^{\prime} \;=\; k\,\left[\mathcal{D} + 2(s+k-1)\right]\eta^{k-1}\,T_{(s)} \;+\; \eta^{k}\,T^{\prime}_{(s)}$$

Fronsdal equations: spin s

- *Fronsdal construction:*

$$\mathcal{F} \equiv \Box\varphi - \partial\partial\cdot\varphi + \partial^2\varphi' = 0$$
$$\delta\varphi = \partial\Lambda$$

- *Constraints:*

$$1.\ \Lambda' = 0$$
$$2.\ \varphi'' = 0$$

- *Lagrangians:*

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{s}{2}(\partial\cdot\varphi)^2 + \frac{s(s-1)}{2}\varphi'\partial\cdot\partial\cdot\varphi + \frac{s(s-1)(s-2)}{8}(\partial\cdot\varphi')^2$$

Kinetic operators for integer spin

Index-free notation:

$$F^{(1)} \equiv \Box\varphi - \partial\,\partial\cdot\varphi + \partial^2\varphi' = 0$$

$$\left(e.g. \quad \partial^p\,\partial^q \equiv \frac{(p+q)!}{p!\,q!}\,\partial^{p+q} \right)$$

Now define:

$$F^{(n+1)} = F^{(n)} + \frac{1}{(n+1)(2n+1)}\frac{\partial^2}{\Box}F^{(n)\,\prime} - \frac{1}{n+1}\frac{\partial}{\Box}\partial\cdot F^{(n)}$$

Then:

$$\delta F^{(n)} = (2n+1)\frac{\partial^{2n+1}}{\Box^{n-1}}\Lambda^{[n]}$$

Kinetic operators for integer spin

Defining:

$$\Phi(x,\xi) = \frac{1}{s!}\,\xi^{\mu_1}\ldots\xi^{\mu_s}\,\varphi_{\mu_1\ldots\mu_s}$$

$$\partial_\xi = \frac{\partial}{\partial\xi}$$

$$\prod_k \left[1+\frac{1}{(k+1)(2k+1)}\frac{\partial^2}{\Box}\partial_\xi\cdot\partial_\xi - \frac{1}{k+1}\frac{\partial}{\Box}\partial_\xi\cdot\partial\right]F^{(1)}(\Phi)=0$$

- *is the generic kinetic operator for higher spins*
- *when combined with traces can be reduced to*

$$F = \partial^3 H \qquad (\delta H = 3\Lambda')$$

Kinetic operators for integer spin

The $F^{(n)}$:

- *Are gauge invariant for $n > [(s-1)/2]$*
- *Satisfy the Bianchi identities*

$$\partial \cdot F^{(n)} - \frac{1}{2n}\partial F^{(n)\,\prime} = -\left(1+\frac{1}{2n}\right)\frac{\partial^{2n+1}}{\Box^{n-1}}\varphi^{[n+1]}$$

- *For $n > [(s-1)/2]$ allow Einstein-like operators*

$$G^{(n)} = \sum_{p=0}^{n-1} \frac{(-1)^p (n-p)!}{2^p\, n!}\,\eta^p F^{(n)[p]}$$

Connections

Christoffel connection:

$$\delta h_{\mu\nu} = \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu \quad \Rightarrow \quad \delta \Gamma^{\mu}_{\nu\rho} = \partial_\nu \partial_\rho \varepsilon^\mu$$

Generalizes to ALL symmetric tensors

(De Wit and Freedman, 1980)

$$\Gamma_{\mu;\nu_1\nu_2} \Rightarrow \Gamma_{\mu_1\ldots\mu_{s-1};\nu_1\ldots\nu_s}$$

$$R_{\mu_1\mu_2;\nu_1\nu_2} \Rightarrow R_{\mu_1\ldots\mu_s;\nu_1\ldots\nu_s}$$

Connections

$$\underbrace{\delta\left(\partial^{s-1}\varphi_s\right)}_{\binom{2s-1}{s-1}} = \underbrace{\partial^s \Lambda_{s-1}}_{\binom{2s-1}{s}}$$

$$\Gamma^{(s-1)} \sim \partial^{s-1}\varphi$$
$$R \sim \Gamma^{(s)} \sim \partial^s \varphi$$

In general:

$$\Gamma^{(m)} = \frac{1}{m+1}\sum_{k=0}^{m} \frac{(-1)^k}{\binom{m}{k}} \partial^{m-k}\nabla^k \varphi$$

$\partial, \nabla :$ *Derivatives w.r.t. two sets of sym. indices*

Connections

- *In Einstein gravity: metric (vielbein) postulate*

$$\nabla_\rho g_{\mu\nu} \equiv \partial_\rho g_{\mu\nu} - \Gamma^\alpha_{\rho\mu} g_{\alpha\nu} - \Gamma^\alpha_{\rho\nu} g_{\mu\alpha} = 0$$

- *Linearizing:*

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$$

$$\partial_\rho h_{\mu\nu} = \Gamma_{\nu;\rho\mu} + \Gamma_{\mu;\rho\nu}$$

- *For spin 3 (linearized):*

$$\partial_\sigma \partial_\tau \phi_{\mu\nu\rho} = \Gamma_{\nu\rho;\sigma\tau\mu} + \Gamma_{\rho\mu;\sigma\tau\nu} + \Gamma_{\mu\nu;\sigma\tau\rho}$$

Glimpses of HS Geometry

1. *Odd spins (s=2n+1):*

$$\partial_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{1}{\Box^n} \partial_\mu R^{[n]\mu;\nu_1 \dots \nu_s} = 0$$

2. *Even spins (s=2n):*

$$R^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{1}{\Box^{n-1}} R^{[n];\nu_1 \dots \nu_s} = 0$$

Summary, II

- *Relation with String Theory*
- *Removal of trace constraints via local terms*
- *Compensator equations for Higher Spins & (A)dS extensions*
- *External currents*
- *The Vasiliev construction (and the compensator)*

[NO FERMI FIELDS FOR BREVITY]

Bosonic string: BRST

- The starting point is the Virasoro algebra:

$$
\begin{aligned}
L_k &= \frac{1}{2} \sum_{l=-\infty}^{+\infty} \alpha^{\mu}_{k-l}\, \alpha_{\mu l} \\
[L_k, L_l] &= (k-l)\, L_{k+l} + \frac{\mathcal{D}}{12}\, m\,(m^2 - 1)
\end{aligned}
$$

- In the low-tension limit, one is left with:

$$
\begin{aligned}
\ell_0 &= p^2 \\
\ell_k &= p \cdot \alpha_k
\end{aligned}
$$

- Virasoro contracts (no c. charge):

$$
[\ell_k, \ell_l] = k\, \delta_{k+l,\,0}\ \ell_0
$$

Low-tension limit

- *Similar simplifications for BRST charge:*

$$\mathcal{Q} = \sum_{-\infty}^{+\infty} \left[\mathcal{C}_{-k}\, \mathcal{L}_k - \frac{1}{2}(k-l) : \mathcal{C}_{-k}\, \mathcal{C}_{-l}\, \mathcal{B}_{k+l} : \right] - \mathcal{C}_0$$

$$Q = \sum_{-\infty}^{+\infty} \left[c_{-k}\, \ell_k - \frac{k}{2} b_0\, c_{-k}\, c_k \right]$$

$$(Q^2 = 0 \quad \forall \mathcal{D})$$

- *Making zero-modes manifest:*

$$Q = c_0\, \ell_0 - b_0\, M + \tilde{Q}$$

$$|\Phi\rangle = |\varphi_1\rangle + c_0\, |\varphi_2\rangle$$

$$|\Lambda\rangle = |\Lambda_1\rangle + c_0\, |\Lambda_2\rangle$$

$$\tilde{Q} = \sum_{k \neq 0} c_{-k}\, \ell_k$$

$$M = \frac{1}{2} \sum_{-\infty}^{+\infty} k\ c_{-k}\, c_k$$

Symmetric triplets

(A. Bengtsson, 1986)
(Henneaux, Teitelboim, 1987)
(Pashnev, Tsulaia, 1998)
(Francia, AS, 2002)
(Bonelli, 2003)
....

- *Emerge from* $\alpha_{-1}, b_{-1}, c_{-1}$

$$\mathcal{Q}\,|\Psi\rangle = 0$$
$$\delta\,|\Psi\rangle = \mathcal{Q}\,|\Lambda\rangle$$

(Kato and Ogawa, 1982; Witten; Neveu, West et al, 1985)

$$|\varphi_1\rangle = \frac{1}{s!}\,\varphi_{\mu_1\ldots\mu_s}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_s}\,|0\rangle + \frac{1}{(s-2)!}\,D_{\mu_1\ldots\mu_{s-2}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-2}}\,c_{-1}\,b_{-1}\,|0\rangle$$
$$|\varphi_2\rangle = \frac{-i}{(s-1)!}\,C_{\mu_1\ldots\mu_{s-1}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-1}}\,b_{-1}\,|0\rangle$$
$$|\Lambda\rangle = \frac{i}{(s-1)!}\,\Lambda_{\mu_1\mu_2\ldots\mu_{s-1}}(x)\,\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-1}}b_{-1}\,|0\rangle$$

- *The triplets are:*

$$\Box\,\varphi = \partial\,C\,,\qquad \partial\cdot\varphi - \partial\,D = C\,,\qquad \Box\,D = \partial\cdot C$$

$$\delta\varphi = \partial\,\Lambda\,,\qquad \delta C = \Box\,\Lambda\,,\qquad \delta D = \partial\cdot\Lambda$$

Symmetric triplets

- *Can also eliminate C:*

$$\mathcal{F} \;=\; \partial^2 \left(\varphi' - 2D\right)$$
$$\Box\, D \;=\; \frac{1}{2}\,\partial\cdot\partial\cdot\varphi \;-\; \frac{1}{2}\,\partial\,\partial\cdot D\;,$$

$$\mathcal{L} \;=\; -\frac{1}{2}\,(\partial_\mu\varphi)^2 \;+\; s\,\partial\cdot\varphi\, C \;+\; s(s-1)\,\partial\cdot C\, D$$
$$+\;\frac{s(s-1)}{2}\,(\partial_\mu D)^2 \;-\; \frac{s}{2}\, C^{\,2}\;,$$
$$\mathcal{L} \;=\; -\frac{1}{2}\,(\partial_\mu\varphi)^2 \;+\; \frac{s}{2}(\partial\cdot\varphi)^2 \;+\; s(s-1)\,\partial\cdot\partial\cdot\varphi\, D$$
$$+\;s(s-1)\;(\partial_\mu D)^2 \;+\; \frac{s(s-1)(s-2)}{2}\,(\partial\cdot D)^2$$

- *Gauge theories of* $\ell_0,\ \ell_{\pm 1}$
- *Physical state conditions:*
- *Propagate spins s,s-2, …, 0 or 1*

$$\Box\varphi = 0$$
$$\partial\cdot\varphi = 0$$
$$\left[\varphi' = 0\right]$$

Compensator Equations

- *Triplet eqs (eliminating C):*

$$\mathcal{F} = \partial^2 \left(\varphi' - 2D\right)$$

$$\Box D = \frac{1}{2}\partial \cdot \partial \cdot \varphi - \frac{1}{2}\partial\, \partial \cdot D$$

$$\varphi' - 2D = \partial \alpha \quad \Rightarrow$$

$$\mathcal{F} = 3\,\partial^3 \alpha$$

$$\varphi'' = 4\partial \cdot \alpha + \partial \alpha'$$

$$\delta\varphi = \partial\Lambda\,, \qquad \delta\alpha = \Lambda'$$

- *Describe a spin-s gauge field with:*
  - *NO trace constraints on the gauge parameter or gauge field*
  - *First can be reduced to minimal non-local form*
  - *simple (A)dS extension*
- *NOT Lagrangian equations*

$$\mathcal{F}^{(k)} = (2k + 1)\,\frac{\partial^{2k+1}}{\Box^{k-1}}\,\alpha^{[k-1]}$$

Minimal local Lagrangians

(Francia, AS, 2005; Francia, Mourad and AS, 2007)

- *"Minimal" local Lagrangians with unconstrained gauge symmetry:*

$$\left[\mathcal{F} = \Box\varphi - \partial\partial\cdot\varphi + \partial^2\varphi'\right]$$
$$\mathcal{A} = \mathcal{F} - 3\,\partial^3\alpha$$
$$\partial\cdot\mathcal{A} - \frac{1}{2}\partial\mathcal{A}' = -\frac{3}{2}\partial^3\left(\varphi'' - 4\partial\cdot\alpha - \partial\alpha'\right)$$

- *The Lagrangians are:*

$$\mathcal{L} = \frac{1}{2}\varphi\left(\mathcal{A} - \frac{1}{2}\,\eta\,\mathcal{A}'\right) - \frac{3}{4}\binom{s}{3}\alpha\,\partial\cdot\mathcal{A}' + 3\binom{s}{4}\beta\left[\varphi'' - 4\,\partial\cdot\alpha - \partial\alpha'\right]$$

Can be nicely extended to (A)dS backgrounds

BRST and Compensator Equations

- *It is also possible to obtain a Lagrangian form of the compensator equations, using BRST techniques* *(Pashnev, Tsulaia, 1998)*
- *Formulation involves number of fields ~ s*
- *Interesting BRST subtleties*
- *Can be reduced to "minimal" compensator equations*

(AS and Tsulaia, 2003)

- *e.g. s=3 Fields:*

$$\varphi, C, D, \alpha, \left[\varphi^{(1)}, C^{(1)}, E, F\right]$$
$$\Lambda, \left[\Lambda^{(1)}, \mu\right]$$

Off-Shell truncation of triplets

(Buchbinder, Krykhtin, Reshetnyak 2007)

Off-shell reduction of triplets :

- *start from a triplet (s,s-2,…)*

$$\Box\,\varphi = \partial\, C\,,$$
$$\partial\cdot\varphi - \partial\, D = C$$
$$\Box\, D = \partial\cdot C$$

- *add two (gauge invariant) Lagrange multipliers :*

$$\lambda : \quad \varphi\,' - 2\,D - \partial\alpha = 0$$
$$\mu : \quad D\,' - \partial\cdot\alpha = 0$$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\,(\partial_\mu\varphi)^2 + s\,\partial\cdot\varphi\, C + s(s-1)\,\partial\cdot C\, D \\ & + \frac{s(s-1)}{2}\,(\partial_\mu D)^2 - \frac{s}{2}\,C^{\,2} \\ & + \lambda\left(\varphi\,' - 2\,D - \partial\alpha\right) + \mu\left(D\,' - \partial\cdot\alpha\right)\end{aligned}$$

λ and μ : set to zero by the field equations

External currents

- *Residues of current exchanges reflect the degrees of freedom*

- *For s=1 :*

$$p^2 A_\mu - p_\mu p \cdot A = J_\mu$$

$$p^2 J^\mu A_\mu = J^\mu J_\mu \Rightarrow J_i J_i$$

- *For all s :*

$$\mathcal{A} - \frac{1}{2}\eta\,\mathcal{A}' + \eta^2 \mathcal{B} = J$$
$$\partial \cdot \mathcal{A}' - (2\,\partial + \eta\,\partial\cdot)\,\mathcal{B} = 0$$
$$\varphi'' - 4\,\partial \cdot \alpha - \partial\,\alpha' = 0$$

External currents : local case

$$\varphi'' - 4\,\partial\cdot\alpha - \partial\alpha' = 0$$

- *$\mathcal{K}$ "doubly traceless" using double trace constraint*
- *$\mathcal{B}$: determines multiplier β for double trace constraint*

$$\mathcal{A} - \frac{1}{2}\eta\,\mathcal{A}' = J - \eta^2\,\mathcal{B} \equiv \mathcal{K}$$

$$\varphi'' - 4\,\partial\cdot\alpha - \partial\,\alpha' = 0$$

$$Traceless\ Proj\ :\ \sum_0^N \rho_n(D,s)\,\eta^n\,V^{[n]}$$

$$\rho_{n+1}(D,s) = -\frac{\rho_n(D,s)}{D+2(s-n-2)}$$

$$\mathcal{K} = J + \sum_{n=2}^N \sigma_n\,\eta^n\,J^{[n]}$$

$$\sigma_n + [D+2(s-n-3)]\,\{2\sigma_{n+1} + [D+2(s-n-4)]\,\sigma_{n+2}\} = 0$$

$$\sigma_n = (-n+1)\,\rho_n(D-2,s)$$

$$\mathcal{A} = \mathcal{K} + \rho_1(D-2,s)\,\eta\,\mathcal{K}' = \sum_n \rho_n(D-2,s)\,\eta^n\,J^{[n]}$$

$$\sum_{n=0}^N \rho_n(D-2,s)\,\frac{s!}{n!\,(s-2n)!\,2^n}\,J^{[n]}\cdot J^{[n]}$$

The exchange involves, correctly, a traceless conserved current

External currents : non-local case

How about the non-local version of the theory?

Apparently: different choices for the field equation, EQUIVALENT without currents

S=3 :

$$\mathcal{F}_{\mu\nu\rho} - \frac{1}{3}\frac{1}{\Box}\left(\partial_\mu\partial_\nu\mathcal{F}'_\rho + \dots\right) = 0$$

$$\mathcal{F}_{\mu\nu\rho} - \frac{\partial_\mu\partial_\nu\partial_\rho}{\Box^2}\,\partial\cdot\mathcal{F}' = 0$$

→

$$\frac{1}{\Box}\,\eta_{\alpha\beta}\,\partial_\mu\,\mathcal{R}^{\mu\alpha\beta}{}_{;\nu_1\nu_2\nu_3} = 0$$

$$\mathcal{F} = 3\,\partial^3\alpha$$

→

$$\mathcal{F}^{(n)} = (2n+1)\,\frac{\partial^{2n+1}}{\Box^{n-1}}\,\alpha^{[n-1]}$$

$$\mathcal{F}^{(n+1)} = \mathcal{F}^{(n)} + \frac{1}{(n+1)(2n+1)}\frac{\partial^2}{\Box}\mathcal{F}^{(n)\,\prime} - \frac{1}{n+1}\frac{\partial}{\Box}\,\partial\cdot\mathcal{F}^{(n)}$$

$$\delta\mathcal{F}^{(n)} = (2n+1)\frac{\partial^{2n+1}}{\Box^{n-1}}\Lambda^{[n]}$$

$$\partial\cdot\mathcal{F}^{(n)} - \frac{1}{2n}\,\partial\mathcal{F}^{(n)\,\prime} = -\left(1+\frac{1}{2n}\right)\frac{\partial^{2n+1}}{\Box^{n-1}}\,\varphi^{(n+1)}$$

Bianchi identity: changes after every iteration

External currents : non-local case

Naively:

$$\mathcal{G}^{(n)} \equiv \sum_{p=0}^{n} (-1)^p \frac{(n-p)!}{2^p\, n!} \eta^p \mathcal{F}^{(n)\,[p]} = \mathcal{J}$$

$$\mathcal{F}^{(n)} = \sum_{n=0}^{n} \rho_n(D-2n, s)\, \eta^n \mathcal{J}^{[n]}$$

Incorrect current exchange !

Solution: modify the non-local Lagrangian equation

$$\mathcal{A}_{nl} = \mathcal{F} - 3\,\partial^3 \alpha_{nl} \; ; \quad \mathcal{C}_{nl} \equiv \varphi'' - 4\,\partial \cdot \alpha_{nl} - \partial\, \alpha'_{nl} = 0$$

$$\partial \cdot \mathcal{A}_{nl} - \frac{1}{2}\, \partial \mathcal{A}'_{nl} = 0 \;\; (\text{Mod } \mathcal{C}_{nl}) \; ; \quad \partial \cdot \mathcal{A}_{nl} = 2\,\partial\, \mathcal{D}_{nl}$$

$$\mathcal{G}_{nl} = \mathcal{A}_{nl} - \frac{1}{2}\, \eta\, \mathcal{A}'_{nl} + \eta^2 \mathcal{D}_{nl} + \ldots + \eta^{n+2} \frac{\mathcal{D}_{nl}^{[n]}}{2^n\, n!} \qquad (s = 2n+4 \text{ or } s = 2n+5)$$

$$\mathcal{A}_{nl} = \sum_{k=0}^{n+1} a_k \frac{\partial^{2k}}{\Box^k} \mathcal{F}^{(n+1)\,[k]} \; ; \quad a_k = (-1)^{k+1} (2k-1) \frac{n+2}{n-1} \prod_{j=-1}^{k-1} \frac{n+j}{n-j+1}$$

$$\mathcal{D}_{nl} = \frac{1}{2} \sum_{k=2}^{n+1} a_k \left\{ \frac{1}{2k-3} \frac{\partial^{2(k-2)}}{\Box^{k-2}} \mathcal{F}^{(n+1)\,[k]} + \frac{2n+4k+1}{2(2k-1)(n-k+1)} \frac{\partial^{2(k-1)}}{\Box^{k-1}} \mathcal{F}^{(n+1)\,[k+1]} + \frac{n+k+1}{2(n-k)(n-k+1)} \frac{\partial^{2k}}{\Box^k} \mathcal{F}^{(n+1)\,[k+2]} \right\}$$

For instance :

$$\mathcal{A}_3 = \frac{1}{\Box} \partial \cdot \mathcal{R}' + \frac{\partial^2}{2\,\Box^2} \partial \cdot \mathcal{R}''$$

$$\mathcal{A}_4 = \frac{1}{\Box} \mathcal{R}'' + \frac{1}{2} \frac{\partial^2}{\Box^2} \mathcal{R}''' - 3 \frac{\partial^4}{\Box^3} \mathcal{R}^{[4]}$$

$$\mathcal{A}_5 = \frac{1}{\Box^2} \partial \cdot \mathcal{R}'' + \frac{2}{3} \frac{\partial^2}{\Box^3} \partial \cdot \mathcal{R}''' - 3 \frac{\partial^4}{\Box^4} \partial \cdot \mathcal{R}^{[4]}$$

$$\mathcal{D}_4 = -\frac{3}{8} \frac{1}{\Box} \mathcal{R}^{[4]}$$

$$\mathcal{D}_5 = -\frac{5}{8\,\Box^2} \partial \cdot \mathcal{R}^{[4]}$$

vD-V-Z Discontinuity for HS

J ———— J

$$m = 0 \; : \quad T_{\mu\nu}T^{\mu\nu} - \frac{1}{2}(T\,')^2$$

(van Dam, Veltman; Zakharov, 1970)

$$m \neq 0 \; : \quad T_{\mu\nu}T^{\mu\nu} - \frac{1}{3}(T\,')^2$$

For all s and D, m=0 :

$$\sum_{n=0}^{N} \rho_n(D-2,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]}$$

$$\rho_{n+1}(D,s) = -\frac{\rho_n(D,s)}{D+2(s-n-2)}$$

$$\Rightarrow \quad s = 2 \; : \quad T_{\mu\nu}T^{\mu\nu} - \frac{1}{D-2}(T\,')^2$$

- *VDVZ discontinuity follows in general comparing D and (D+1) massless exchanges*
- *First present for s=2*
- *For all s: can describe irreducibly a massive field a' la Scherk-Schwarz from (D+1) dimensions :*

$$\sum_{n=0}^{N} \rho_n(D-2,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]} \quad \Longleftrightarrow \quad \sum_{n=0}^{N} \rho_n(D-1,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]}$$

(A)dS extension, first discussed, for s=2, by Higuchi and Porrati
Discontinuity → smooth interpolation in $(mL)^2$
(Francia, Mourad, AS, to appear)

HS Interactions: problems

Problems with interacting higher spins :

- *Inconsistent equations (derivatives imply further conditions)*
- *Coupling with gravity leaves "naked" Weyl tensors* *(Aragone, Deser,1979)*
- *Coleman - Mandula*

Way out:

(Berends, Burgers, van Dam, 1982)
(Bengtsson[2], Brink, 1983)
(Fradkin and Vasiliev, 1980's)

- *Infinitely many interacting fields*
- *Non-vanishing "cosmological constant"* Λ
- *Vasiliev equations: paradigmatic example*

(Vasiliev, 1990, 2003)
(Sezgin, Sundell, 2001)

HS Interactions: Vasiliev's setting

Vasiliev's setting:

$$g_{\mu\nu} \;\longrightarrow\; e_\mu^a \;,\; \omega_\mu^{ab} \;\longrightarrow\; \omega_\mu^{A;B}$$

1. *Extend the frame formulation of gravity :*

$$\omega_\mu^{A;B} \; : \; \begin{array}{|c|}\hline A \\ \hline B \\ \hline \end{array}$$

$$\varphi_{\mu_1 \dots \mu_s} \;\longrightarrow\; \omega_\mu^{A_1 \dots A_{s-1}; B_1 \dots B_{s-1}}$$

- *For spin-s:*

$$\omega_\mu^{A_1 \dots A_{s-1}; B_1 \dots B_{s-1}} \; : \; \begin{array}{|c|c|c|}\hline A_1 & \cdots & A_{s-1} \\ \hline B_1 & \cdots & B_{s-1} \\ \hline \end{array}$$

2. ∞ - *dim. HS-algebra via oscillators (coordinates and momenta):*

$$M_{A;B} \rightarrow M_{A_1 \dots A_s; B_1 \dots B_s}$$

$$M_{A;B} = x_A p_B - x_B p_A$$

$$M_{A_1 \dots A_s; B_1 \dots B_s} = x_{A_1} \cdots x_{A_s} p_{B_1} \cdots p_{B_s} \pm \dots$$

HS Interactions: (twisted) adjoint

3. [Weyl ordered (symmetric) polynomials in (x,p) or $*$- products]

4. A one-form $\mathcal{A}$ in adjoint of HS algebra :
   (all ω's : HS vielbeins and connections)

$$\mathcal{A}(x^\mu | x_A, p_A)$$
$$F = dA + A \wedge \star A$$

5. A zero-form Φ in the "twisted adjoint":

- Write spin-2 equation in the form:
  - trace gives the familiar "Ricci=0"
- "Weyl" (+ derivatives) :

"Riemann = Weyl"

$\Phi^{A_1 \ldots A_{s+k}; B_1 \ldots B_s}$:

	⋯		s	⋯	s+k
	⋯	s			

→ Scalar : φ

HS Interactions: oscillators

- TWO forms of Vasilev's oscillators :

1. 4-dim spinors :

$$\left[\xi_\alpha , \xi_\beta\right] = i\,\epsilon_{\alpha\beta}\ ; \qquad \left[\bar{\xi}_{\dot\alpha} , \bar{\xi}_{\dot\beta}\right] = i\,\epsilon_{\dot\alpha\dot\beta}$$

2. D-dim vectors :

$$[Y_{iA}, Y_{jB}] = 2i\epsilon_{ij}\ \eta_{AB}$$

$$\widehat{f}(Z;Y)\star\widehat{g}(Z;Y) = \int \frac{d^{2(D+1)}S\, d^{2(D+1)}T}{(2\pi)^{2(D+1)}}\ \widehat{f}(Z+S;Y+S)\,\widehat{g}(Z-T;Y+T)\, e^{iT^{iA}S_{iA}}$$

$$Z^{iA} \equiv (Z^{ia}, Z^i)$$

HS Interactions: Vasiliev equations

$$\widehat{F} \;=\; \frac{i}{2}\, dZ^i \wedge dZ_i \; \widehat{\Phi} \star k$$

$$\widehat{D}\,\widehat{\Phi} \;=\; 0$$

- *Background independent (non-Lagrangian) !*
- *A: one field for every (even) rank s ("adjoint" of HS algebra)*
  (Generalized vielbeins and connections)
- *[Chan-Paton extension to all (even and odd) ranks]*
- *Φ: ∞ fields for every rank s ("twisted adjoint" of HS algebra)*
  (Generalized Weyl and their covariant derivatives)
- *Φ*κ: converts "twisted adjoint" Φ to adjoint*
- *Consistent (almost by inspection): Bianchi for F implies second eq !*

HS Interactions: internal expansion

$$\widehat{F} \;=\; \frac{i}{2}\, dZ^i \wedge dZ_i \; \widehat{\Phi} \star k$$
$$\widehat{D}\,\widehat{\Phi} \;=\; 0$$

- *Gauge field $\mathcal{A}$ in (x,z) space:* $\widehat{A}(x^\mu|Y^{iA}, Z^{iA}) \;=\; \widehat{A}_\mu dx^\mu \;+\; \widehat{A}_{ia} dZ^{ia} \;+\; \widehat{A}_i dZ^i$

$$\widehat{A}_{ia} = 0 \;,\; \partial_{ia}\widehat{\Phi} = 0$$
$$\partial_{ia}\widehat{A}_i = 0 \;,\; \partial_{ia}\widehat{A}_\mu = 0$$

$$\widehat{F}_{ij} \;=\; -\, i\; \epsilon_{ij}\; \widehat{\Phi} \star k$$
$$\widehat{F}_{\mu\nu} = 0 \;, \qquad \widehat{F}_{\mu i} = 0$$
$$D_\mu \widehat{\Phi} = 0 \;,\; D_i \widehat{\Phi} = 0$$

Internal equations: power series in Φ by successive iterations

HS Interactions: unfolding

- *Linearized Φ – equation :* $$D_\mu \Phi + \frac{1}{2i}\{P_a, \Phi\} = 0$$

- *Unfolding :*
$$\varphi_\mu = \partial_\mu \varphi$$
$$\varphi_{\mu\nu} = \partial_\mu \varphi_\nu$$
$$\dots$$
$$\varphi_{\mu(n+1)} = \partial_\mu \varphi_{\mu(n)}$$
$$\Rightarrow \quad \eta^{\mu\nu}\varphi_{\mu\nu} = 0 \rightarrow \Box\varphi = 0$$

Uniform description of HS interactions

HS Interactions: Cartan I.S.

$$\widehat{F} \;=\; \frac{i}{2}\, dZ^i \wedge dZ_i\; \widehat{\Phi} \star k$$
$$D\widehat{\Phi} \;=\; 0$$

Cartan Integrable System :

(Sullivan, 1977; D'Auria, Fre', 1982)

- *e.g. Chern-Simons theory*
- *manifestly consistent eqs*
- *gauge covariance*
- *manifest diff covariance*
- *non-Lagrangian*

$$R^i \equiv dW^i + f^i(W) = 0 \quad \text{with} \quad f^i(W)\,\frac{\partial f^i(W)}{\partial W^j} = 0$$
$$\delta W^i = d\Lambda^i - \Lambda^j\,\frac{\partial f^i(W)}{\partial W^j}$$
$$\delta R^i = (-)^j\,\Lambda^j\, R^k\,\frac{\partial^2 f^i(W)}{\partial W^k\,\partial W^j}$$

- *NEW INGREDIENT : 0-form Φ*

HS Interactions: projections

$$\widehat{F} = \frac{i}{2}\, dZ^i \wedge dZ_i\, \widehat{\Phi} \star k$$
$$D\widehat{\Phi} = 0$$

$$\left[Y^{iA}, Y^{jB}\right] = i\,\epsilon^{ij}\,\eta^{AB}$$

Some missing ingredients :

- Y^{iA}, Z^{iA} to build HS algebra extending SO(2,D)
- must select Sp(2,R) singlets
- K_{ij}: Sp(2,R) generators (bilinears in Y, Z)

$$\widehat{D}\,\widehat{K}_{ij} = 0$$
$$\left[\widehat{K}_{ij}, \widehat{\Phi}\right]_\star = 0$$

- *REMOVE TRACES to obtain dynamical equations*
  - *Weak projection* : remove traces symmetrically from $\mathcal{A}$ and Φ (Vasiliev, 2003)
  - *Strong* : leave traces in $\mathcal{A}$ (AS,Sezgin,Sundell, 2004)

$$\widehat{K}_{ij} \star \widehat{\Phi} = 0$$

HS Interactions: linearization

Free flat limit (w. Strong projection):

$$\partial_{[\mu}\,\omega^{(s-1,k)}_{\nu],a(s-1),b(k)} + \omega^{(s-1,k+1)}_{[\mu|,a(s-1),|\nu]b(k)} = \delta_{k,s-1}\,\Phi_{[\mu|a(s-1),|\nu]b(s-1)}$$

- $s = 2$:

$$\partial_{[\mu}\,e^{a}_{\nu]} - \omega_{[\mu\nu]}{}^{a} = 0 \rightarrow \omega(e)$$

$$R_{\mu\nu}{}^{ab} = \partial_{[\mu}\,\omega_{\nu]}{}^{ab} = \Phi_{\mu a,\nu b} \rightarrow R_{\nu b} = 0$$

- $s = 3$:
  - *a first equation, analogous of the vielbein postulate giving* $\omega(e)$
  - *a second equation, defining a second-order kinetic operator*

$$\omega^{(2,2)\;c}_{\mu,ab,c} = \Box\,\varphi_{\mu ab} - \partial_a\,\partial\cdot\varphi_{\mu b} - \partial_b\,\partial\cdot\varphi_{\mu a} + \partial_a\,\partial_b\,\varphi'_{\mu}$$

  - *a third equation giving the constraint*

$$\partial_{[\mu}\omega^{(2,2)\;c}_{\nu],ab,c} = 0 \rightarrow \omega^{(2,2)\,c}_{\mu,ab,c} = \partial_\mu\beta_{ab}$$

HS Interactions: the compensator

(AS, Sezgin, Sundell, 2004)

$$\Box\, \varphi_{\mu ab} - \partial_a\, \partial\cdot\varphi_{\mu b} - \partial_b\, \partial\cdot\varphi_{\mu a} + \partial_a\, \partial_b\, {\varphi'}_{\mu} \;=\; \partial_\mu\, \beta_{ab}$$

$$\partial_{[\mu}(\partial\cdot\varphi_{a]b} - \partial_{a]}\varphi'_b) \;=\; \partial_{[\mu}\beta_{b]a}$$

$$\beta_{ab} = (\partial\cdot\varphi_{ab} - \partial_a\varphi'_b - \partial_b\varphi'_a) + 3\,\partial_a\partial_b\alpha$$

(Dubois-Violette, Henneaux, 1999)

$$\mathcal{F} \;=\; 3\,\partial^3\,\alpha$$

Conclusions

- *Why Higher Spins?*
  - *Field Theory*
  - *String Theory (an instance with "spontaneous breaking")*

- *Here:*
  - *Unconstrained HS fields [Bose for brevity]*
    1. *Non-local (geometry)*
    2. *Local (compensator)*
  - *Vasiliev' construction and the compensator*
  - *(flat-space) current exchanges (vDVZ & non-local actions)*

HS Interactions

"Internal" equations: power series in Φ

$$\partial_i \widehat{A}_j - \partial_j \widehat{A}_i + \left[\widehat{A}_i, \widehat{A}_j\right] = -i\epsilon_{ij}\widehat{\Phi} \star k$$
$$\partial_i \widehat{\Phi} + \widehat{A}_i \star \widehat{\Phi} - \widehat{\Phi} \star \pi\left(\widehat{A}_i\right) = 0$$

with

$$\pi(Y^{ia}) = Y^{ia}\ ,\ \pi(Y^i) = -Y^i$$
$$\pi(Z^{ia}) = Z^{ia}\ ,\ \pi(Z^i) = -Z^i$$

$$\widehat{A}_i = \partial_i \zeta + Z_i \int_0^1 t dt \left[-i\widehat{\Phi} \star k + \widehat{A}^i \wedge \widehat{A}_i\right]_{Z \to tZ}$$
$$\widehat{\Phi} = -Z^i \int_0^1 dt \left[\widehat{A}_i \star \widehat{\Phi} + \widehat{\Phi} \star \pi\left(\widehat{A}_i\right)\right]_{Z \to tZ}$$

Lowest order in Φ → "Riemann = Weyl" + …

$$D_\mu A_\nu - D_\nu A_\mu \ = \ - \ 2\, i\, e_\mu^a\ e_\nu^b\ \frac{\partial^2 \Phi}{\partial Y^{ia} \partial Y^{jb}}$$
$$D_\mu \Phi + \frac{1}{2i}\{P_a, \Phi\} = 0$$