Subdivision schemes are a powerful tool for the fast generation of curves and surfaces in computer-aided geometric design. In such algorithms discrete data are recursively generated from coarse to fine scale by means of local rules. The stability and the convergence of such refinement process, as well as the smoothness properties of its limits function if it exists, they have been the subject of active research in recent years.

Marquina and Serna [4] introduced a new class of ENO reconstruction techniques, called Power ENO methods, which were considered and analyzed the subdivision context by K. Dadaudron [3]. In this work, we analyze a generalization of the Power subdivision schemes, in [3], based on an harmonic weighted average instead of the plain harmonic average considered in [3]. Our development is motivated by the more complicated analysis of WENO subdivision schemes [1, 2].

The poster is organized of the following way. In the section 2 we describe briefly the scheme $\text{power}_p$. In the section 3 we present the schemes $\text{power}_{p,t}$. Finally, in the section 4, we compare the efficiency these schemes with a type of signal and in the section 5 we explain our conclusions and perspectives.

2. SCHEMES $\text{power}_p$

The average $\text{power}_p$ [4] uses the arithmetic average of two values $x$ and $y$ ($M(x,y) = \frac{x+y}{2}$), a possible extension of this average is realized using the an harmonic weighted average, $M_h(x,y) = \frac{x+y}{a(x+y)}$ with $a > 0$. The alternative averages are the following ones for $p \geq 1$, $p \in \mathbb{N}$:

$$\text{power}_p(x,y) = \frac{\frac{a(x+y)}{p} + \frac{a(x+y)}{p+1}}{2} = \frac{\frac{p+1}{a(x+y)} + \frac{p}{a(x+y)}}{2} = \frac{p+2}{a(x+y)}$$

where $z = \max(|x|,|y|)$, $w = \min(|x|,|y|)$, $c = \max(a,b)$, $y = \min(a,b)$.

It is trivial to see that $\text{power}_p(x,y) = \text{power}_p(y,x)$, therefore we suppose that $0 < a, b < 1$ and that $a + b = 1$.

The scheme $\text{power}_{p,t}$ can define from Lagrange’s scheme $S_{1,2}$. This scheme can rewrites as the disturbance of the linear scheme $S_{1,1}$:

$$(S_{1,1})_n = f_n$$

$$(S_{1,2})_{n+1} = \frac{f_{n+1} + f_n}{2} + \frac{d f_n}{2}$$

If the arithmetic mean is replaced by the average $\text{power}_p$ the definition of the scheme $\text{power}_{p,t}$ is obtained:

$$(S_{1,2})_{n+1} = f_n$$

$$(S_{2,2})_{n+1} = \frac{f_{n+1} + f_n}{2} + \frac{d f_n}{2} - \text{power}_p(d f_n, f_n)$$

Algebraic properties of the average $\text{power}_p$

The average $\text{power}_p$ with $p \geq 1$ satisfies the following properties $V(x,y) \in \mathbb{R}$:

$E_1$: $\text{power}_p(x,y) = \text{power}_p(y,x)$,

$E_2$: $\text{power}_p(x,y) = 0$ if $x \neq y$.

$E_3$: $\text{power}_p(x,-y) = -\text{power}_p(x,y)$.

$E_4$: $\text{power}_p(x,\pm x) = \text{power}_p(x,y)$, $\forall y \in \mathbb{R}$.

$E_5$: $\text{power}_p(x,y) = \text{power}_p(x, y)$, $\forall y \in \mathbb{R}$ \{0\}.

$E_6$: $\text{power}_p(x,y) = x \forall x \in \mathbb{R}$.

Estimations for the average $\text{power}_p$

$P_1$: $p \geq 2$ satisfies:

$C_1$: $\text{power}_p(x,y) \leq \max(|x|,|y|)$ $\forall (x,y) \in \mathbb{R}$.

$C_2$: $\text{power}_p(x,y) \leq 2 \max(|x|,|y|) \forall (x,y) \in \mathbb{R}$.

$C_3$: $\text{power}_p(x,y) \leq 2 \max(|x|,|y|) \max(|x|,|y|)$ $\forall (x,y) \in \mathbb{R}$.

$C_4$: $\text{power}_p(x,y) \leq \frac{1}{2} \max(|x|,|y|) \leq \frac{1}{2} \min(|x|,|y|)$ $\forall (x,y) \in \mathbb{R}$.

For $x, y > 0$, $\frac{1}{2} \min(|x|,|y|) \leq \text{power}_p(x, y) \leq \frac{1}{2} \max(|x|,|y|)$. $\forall (x,y) \in \mathbb{R}$.

The indicators of smoothness can be chosen of two forms:

- Indicators type I $f_{n+1} = (df_0 + f_0) + f_{n+1}$ and $f_{n+2} = (df_{n+1} + f_{n+1})$.

- Indicators type II We define also $I_f = (\alpha_1 - \beta_1)^2 + 2 \beta_1$.

Where the coefficients $\alpha_1$ and $\beta_1$ of $I_f$ are:

$$\alpha_1 = \frac{1}{(1 + f_0)}$$

$$\beta_1 = \frac{1}{1 + f_0}$$

We have demonstrated that:

1. The scheme $\text{power}_{p,t}$ is convergent and the regularity $C^{p+1}$ for all $p \geq 1$.

2. $\text{power}_{p,t}$ reproduces exactly polynomials of degree 2.

3. $\text{power}_{p,t}$ with $\text{power}_{p,weno}$ with the indicators smoothness type II, weno3 with the indicators of smoothness type II, $\varphi$, $\text{power}_{p,weno}$ with the indicators of smoothness type I, $\varphi$, and weno3 with the indicators of smoothness type II, $\varphi$.

4. EXPERIMENTS

We apply to an initial information, presented in $\varphi$, the methods $\text{lin}_0$, $\text{power}_{p,\varphi}$, $\text{power}_{p,weno}$ with type II, $\varphi$, $\text{power}_{p,weno}$ with type I, $\varphi$, $\text{power}_{p,\varphi}$ with type II, $\varphi$, $\text{power}_{p,weno}$ with type II, $\varphi$ with $\text{power}_{p,weno}$. For the type of information, figure 1, all the methods coincide with the exception of $\varphi$ $\text{power}_{p}$ and $\text{lin}_0$ in the singular regions (a) and (d). There exists a light difference between all the methods in the singular region (b). The worst approximation is obtained with $\text{lin}_0$. The methods presented in this paper improve the obtained results, since they are not affected of Gibb’s phenomenon. Also we observe that these new schemes obtain similar results for this type of signals.

5. CONCLUSIONS AND PERSPECTIVES

We have presented two types of schemes of subdivision and his properties. These schemes are of usefulness for the recovery of signs, but we do not observe very big differences. These methods improve to the linear scheme of 4 points. Nowadays we are using these schemes of subdivision for the compression of images. The first results that we have obtained are encouraging.

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References


Figure 1: (a) Initial information, $\varphi$, $\text{lin}_0$, $\text{power}_{p,\varphi}$, $\text{power}_{p,weno}$ with type II, $\varphi$, $\text{power}_{p,weno}$ with type I, $\varphi$, $\text{power}_{p,\varphi}$ with type II, $\varphi$, $\text{power}_{p,weno}$ with type II, $\varphi$ with $\text{power}_{p,weno}$.