Three-Dimensional Scattering of Dielectric Gratings Under Plane-Wave Excitation

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***Abstract*—In this letter, the problem of scattering of electromagnetic plane waves by one-dimensional (1-D) periodic dielectric gratings, under the most general condition of oblique incidence [three-dimensional (3-D) incidence], is rigorously solved. A recently developed vectorial modal method for obtaining the modal spectrum of 1-D dielectric periodic guiding media has been extended for considering 3-D incidence. Polarization coupling effects are included in this analysis, just demonstrating the impossibility of the separation between the transverse electric and transverse magnetic polarizations traditionally employed in the two-dimensional (2-D) case. A study of the scattering parameters of a multilayered dielectric periodic structure is accomplished by imposing the boundary conditions in terms of the multimode scattering matrix. The effect of changing the azimuthal angle of excitation is shown in the dispersion curves of the periodic dielectric medium. For 3-D incidence on a dielectric waveguide grating, it is found that total reflection can be predicted by imposing the phase-match condition, analogous to the bi-dimensional incidence case.**

***Index Terms*—Dielectric waveguide grating, frequency-selective surfaces, hybrid modes, resonance frequency, three-dimensional (3-D) incidence.**

I. Introduction

DIELECTRIC frequency selective surfaces (DFSS) are structures containing a one-dimensional (1-D) periodic array of dielectric slabs, known as dielectric waveguide gratings, which have a frequency selective behavior. When an electromagnetic plane wave is incident on a dielectric waveguide grating of the type shown in Fig. 1, this structure resonates at certain frequencies, thus providing filtering characteristics. Due to this property, the electromagnetic behavior of such structures has received considerable attention in the technical literature. In the past, the incident wave on the periodic grating was assumed to have a transverse component only in the periodic direction. Under such restrictive condition, the guidance and scattering of waves by DFSS is analyzed as a two-dimensional (2-D) boundary-value problem, and the polarization of electromagnetic fields in the structure is always preserved [1]–[5], so the transverse electric and transverse magnetic polarizations can be analyzed separately. While many efforts have been devoted to the 2-D incidence to the authors knowledge, there are few technical works considering the three-dimensional (3-D) case [6]. However, for many applications, the electromagnetic fields are varying in all three mutually perpendicular directions, and the restrictive conditions previously imposed on the structure, as well as the excitation conditions, are no longer valid. Therefore, it becomes necessary to analyze the general case of 3-D incidence, which requires to consider the simultaneous presence of mixed polarizations.

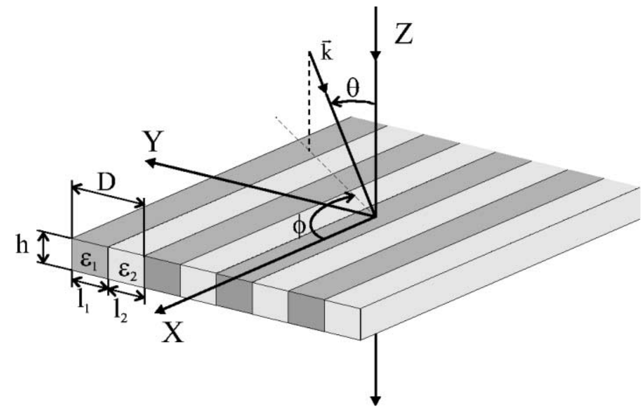

Fig. 1. Characteristic unit cell of a periodic dielectric medium.

This letter describes the guidance characteristics and scattering of a plane wave by a dielectric waveguide grating in the general case of 3-D incidence (also called conical incidence). The method used for obtaining the modal spectrum in 1-D periodic dielectric media is an extension of the method recently published in [3], [7], and [8] to the 3-D incidence case. In Section II we first present a rigorous formulation for the study of guidance of waves by an infinite dielectric periodic structure with a rectangular profile under 3-D excitation. The results are then used as the basis for the analysis of the scattering problem of a uniform plane wave by a DFSS under the most general incidence condition. Finally, in Section III, numerical results are presented showing the effect of the 3-D incidence in both the propagation constant of the periodic region, and also in the location of the resonant frequencies of a dielectric waveguide grating under 3-D plane-wave excitation.

II. Theory

The rigorous solution of the electromagnetic scattering by a dielectric waveguide grating as the one shown in Fig. 1 starts with the evaluation of the electromagnetic fields in all regions

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of the structure. Then the general solutions are requested to satisfy the boundary conditions at the plane interfaces separating the constituent layers. As shown in Fig. 1, the DFSS is taken as a dielectric waveguide grating consisting of alternating bars having relative dielectric permittivities ε_1 and ε_2, and widths l_1 and l_2, respectively, arranged periodically along the Y axis with period D and thickness h. Dielectric media are assumed as nonmagnetic ($\mu = \mu_0$), and a harmonic time dependence $e^{j\omega t}$ is considered for the electromagnetic fields, and there is also an implicit dependence with the z coordinate of the form $e^{\mp j\beta z}$ for waves propagating in the $\pm z$ direction, respectively. The grating is illuminated from the air region $z < 0$ by an arbitrary linearly polarized plane wave with wave vector in the rectangular coordinate system given by

$$\mathbf{k} = k_x\widehat{\mathbf{x}} + k_y\widehat{\mathbf{y}} + k_z\widehat{\mathbf{z}} \tag{1}$$

being

$$k_x = k_0 \sin\theta\cos\phi, \quad k_y = k_0\sin\theta\sin\phi, \quad k_z = k_0\cos\theta \tag{2}$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the wavenumber in free space and θ and ϕ are the elevation and azimuthal angles of the incident plane wave, respectively. Due to the existence of a preferred direction (Y-axis) in the transverse plane (see Fig. 1), both electric and magnetic fields are decomposed into E-type (denoted as $'$) and H-type modes (denoted as $''$) [9]. Therefore, following [10], the transverse components of the vector Floquet harmonics in both air regions situated above and below of the grating are described as follows:

$$\tilde{\mathbf{e}}'_p = \frac{1}{\sqrt{D}} e^{-j(k_x x + k_{y_p} y)} \left[\frac{-k_x k_{y_p}}{k_0^2 - k_{y_p}^2}\widehat{\mathbf{x}} + \widehat{\mathbf{y}}\right] \tag{3}$$

$$\tilde{\mathbf{h}}'_p = \frac{-1}{\sqrt{D}} e^{-j(k_x x + k_{y_p} y)}\widehat{\mathbf{x}} \tag{4}$$

$$\tilde{\mathbf{e}}''_p = \frac{1}{\sqrt{D}} e^{-j(k_x x + k_{y_p} y)}\widehat{\mathbf{x}} \tag{5}$$

$$\tilde{\mathbf{h}}''_p = \frac{1}{\sqrt{D}} e^{-j(k_x x + k_{y_p} y)} \left[\frac{-k_x k_{y_p}}{(k_0^2 - k_{y_p}^2)}\widehat{\mathbf{x}} + \widehat{\mathbf{y}}\right] \tag{6}$$

where k_{y_p} is the Floquet wavenumber, given by

$$k_{y_p} = k_y + \frac{2\pi}{D}p = k_0\sin\theta\sin\phi + \frac{2\pi}{D}p;\ p = 0, \pm 1, \pm 2, \ldots. \tag{7}$$

The modes inside the periodic layer are obtained using a vectorial modal method recently developed in [3], [7], and [11]. In such a method, the vector wave equation satisfied by the transverse components of the magnetic field in the periodic medium is expressed as an eigenvalue problem as follows:

$$L\mathbf{h}_n = \beta_n^2\mathbf{h}_n \tag{8}$$

where L represents the differential operator governing the evolution of the transverse magnetic field along the Z axis, $\mathbf{h}_n$ is the nth magnetic vector mode function, and β_n is the propagation constant of such mode. This eigenvalue equation can be expressed in a matrix form when the modes of the periodic medium are expanded in terms of an auxiliary system whose eigenvectors satisfy an orthogonality relationship of the form

$$\langle\tilde{\mathbf{e}}_{\mathbf{p}}|\tilde{\mathbf{h}}_{\mathbf{q}}\rangle = \delta_{pq}. \tag{9}$$

For the auxiliary system we have used the modes corresponding to a homogeneous medium (3)–(6) as the auxiliary basis. These modes satisfy the orthogonality relationship (9)

$$\langle\tilde{\mathbf{e}}_{\mathbf{p}}^{\alpha}|\tilde{\mathbf{h}}_{\mathbf{q}}^{\beta}\rangle = \int_0^D (\tilde{\mathbf{e}}_{\mathbf{p}}^{\alpha *} \times \tilde{\mathbf{h}}_q^{\beta})\cdot\widehat{\mathbf{z}}dy = \delta_{\alpha\beta}\delta_{pq} \tag{10}$$

where α and β indicate the polarization state, i.e., E-type ($'$) or H-type ($''$) modes. Then, the application of the standard Galerkin moment method yields the following linear matrix eigenvalue problem:

$$\sum_q L_{pq}c_{qn} = \beta_n^2 c_{pn} \tag{11}$$

where c_{qn} are the complex coefficients of the modal expansion for the transverse magnetic field of the nth mode, and L_{pq} are the matrix elements of the L operator, which are obtained as follows:

$$L_{pq}^{\alpha\beta} = \langle\tilde{\mathbf{e}}_p^{\alpha}|L\tilde{\mathbf{h}}_q^{\beta}\rangle = \int_0^D (\tilde{\mathbf{e}}_p^{\alpha *} \times L\tilde{\mathbf{h}}_q^{\beta})\cdot\widehat{\mathbf{z}}dy. \tag{12}$$

For the particular case of a periodic dielectric medium with rectangular profile under general 3-D excitation, these integrals have been analytically calculated obtaining, as shown in (13)–(16) at the bottom of the page.

$$\langle\widetilde{\mathbf{e}}'_p|L\widetilde{\mathbf{h}}'_q\rangle = \begin{cases} \widetilde{\beta}_p^2 + \sum_{i=1}^{2} k_0^2(\varepsilon_i - \widetilde{\varepsilon}_b)\frac{l_i}{D} & p = q \\ \sum_{i=1}^{2}\left[\frac{2k_0^2}{(k_{y_p}-k_{y_q})} + \frac{4k_{y_q}}{(\varepsilon_i+\widetilde{\varepsilon}_b)}\right]\frac{(\varepsilon_i-\widetilde{\varepsilon}_b)}{D}\sin\left((k_{y_p}-k_{y_q})\frac{l_i}{2}\right)e^{j(k_{y_p}-k_{y_q})y_{0i}} & p \neq q \end{cases} \tag{13}$$

$$\langle\widetilde{\mathbf{e}}'_p|L\widetilde{\mathbf{h}}''_q\rangle = \begin{cases} 0 & p = q \\ \sum_{i=1}^{2}\frac{4k_x(\varepsilon_i-\widetilde{\varepsilon}_b)}{D(\varepsilon_i+\widetilde{\varepsilon}_b)}\left(1 + \frac{k_{y_q}^2}{(k_b^2-k_{y_q}^2)}\right)\sin\left((k_{y_p}-k_{y_q})\frac{l_i}{2}\right)e^{j(k_{y_p}-k_{y_q})y_{0i}} & p \neq q \end{cases} \tag{14}$$

$$\langle\widetilde{\mathbf{e}}''_p|L\widetilde{\mathbf{h}}'_q\rangle = 0 \tag{15}$$

$$\langle\widetilde{\mathbf{e}}''_p|L\widetilde{\mathbf{h}}''_q\rangle = \begin{cases} \widetilde{\beta}_p^2 + \sum_{i=1}^{2} k_0^2(\varepsilon_i - \widetilde{\varepsilon}_b)\frac{l_i}{D} & p = q \\ \sum_{i=1}^{2}\frac{2k_0^2(\varepsilon_i-\widetilde{\varepsilon}_b)}{(k_{y_p}-k_{y_q})D}\sin\left((k_{y_p}-k_{y_q})\frac{l_i}{2}\right)e^{j(k_{y_p}-k_{y_q})y_{0i}} & p \neq q. \end{cases} \tag{16}$$

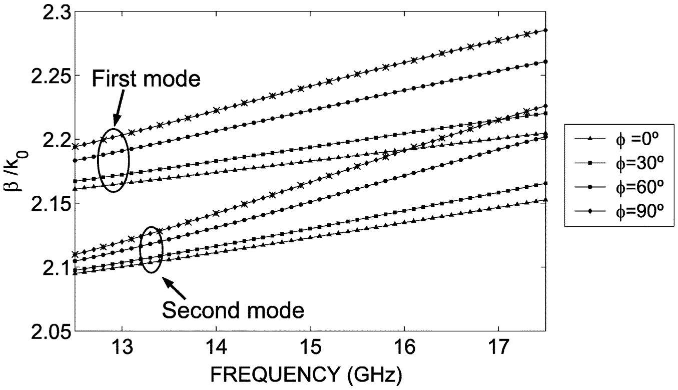

Fig. 2. Dispersion curves of the first and second mode for a periodic dielectric medium with parameters $D = 11.28$ mm, $\varepsilon_1 = 6.13$, $\varepsilon_2 = 3.7$, $l_1 = l_2 = D/2$ for 3-D propagation ($\theta = 45°$). Results obtained with the software developed in [3] and [7] are represented with crosses for comparison.

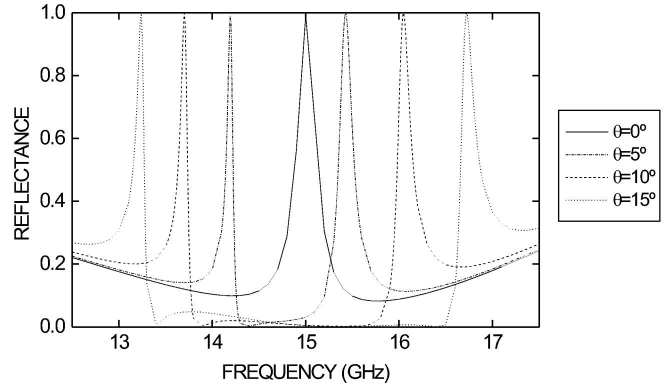

Fig. 3. Spectral response of the dielectric waveguide grating of Fig. 2 for transverse electric incident polarization ($\phi = 90°$) at four different values of the angle of incidence θ.

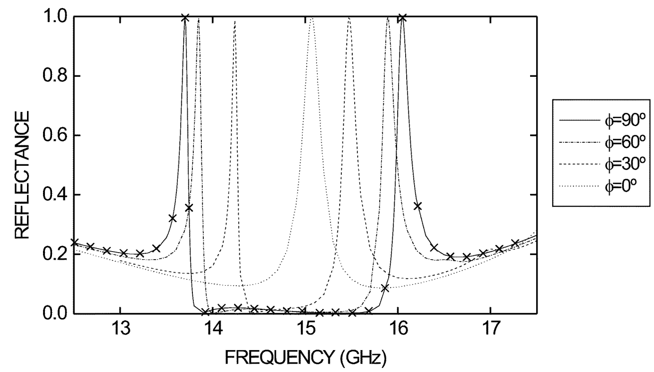

Fig. 4. Spectral response of the dielectric waveguide grating of Fig. 2 for H-type incident polarization at four different azimuthal angles and an elevation angle $\theta = 10°$. Results obtained with the software developed in [3] and [7] are represented with crosses for comparison.

From (13)–(16), it becomes evident that for 3-D excitation, the polarization of electromagnetic fields in the periodic medium is not yet preserved, i.e., the guiding problem requires the simultaneous presence of mixed polarizations. It can be noticed the disappearance of the coupling term of (14) for the the 2-D incidence case $\phi = 90°$ ($k_x = 0$), as it should be expected from [3], [7]. The numerical diagonalization of (11) yields the propagation constants and the magnetic fields of the modes in the periodic medium at each frequency point. Finally, the transverse electric fields of the modes are related to the magnetic ones through constraints directly derived from Maxwell's equations [12].

After specifying the fields outside and inside the grating layer, the problem is reduced to obtain the scattering parameters of the structure. To this end, the boundary conditions along the interfaces between adjacent layers are imposed for the tangential components of the electric and magnetic fields, thus obtaining the generalized scattering matrix (GSM) of the structure. Finally, the reflection and transmission coefficients of the propagation modes are calculated by considering an incident linearly polarized plane wave of any of both E-type or H-type polarization states with a unit amplitude.

III. Numerical Results

As an example, we have first examined the modal spectrum of a dielectric waveguide grating with two dielectric slabs within the unit cell (see Fig. 1), with the following parameters: period $D = 11.28$ mm, thickness $h = 4.37$ mm, $\varepsilon_1 = 6.13$ (E-glass), $\varepsilon_2 = 3.7$ (silica) and $l_1 = l_2 = D/2$. This structure has been analyzed in a previous work from the authors under a transverse electric-polarized normal incident wave for the case $\phi = 90°$ [7]. Fig. 2 shows the curves of the normalized propagation constant β/k_0 as a function of frequency of the first and second mode at an angle of incidence $\theta = 45°$, for different azimuthal angles. In this figure it can be observed an increase of the propagation constant with the angle ϕ. For the particular incidence $\phi = 90°$, the dispersion curve of the first mode corresponds to the first propagation mode under transverse electric excitation for 2-D incidence. In the same way, the normalized propagation constant of the second mode in Fig. 2 at $\phi = 90°$ corresponds to the first propagation mode under TM excitation in the case of bi-dimensional incidence [7]. In Fig. 2 we have also represented with crosses the results obtained with the software developed in [3], [7] for the 2-D incidence case, showing a total concordance.

The mentioned dielectric waveguide grating was originally designed in [7] as a reflection filter at normal incidence ($\theta = 0°$), with a resonance frequency peak centered at 15 GHz. The effect of increasing the angle of incidence θ in the spectral response for 2-D incidence, i.e., for an azimutal angle $\phi = 90°$, is shown in Fig. 3 for transverse electric polarization. In this figure it can be appreciated that, for oblique incidence, there is a splitting of the guided-mode resonance due to the asymmetry of the structure, as it has been experimentally shown in [2] for a reflection filter. This behavior can be predicted by applying the phase-match condition [1]

$$k_0 \sin\theta \sin\phi = \left|\beta_g + \frac{2\pi}{D} n\right|;\ n = 0, \pm 1, \pm 2, \ldots \tag{17}$$

where β_g is the propagation constant of the unmodulated waveguide in the Y direction and $2\pi n/D$ is the wave vector provided by the grating.

When the angle θ is fixed and the angle ϕ is varied, a similar behavior is observed. In Fig. 4 it is represented the frequency dependence of the reflectance of the grating at a fixed angle of

TABLE I
ESTIMATED AND CALCULATED PEAK RESONANT FREQUENCIES IN GHz ($D = 11.28$ mm, $h = 4.37$ mm, $\varepsilon_1 = 6.13$, $\varepsilon_2 = 3.7$, $l_1 = l_2 = D/2, \theta = 10°$)

$\phi(°)$	n	Data from Fig. 4	Data from (17)	Rel. Error (%)
90	-1	13.70	13.52	1.3
90	+1	16.05	15.93	0.7
60	-1	13.85	13.53	2.3
60	+1	15.89	15.75	0.9
30	-1	14.24	14.15	0.6
30	+1	15.47	15.30	1.1
0	-1	15.07	14.70	2.4

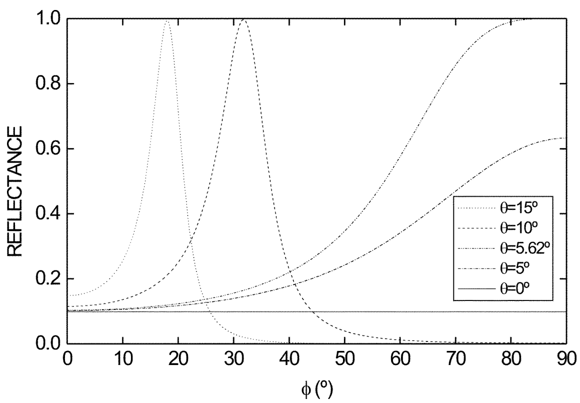

Fig. 5. Reflectance of the dielectric waveguide grating of Fig. 2 for H-type incident polarization as a function of the azimuthal angle ϕ, for different values of the angle θ, at a frequency of 15.5 GHz.

incidence $\theta = 10°$, at four different azimuthal angles, $\phi = 90°$, $60°$, $30°$, and $0°$, for H-type incident polarization. We have represented for comparison the data corresponding to the case $\phi = 90°$ obtained with the software developed in [3] and [7] for transverse electric polarization, showing an excellent agreement. For oblique incidence, the higher frequency peaks correspond to the excitation of the $n = +1$ space harmonic and the lower frequency peaks correspond to the $n = -1$ space harmonic. Once again, the location of the resonance frequencies obeys the mentioned phase-match condition (17). It is interesting to verify that, for the particular case of $\phi = 0°$, there exists only one resonance frequency peak in the range analyzed (corresponding to the $n = -1$ space harmonic), because in this case, the term $k_0 \sin\theta \sin\phi$ is canceled. Table I compares the estimated values of the resonance frequencies of the waveguide grating for the angles of incidence considered in Fig. 4, provided by (17), with the values numerically obtained with the present method. Although (17) is an approximate expression, it is very useful for the design of these structures, helping us to explain the effect of both azimuthal and elevation angles of incidence in the location of the resonant frequencies. In Fig. 5 it has been represented the reflectance of the waveguide grating under H-type incident polarization as a function of the azimuthal angle ϕ for different values of the angle θ. In this case, we have fixed the parameters of the grating, and the working frequency to a value of 15.5 GHz, so the only variables are the angles θ and ϕ. For a fixed value of the angle θ, it appears a resonance peak when the angle ϕ satisfies the phase-match condition (17). For $\theta = 5.62°$, the resonance peak occurs at $\phi = 90°$, and for lower values of θ, no value of ϕ satisfies (17), as can be observed in Fig. 5.

IV. CONCLUSION

In this letter, we have solved the problem of scattering of electromagnetic plane waves by 1-D periodic dielectric gratings under the most general condition of oblique incidence (3-D incidence). The propagation constant and the fields in the periodic medium are determined as the numerical solution of a linear eigenvalue problem. The scattering parameters of the grating are then obtained by imposing the boundary conditions in terms of the GSM formulation. The numerical results, apart from reproducing those obtained previously for the 2-D incidence, give a novel insight in the prediction of total reflection produced by DFSS in the 3-D incidence case.

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