Full-vector analysis of a realistic photonic crystal fiber

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We analyze the guiding problem in a realistic photonic crystal fiber, using a novel full-vector modal technique. This is a biorthogonal modal method based on the non-self-adjoint character of the electromagnetic propagation in a fiber. Dispersion curves of guided modes for different fiber structural paremeters are calculated, along with the two-dimensional transverse intensity distribution of the fundamental mode. Our results match those achieved in recent experiments in which the feasibility of this type of fiber was shown.

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Periodic dielectric structures (photonic crystals) have engendered growing interest in recent years because they exhibit interesting optical features. The most relevant property of a photonic crystal is the possibility that it can generate photonic bandgaps for certain geometries.[1] This effect has been observed in both two- and three-dimensional structures in the form of the absence of light propagation for a specific set of frequencies (see Ref. 2 and references therein). A related phenomenon that occurs in photonic crystal structures is light localization at defects.[3] The breakdown of dielectric periodicity at a defect generates a local variation of the effective refractive index that can cause the localization of light in its vicinity. Although the previous phenomenon of light confinement at defects has already been analyzed for two-dimensional (2D) structures,[4] a rigorous study of the guiding properties of dielectric crystals that have a 2D periodicity in the $x-y$ plane broken by the presence of a defect but that are continuous and infinitely long in the z direction (so-called photonic crystal fibers) has not been performed. Our aim is to describe accurately the propagation of guided modes, including their nontrivial dispersion relations and amplitudes, in this new kind of fiber. To our knowledge, this is the first report of the dispersion characteristics of a photonic crystal fiber.

The physical realization of photonic crystal fiber is a thin silica fiber that has a regular structure of holes that extend along the whole fiber length. If one of these holes is absent, the transverse dielectric periodicity is broken, and a defect appears. The fact that light may be trapped at defects becomes a propagation feature. Consequently, the bound states of the 2D transverse problem (2D trapped states of light) become the guided modes of the fiber propagation problem. The experimental feasibility of these fibers was proved recently.[5] A robust single-mode structure was observed for an unusually wide range of wavelengths, a remarkable property that is not present in ordinary fibers. A preliminary interpretation of the behavior of photonic crystal fibers involving the concept of effective refractive index is presented in Ref. 6. The confinement mechanisms can be thought of as being produced by the existence of a homogeneous material with a specific average index.[7]

Our interest lies in formulating an appropriate treatment of the realistic problem of a photonic crystal fiber by modeling and solving efficiently the transverse 2D structure of the crystal. We present an approach in which the full-vector character of light propagation in fibers is taken into account. It is an adapted version of our biorthonormal-basis modal method.[8] In this way, a realistic 2D periodic structure with a central defect is properly implemented, allowing us to analyze different fiber designs. As we shall see, our results agree with those experimentally measured and at the same time predict different interesting behaviors for some designs.

Guided modes in an inhomogeneous fiber verify a set of dimensionally reduced equations involving the transverse coordinates x and y exclusively.[9] We obtain this set of equations from Maxwell's equations by assuming that the electromagnetic field is monochromatic in time and has a harmonic dependence on z (i.e., the field has a well-defined propagation constant β). In terms of transverse components of the magnetic and electric fields $h_t = \binom{h_x}{h_y}$ and $e_t = \binom{e_x}{e_y}$, these equations can be rewritten as[8]

$$L h_t = \beta^2 h_t, \qquad L^\dagger \bar{e}_t = \beta^{*2} \bar{e}_t\,, \tag{1}$$

where $\bar{e}_t = \binom{e_y{}^*}{-e_x{}^*}$, $L^\dagger$ is the adjoint operator of L, $*$ denotes the complex-conjugate operation, and each element $L_{\alpha\gamma}$ of the matrix differential operator L has the form

$$L_{\alpha\gamma} \equiv (\nabla^2 + k^2 n^2)\delta_{\alpha\gamma} - \left(\epsilon_{\alpha\eta} \frac{\nabla_\eta n^2}{n^2}\right)(\epsilon_{\gamma\zeta}\nabla_\zeta)\,, \qquad \alpha, \gamma, \zeta, \eta = x, y\,, \tag{2}$$

where $\epsilon_{\alpha\gamma}$ is the completely antisymmetric tensor in two dimensions, n is the refractive index of an isotropic

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medium, and k is the free-space wave number. Of course, ∇^2 is the Laplacian operator and ∇_α are the transverse components of the gradient operator. Note that the general problem of light propagation in a fiber, even for nonabsorbing materials (when n^2 is real), involves the non-Hermitian operator L.

The most relevant property of Eqs. (1) is that they constitute a system of eigenvalue equations for the L operator and its adjoint $L^\dagger$ (something that is far from obvious when one starts from the reduced equations written in terms of h_t and e_t instead of $\bar{e}_t$; see, for instance, Ref. 9). Because h_t and $\bar{e}_t$ are the eigenfunctions of the L and $L^\dagger$ operators, respectively, they are closely related. In fact, they verify what is called the biorthogonality relation, $\langle \bar{e}_t{}^n | h_t{}^m \rangle = \delta_{nm}$. This property is crucial in our approach to the full-vector problem, as explained in detail in Ref. 8.

The main goal of our approach is to transform the problem of solving the system of differential equations (1) (sometimes including highly nontrivial boundary conditions) into an algebraic problem involving the diagonalization of the L matrix. The spectrum of the L matrix will be formed in general by 2D bound states and continuum states. In terms of fiber propagation, the bound states of the L spectrum are guided modes because, despite the finite width of the fiber, the fields exhibit a strong decay in the transverse direction. States from the continuum, however, radiate radially, and thus they are not guided by the fiber.

The choice of an appropriate auxiliary basis is important for efficient implementation of our method. In the particular case of a photonic crystal fiber this selection must be especially accurate. The main reason for this is that the complicated spatial structure of the refractive index in a realistic case can transform the actual computation of the L-matrix elements into an impossible task. Realistic simulations must include almost 100 2D step-index individual structures (the air holes of the photonic crystal fiber). Therefore a brute force computation of the matrix elements becomes useless in practice because of a critical loss of precision.

One carries out the implementation of the dielectric structure by putting the system into a finite 2D box (of dimensions D_x and D_y) and requiring the fields to fulfill periodic boundary conditions in the x and y directions. So we create an artificial lattice by replicating the original almost periodic structure, including the central defect, in both transverse directions. This new superlattice is made from copies of the original cell covering the entire 2D transverse plane. Although the original cell is not periodic, the entire superlattice really is. The periodicity requirement implies that we can expand the 2D electromagnetic fields in a discrete Fourier series in terms of the exponential functions $f_{\mathbf{n}}(\mathbf{x}_t) = \exp(i\mathbf{k}_{\mathbf{n}} \cdot \mathbf{x}_t)$, where $\mathbf{k}_{\mathbf{n}} = 2\pi(n_x/D_x, n_y/D_y)$ is the discretized transverse wave vector.

A crucial property of this basis is that, because of the periodicity of the superlattice and despite the fact that it is defined in a finite volume—the unit cell of the superlattice of size D_x times D_y—it is translationally invariant. The presence of this symmetry shown by our auxiliary basis turns out to be critical for the feasibility of the method. The advantage of the translation symmetry is twofold: On the one hand, it allows us to relate easily any matrix element of the operator that represents a hole at an arbitrary position to that which represents a hole at the origin of coordinates. Because the whole matrix of the photonic crystal fiber structure can be written as a sum over all matrices that represent each one of the substructures (holes), and because these substructures are identical (although they are differently located), it is possible to reduce the problem to the calculation of one single matrix. On the other hand, the calculation of any element of this single matrix can be worked out analytically on this basis (we assume a circular step-index profile for the hole). In addition, and because of the symmetry properties of a realistic hexagonally centered configuration of holes, the sum over the set of points where the holes are located can also be analytically solved. Consequently, the choice of the exponential functions as a basis for defining the matrix elements of the realistic photonic fiber operator L leads to a crucial simplification. The problem of a critical loss of precision owing to the complex spatial structure of the photonic crystal fiber is, in this way, overcome.

We simulated a realistic photonic crystal fiber characterized by a hexagonal distribution of air holes with a central defect. The hole radius a, the horizontal distance between the center of two consecutive holes—or pitch—Λ, and the wavelength of light λ are free parameters that we have changed at will. The height of the refractive-index step is also free, although we have kept it constant for comparison purposes. We first simulated a realistic air-filled fiber with parameters $a = 0.3$ μm and $\Lambda = 2.3$ μm. The dimensions of the superlattice unit cell are $D_x = 8\Lambda$ and $D_y = 5\sqrt{3}\,\Lambda$, and the number of modes of the auxiliary basis considered is 1224. We focused on this particular design because the intensity distribution for the guided mode in this structure has been measured experimentally for a wavelength of $\lambda = 632.8$ nm. Experimental measures also show that the guided mode in this fiber remains single in a remarkably wide wavelength range, extending from 337 to 1550 nm.[5] Our simulation allows us to evaluate the eigenvalues of the L operator at any wavelength and thus to calculate the modal dispersion curves for the fiber under consideration in an even wider range of wavelengths (see Fig. 1). The single-mode structure is formed by a polarization doublet. Our results completely agree with the previous experimental results, as they account for the existence of a robust single-mode structure at nearly all wavelengths for the above fiber parameters. We include in Fig. 1 the envelope of the radiation modes, which is referred to as the cladding index (i.e., the effective refractive index of the photonic crystal).

Inasmuch as the diagonalization procedure of the full-vector operator L generates not only the set of eigenvalues but also their respective eigenvectors, we can also evaluate the transverse intensity distribution of the electromagnetic field for the guided mode. The result for one of the polarizations is shown in Fig. 2 for $\lambda = 632.8$ nm and reproduces, with excellent accuracy, that which was experimentally measured.[5] We also calculated the transverse intensity of the guided mode

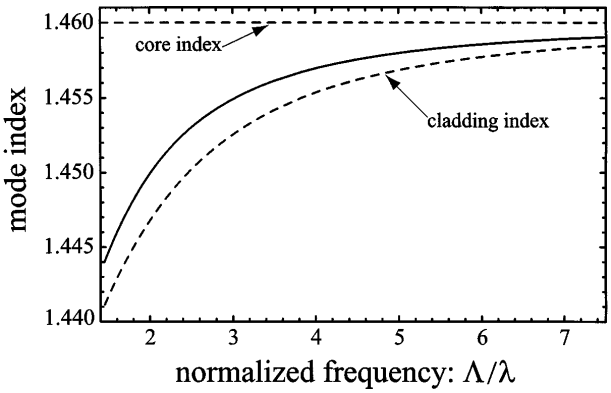

Fig. 1. Modal dispersion curves extending from $\lambda = 300$ nm to $\lambda = 1600$ nm for a single-mode photonic crystal fiber structure with $a = 0.3$ μm and $\Lambda = 2.3$ μm. Here the variations of the mode index for both polarizations coalesce in a single curve.

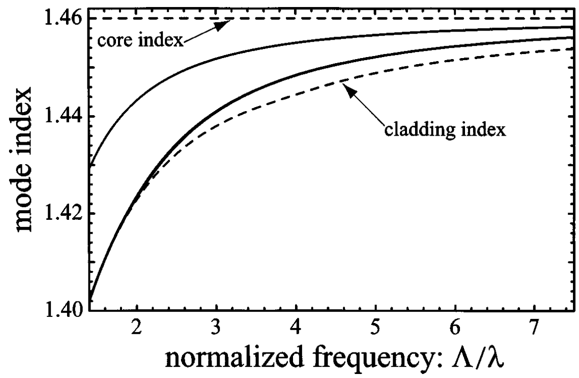

Fig. 3. Same as in Fig. 1 but with $a = 0.6$ μm. Here the two higher-order polarization doublets are slightly shifted.

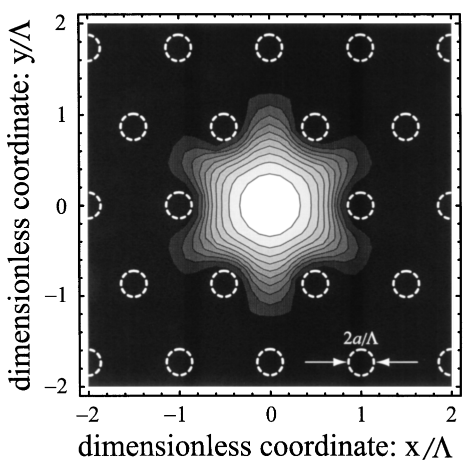

Fig. 2. Transverse intensity distribution for the x-polarized guided mode of the photonic crystal fiber described in Fig. 1 for $\lambda = 632.8$ nm.

at widely different wavelengths and for both polarizations. In all cases the intensity profile is similar to that shown in Fig. 2. In this way, we verified the robust character of the single-mode structure to changes in the wavelength of light. This fact agrees with the behavior of the dispersion curves mentioned above.

Besides simulating this remarkable structure, we also simulated a number of different fiber designs by changing the pitch Λ and the hole radius a. By analyzing the dispersion curves of these different fibers, we found a richer modal structure in some of them. In the example shown in Fig. 3 there are, besides the fundamental doublet, two other polarization doublets. Unlike for conventional fibers, the number of modes does not increase with the light wave number k. The number of guided modes becomes stabilized above a k threshold, or, equivalently, it remains constant for wavelengths smaller than a threshold wavelength. For particular designs one can get guiding structures in which this constant number is just 1. In such a case one obtains an "endlessly" single-mode fiber such as the one reported in Ref. 5. This is an unconventional property of photonic crystal fibers.

In a conventional fiber the cladding refractive index is nearly constant; its V value, the optical volume (or phase space) of the fiber, grows with k. This fact permits us to accomodate an increasing number of guided modes inside the fiber as the wavelength is reduced. In a photonic crystal fiber the periodic structure responsible for light trapping at the central defect creates a dependence on the effective refractive index of the cladding such that a much more weakly k-dependent V value is generated. The optical volume then becomes practically independent of the wavelength for large values of k, and, consequently, so do the number of guided modes.

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