

# Elementos matemáticos

## Potencias y logaritmos.

$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln x^b = b \ln x$$

## Derivadas y diferenciales.

$$(cu)' = cu'$$

$$(u + v)' = u' + v'$$

$$\frac{d(u(v(x)))}{dx} = \frac{d(u(v))}{dv} \frac{dv}{dx}$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$d(f(x, y, z, \dots)) = \left(\frac{\partial f}{\partial x}\right)_{y, z, \dots} dx + \left(\frac{\partial f}{\partial y}\right)_{x, z, \dots} dy + \left(\frac{\partial f}{\partial z}\right)_{x, y, \dots} dz + \dots$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$$

$$\frac{d}{dx}(\sqrt[n]{x}) = \frac{1}{n\sqrt[n]{x^{n-1}}}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\operatorname{tg} x) = \frac{1}{\cos^2 x}$$

## Integrales.

$$\int af(x)dx = a \int f(x)dx$$

$$\int (u + v)dx = \int udx + \int vdx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int_{-\infty}^{+\infty} x^{2n} e^{-ax^2} dx = 2 \int_0^{\infty} x^{2n} e^{-ax^2} dx$$

$$\int_{-\infty}^{+\infty} x^{2n+1} e^{-ax^2} dx = 0$$

$$\int_0^{\infty} e^{-ax^2} dx = \frac{\pi^{\frac{1}{2}}}{2a^{\frac{1}{2}}}$$

$$\int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}$$

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{(2n)! \pi^{\frac{1}{2}}}{2^{2n+1} n! a^{n+\frac{1}{2}}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\operatorname{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$$

$$\operatorname{Erf}(0) = 0; \operatorname{Erf}(\infty) = 1$$

$$\int_0^x e^{-at^2} dt = \frac{\sqrt{\pi}}{2a^{\frac{1}{2}}} \operatorname{Erf}(\sqrt{ax});$$

$$\int_0^x t^2 e^{-at^2} dt = \frac{\sqrt{\pi}}{4a^{\frac{3}{2}}} \operatorname{Erf}(\sqrt{ax}) - \frac{x}{2a} e^{-ax^2}$$

## Aproximaciones:

Stirling  $\rightarrow \ln N! \approx N \ln N - N$  (para N grande)

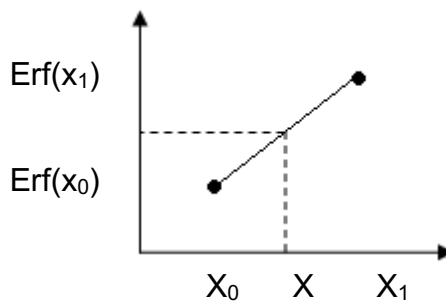
$$e^{-x} \approx 1 - x \text{ (si } |x| \ll 1)$$

$$e^x \approx 1 + x \text{ (si } |x| \ll 1)$$

| FUNCIÓN DE ERROR |          |           |           |           |           |          |           |           |           |          |
|------------------|----------|-----------|-----------|-----------|-----------|----------|-----------|-----------|-----------|----------|
|                  | 0        | 1         | 2         | 3         | 4         | 5        | 6         | 7         | 8         | 9        |
| 0.0              | 0.       | 0.0112834 | 0.0225646 | 0.0338412 | 0.0451111 | 0.056372 | 0.0676216 | 0.0788577 | 0.0900781 | 0.101281 |
| 0.1              | 0.112463 | 0.123623  | 0.134758  | 0.145867  | 0.156947  | 0.167996 | 0.179012  | 0.189992  | 0.200936  | 0.21184  |
| 0.2              | 0.222703 | 0.233522  | 0.244296  | 0.255023  | 0.2657    | 0.276326 | 0.2869    | 0.297418  | 0.30788   | 0.318283 |
| 0.3              | 0.328627 | 0.338908  | 0.349126  | 0.359279  | 0.369365  | 0.379382 | 0.38933   | 0.399206  | 0.409009  | 0.418739 |
| 0.4              | 0.428392 | 0.437969  | 0.447468  | 0.456887  | 0.466225  | 0.475482 | 0.484655  | 0.493745  | 0.50275   | 0.511668 |
| 0.5              | 0.5205   | 0.529244  | 0.537899  | 0.546464  | 0.554939  | 0.563323 | 0.571616  | 0.579816  | 0.587923  | 0.595936 |
| 0.6              | 0.603856 | 0.611681  | 0.619411  | 0.627046  | 0.634586  | 0.642029 | 0.649377  | 0.656628  | 0.663782  | 0.67084  |
| 0.7              | 0.677801 | 0.684666  | 0.691433  | 0.698104  | 0.704678  | 0.711156 | 0.717537  | 0.723822  | 0.73001   | 0.736103 |
| 0.8              | 0.742101 | 0.748003  | 0.753811  | 0.759524  | 0.765143  | 0.770668 | 0.7761    | 0.78144   | 0.786687  | 0.791843 |
| 0.9              | 0.796908 | 0.801883  | 0.806768  | 0.811564  | 0.816271  | 0.820891 | 0.825424  | 0.82987   | 0.834232  | 0.838508 |
| 1.0              | 0.842701 | 0.84681   | 0.850838  | 0.854784  | 0.85865   | 0.862436 | 0.866144  | 0.869773  | 0.873326  | 0.876803 |
| 1.1              | 0.880205 | 0.883533  | 0.886788  | 0.889971  | 0.893082  | 0.896124 | 0.899096  | 0.902     | 0.904837  | 0.907608 |
| 1.2              | 0.910314 | 0.912956  | 0.915534  | 0.91805   | 0.920505  | 0.9229   | 0.925236  | 0.927514  | 0.929734  | 0.931899 |
| 1.3              | 0.934008 | 0.936063  | 0.938065  | 0.940015  | 0.941914  | 0.943762 | 0.945561  | 0.947312  | 0.949016  | 0.950673 |
| 1.4              | 0.952285 | 0.953852  | 0.955376  | 0.956857  | 0.958297  | 0.959695 | 0.961054  | 0.962373  | 0.963654  | 0.964898 |
| 1.5              | 0.966105 | 0.967277  | 0.968413  | 0.969516  | 0.970586  | 0.971623 | 0.972628  | 0.973603  | 0.974547  | 0.975462 |
| 1.6              | 0.976348 | 0.977207  | 0.978038  | 0.978843  | 0.979622  | 0.980376 | 0.981105  | 0.98181   | 0.982493  | 0.983153 |
| 1.7              | 0.98379  | 0.984407  | 0.985003  | 0.985578  | 0.986135  | 0.986672 | 0.98719   | 0.987691  | 0.988174  | 0.988641 |
| 1.8              | 0.989091 | 0.989525  | 0.989943  | 0.990347  | 0.990736  | 0.991111 | 0.991472  | 0.991821  | 0.992156  | 0.992479 |
| 1.9              | 0.99279  | 0.99309   | 0.993378  | 0.993656  | 0.993923  | 0.994179 | 0.994426  | 0.994664  | 0.994892  | 0.995111 |
| 2.0              | 0.995322 | 0.995525  | 0.995719  | 0.995906  | 0.996086  | 0.996258 | 0.996423  | 0.996582  | 0.996734  | 0.99688  |
| 2.1              | 0.997021 | 0.997155  | 0.997284  | 0.997407  | 0.997525  | 0.997639 | 0.997747  | 0.997851  | 0.997951  | 0.998046 |
| 2.2              | 0.998137 | 0.998224  | 0.998308  | 0.998388  | 0.998464  | 0.998537 | 0.998607  | 0.998674  | 0.998738  | 0.998799 |
| 2.3              | 0.998857 | 0.998912  | 0.998966  | 0.999016  | 0.999065  | 0.999111 | 0.999155  | 0.999197  | 0.999237  | 0.999275 |
| 2.4              | 0.999311 | 0.999346  | 0.999379  | 0.999411  | 0.999441  | 0.999469 | 0.999497  | 0.999523  | 0.999547  | 0.999571 |
| 2.5              | 0.999593 | 0.999614  | 0.999635  | 0.999654  | 0.999672  | 0.999689 | 0.999706  | 0.999722  | 0.999736  | 0.999751 |
| 2.6              | 0.999764 | 0.999777  | 0.999789  | 0.9998    | 0.999811  | 0.999822 | 0.999831  | 0.999841  | 0.999849  | 0.999858 |
| 2.7              | 0.999866 | 0.999873  | 0.99988   | 0.999887  | 0.999893  | 0.999899 | 0.999905  | 0.99991   | 0.999916  | 0.99992  |
| 2.8              | 0.999925 | 0.999929  | 0.999933  | 0.999937  | 0.999941  | 0.999944 | 0.999948  | 0.999951  | 0.999954  | 0.999956 |
| 2.9              | 0.999959 | 0.999961  | 0.999964  | 0.999966  | 0.999968  | 0.99997  | 0.999972  | 0.999973  | 0.999975  | 0.999976 |

**Interpolación lineal:**

$$erf(x) = erf(x_0) + \frac{erf(x_1) - erf(x_0)}{x_1 - x_0} (x - x_0)$$



$$\rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

## CONSTANTES FUNDAMENTALES

| Nombre                     | Símbolo                                     | Valor                   |              | Unidades  |
|----------------------------|---|-------------------------|--------------|---|
| Velocidad de la luz        | c   | 2.997 924 58            | $10^{8(*)}$  | $\text{m s}^{-1}$   |
| Carga elemental            | e   | 1.602 177               | $10^{-19}$   | C   |
| Constante de Faraday       | $F=N_A e$                                   | 9.648 53                | $10^4$       | $\text{C mol}^{-1}$   |
| Constante de Boltzmann     | k   | 1.380 66                | $10^{-23}$   | $\text{J K}^{-1}$   |
| Constante de los gases     | $R=N_A k$                                   | 8.314 51                |              | $\text{J K}^{-1} \text{mol}^{-1}$   |
|                            |   | 0.083 145 1             |              | $\text{L bar K}^{-1} \text{mol}^{-1}$                                       |
|                            |   | 0.082 057 8             |              | $\text{L atm K}^{-1} \text{mol}^{-1}$                                       |
| Constante de Planck        | h<br>$\hbar=h/2\pi$                         | 6.626 08                | $10^{-34}$   | J s   |
|                            |   | 1.054 57                | $10^{-34}$   | Js  |
| Número de Avogadro         | $N_A$                                       | 6.022 14                | $10^{23}$    | $\text{mol}^{-1}$   |
| Unidad de masa atómica     | uma   | 1.66054                 | $10^{-27}$   | kg  |
| Masa electrón              | $m_e$                                       | 9.109 39                | $10^{-31}$   | kg  |
| Masa protón.               | $m_p$                                       | 1.672 62                | $10^{-27}$   | kg  |
| Masa neutrón               | $m_n$                                       | 1.674 93                | $10^{-27}$   | kg  |
| Permitividad en el vacío   | $\epsilon_0=1/c^2\mu_0$<br>$4\pi\epsilon_0$ | 8.854 19                | $10^{-12}$   | $\text{J}^{-1}\text{C}^2\text{m}^{-1}$                                      |
|                            |   | 1.112 65                | $10^{-10}$   | $\text{J}^{-1}\text{C}^2\text{m}^{-1}$                                      |
| Permeabilidad en el vacío. | $\mu_0$                                     | $4\pi$                  | $10^{-7(*)}$ | $\text{Js}^2\text{C}^{-2}\text{m}^{-1}(=\text{T}^2\text{J}^{-1}\text{m}^3)$ |
| Magnetón Bohr              | $\mu_B=e\hbar/2m_e$                         | 9.274 02                | $10^{-24}$   | $\text{JT}^{-1}$  |
| Magnetón nuclear           | $\mu_N=e\hbar/2m_p$                         | 5.050 79                | $10^{-27}$   | $\text{JT}^{-1}$  |
| Constante de Rydberg       | $R=m_e e^4/8h^3 c\epsilon_0^2$              | 1.097 377               | $10^5$       | $\text{cm}^{-1}$  |
| Aceleración en caída libre | g   | 9.806 65 <sup>(*)</sup> |              | $\text{ms}^{-2}$  |
| Constante gravitatoria     | G   | 6.672 59                | $10^{-11}$   | $\text{Nm}^2\text{kg}^{-2}$   |

(\*) Valor exacto

### FACTORES DE CONVERSIÓN

|  |
|--|
| 1 cal = 4.184 J(*)                       |
| 1 $\text{cm}^{-1}$ = 1.9864 $10^{-23}$ J |
| 1 eV = 1.60218 $10^{-19}$ J              |
| 1 atm = 1.01325 $10^5$ Pa                |
| 1 atm = 760 Torr                         |
| 1 bar = $10^5$ Pa                        |
| 1 D = 3.335 64 $10^{-30}$ C m            |
| 1 Å = $10^{-10}$ m(*)                    |
| 1 P = 0.1 Pa·s                           |
| 1 J = $10^7$ erg                         |
| 1 N = $10^5$ dina                        |
| 1 erg = 1 dina·cm                        |
| 1 J = 1 N·m                              |

| Submúltiplo | Prefijo | Símbolo | Múltiplo  | Prefijo | Símbolo |
|-------------|---------|---------|-----------|---------|---------|
| $10^{-1}$   | deci    | d       | 10        | deca    | da      |
| $10^{-2}$   | centi   | c       | $10^2$    | hecto   | h       |
| $10^{-3}$   | mili    | m       | $10^3$    | kilo    | k       |
| $10^{-6}$   | micro   | $\mu$   | $10^6$    | mega    | M       |
| $10^{-9}$   | nano    | n       | $10^9$    | giga    | G       |
| $10^{-12}$  | pico    | p       | $10^{12}$ | tera    | T       |
| $10^{-15}$  | femto   | f       | $10^{15}$ | peta    | P       |
| $10^{-18}$  | atto    | a       | $10^{18}$ | exa     | E       |

## Tema 1

$$Q = \sum_j e^{-\frac{E_j}{kT}} \quad p_j = \frac{e^{-\frac{E_j}{kT}}}{Q} \quad U = U(0) + kT^2 \left( \frac{\partial \ln Q}{\partial T} \right)_{N,V} \quad P = kT \left( \frac{\partial \ln Q}{\partial V} \right)_{N,T} \quad S = \frac{U - U(0)}{T} + k \ln Q$$

$$A = U(0) - kT \ln Q \quad G = U(0) - kT \ln Q + kTV \left( \frac{\partial \ln Q}{\partial V} \right)_{N,T} \quad \mu_B = \mu_B(0) - RT \left( \frac{\partial \ln Q}{\partial N_B} \right)_{T,V,N_c \neq N_B}$$

$$\Lambda = \frac{h}{(2\pi mkT)^{1/2}} \quad q_t = \frac{V}{\Lambda^3} \quad \Theta_r = \frac{h^2}{8\pi^2 Ik} = \frac{hB}{k} = \frac{hc\bar{B}}{k} \quad q_r = \frac{T}{\sigma \Theta_r}; \quad q_r = \frac{\sqrt{\pi T}^{3/2}}{\sigma \sqrt{\Theta_A \Theta_B \Theta_C}}$$

$$\Theta_v = \frac{hv}{k} \quad q_v = \prod_{s=1}^{3N-6} \frac{1}{1 - e^{-\frac{\Theta_{v,s}}{T}}} \quad K_p = e^{-\frac{\Delta U_m(0)}{RT}} \prod_i \left( \frac{q_{m,i}^\theta}{N_A} \right)^{v_i} \quad \Delta U_m(0) = -N_A \sum_i v_i D_{0,i}$$

## Tema 2

$$g(v_x) = \left( \frac{m}{2\pi kT} \right)^{1/2} e^{-\frac{mv_x^2}{2kT}} \quad \phi(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} \quad G(v) = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} \quad G(\varepsilon) = 2\pi \left( \frac{1}{\pi kT} \right)^{3/2} \varepsilon^{1/2} e^{-\frac{\varepsilon}{kT}}$$

$$v_{rms} = \langle v^2 \rangle^{1/2} = \left( \frac{3kT}{m} \right)^{1/2} \quad v_p = \left( \frac{2kT}{m} \right)^{1/2}$$

$$\langle v \rangle = \left( \frac{8kT}{\pi m} \right)^{1/2} \quad z_w = \frac{1}{4} \frac{N}{V} \langle v \rangle \quad z_{(1)2} = \pi d_{12}^2 \left( \frac{8kT}{\pi \mu} \right)^{1/2} \frac{N_2}{V} \quad Z_{12} = z_{(1)2} \frac{N_1}{V}$$

$$z_{(1)1} = \sqrt{2} \pi d^2 \left( \frac{8kT}{\pi m} \right)^{1/2} \frac{N_1}{V} \quad Z_{11} = \frac{1}{2} z_{(1)1} \frac{N_1}{V} \quad \lambda = \frac{1}{\sqrt{2} \pi d^2} \frac{V}{N}$$

$$k_r = A \exp\left(-\frac{E_a}{RT}\right); \quad E_a = -R \frac{d \ln k_r}{d(1/T)} = RT^2 \frac{d \ln k_r}{dT}; \quad k_r = \left( \frac{8kT}{\pi \mu} \right)^{1/2} N_A \pi d_{BC}^2 \exp\left(-\frac{\varepsilon_0}{kT}\right)$$

$$E_a = \frac{1}{2} RT + E_0; \quad k_r = \frac{kT}{h} \frac{\bar{q}^\ddagger / N_A V}{q_B / N_A V \cdot q_C / N_A V} \exp\left(-\frac{\Delta \varepsilon_0^\ddagger}{kT}\right); \quad k_r = \frac{kT}{h} \left( \frac{1}{C^0} \right)^{n-1} \exp\left(-\frac{\Delta G_C^{0\ddagger}}{RT}\right);$$

$$k_r = \frac{kT}{h} \left( \frac{RT}{P^0} \right)^{n-1} \exp\left(-\frac{\Delta G_p^{0\ddagger}}{RT}\right); \quad E_a = nRT + \Delta H_p^{0\ddagger}; \quad A = \frac{kT}{h} \left( \frac{RT}{P^0} \right)^{n-1} e^n e^{\frac{\Delta S_p^{0\ddagger}}{R}}$$

### Tema 3

$$J_z = -\kappa \frac{dT}{dz}; \quad J_z = -\eta \frac{dv_x}{dz}; \quad \frac{dV}{dt} = -\frac{\pi r^4}{8\eta} \left( \frac{dP}{dz} \right); \quad J_j = -\left( \frac{c_j D_{jk}}{RT} \right) \nabla \mu_j; \quad J_j = -D_{jk} \nabla c_j;$$

$$J_{D_{jk},z} = -D_{jk} \frac{dc_j}{dz};$$

$$J_z = \frac{I}{A} = -\sigma \frac{d\phi}{dz} = \sigma E_z; \quad R = \rho \frac{\ell}{A}; \quad I = \sum_j I_j = FA \frac{\Delta\phi}{\ell} \sum_j |z_j| u_j c_j;$$

$$\kappa = \frac{25\pi}{64} \langle v \rangle \lambda \rho \frac{C_{v,m}}{N_A}; \quad \eta = \frac{5\pi}{32} \langle v \rangle \lambda \rho \frac{M}{N_A}; \quad D_{jj} = \frac{3\pi}{16} \langle v \rangle \lambda; \quad \sigma = F \sum_j |z_j| u_j c_j; \quad t_j = \frac{|z_j| u_j c_j}{\sum_k |z_k| u_k c_k};$$

$$J_j = -\left( \frac{c_j D_{jk}}{RT} \right) \nabla \bar{\mu}_j + c_j \mathbf{v}; \quad \Lambda_m = \Lambda_m^0 - \Re \sqrt{c}; \quad \Lambda_m^0 = v_+ \lambda_+^0 + v_- \lambda_-^0; \quad \langle u_j \rangle = \frac{\langle v_j \rangle}{|\mathbf{E}|} = \frac{q}{f} = \frac{qD}{k_B T};$$

$$u_{j,\text{disolvente}}^\infty = \frac{|z_j| e}{6\pi\eta_{\text{disolvente}} r_j}; \quad D = \frac{k_B T}{6\pi\eta r_p}; \quad \mathbf{F}_\eta = -f \cdot \mathbf{v} = -6\pi\eta r_p \mathbf{v}$$

### Tema 4

$$p_{in} = p_{ex} + \frac{2\gamma}{r} \quad \ln\left(\frac{p}{p^*}\right) = \frac{V_m^L}{RT} \left[ (p - p^*) + \frac{2\gamma}{r} \right] \quad h = \frac{2\gamma \cos\theta}{(\rho^L - \rho^V) g R}$$

$$d\gamma = -\sum_i \Gamma_i^\sigma d\mu_i \quad \Gamma_{2(1)} = -\frac{C_2^\beta}{RT} \left( \frac{\partial \gamma}{\partial \mathcal{X}_2^\beta} \right)_T$$

$$\theta = \frac{Kp}{1+Kp} \quad \theta = \frac{(Kp)^{1/2}}{1+(Kp)^{1/2}} \quad \left( \frac{\partial \ln p}{\partial 1/T} \right)_\theta = \frac{\Delta H^\circ_{ads}}{R}$$

$$\frac{V}{V_{\text{mon}}} = \frac{cx}{(1-x)(1-x+cx)} \quad c=K/K^* \quad x=p/p^* \quad \theta = kp^{1/n} \quad \theta = A \ln(Bp)$$

$$d\gamma = -\sum_i \Gamma_i d\mu_i - \sigma^\alpha d(\Delta\phi) \quad \left( \frac{\partial \gamma}{\partial V} \right)_{T,\mu} = -\sigma \quad C = \frac{d\sigma}{dV} = -\left( \frac{\partial^2 \gamma}{\partial V^2} \right)_{T,\mu}$$

$$\gamma = \gamma^{\text{max}} - \frac{\varepsilon}{2d} \phi_e^2 \quad x_D = \left( \frac{\varepsilon RT}{2F^2 I} \right)^{1/2} \quad \gamma = \gamma^{\text{max}} - \frac{\varepsilon}{2x_D} \phi_e^2$$

## Tema 5

$$v = \frac{k_1 k_2 [S][C]}{k_1 [S] + k_{-1} + k_2}; \quad v_0 = \frac{k_2 [S]_0 [C]_0}{[C]_0 + [S]_0 + K_m}$$

$$i = nFAv_{total} = nFAk^0 \left( e^{\frac{(1-\alpha)nF(E-E^{0'})}{RT}} [R] - e^{-\frac{\alpha nF(E-E^{0'})}{RT}} [O] \right)$$

$$i_0 = i_{a,eq} = nFAk^0 \cdot e^{\frac{(1-\alpha)nF(E_{eq}-E^{0'})}{RT}} [R]$$

$$i_0 = -i_{c,eq} = nFAk^0 \cdot e^{-\frac{\alpha nF(E_{eq}-E^{0'})}{RT}} [O]$$

$$i = i_0 \left( e^{\frac{(1-\alpha)nF\eta}{RT}} - e^{-\left(\frac{\alpha nF\eta}{RT}\right)} \right) ; \quad j = j_0 \left( e^{\frac{(1-\alpha)nF\eta}{RT}} - e^{-\left(\frac{\alpha nF\eta}{RT}\right)} \right)$$

## Tema 6

$$\bar{x}_n = \frac{\sum_{i=1}^{i=\infty} x_i N_i}{\sum_{i=1}^{i=\infty} N_i} = \sum x_i n_i; \quad \bar{x}_w = \frac{\sum_{i=1}^{i=\infty} x_i^2 N_i}{\sum_{i=1}^{i=\infty} x_i N_i} = \frac{\sum_{i=1}^{i=\infty} x_i^2 n_i}{\sum_{i=1}^{i=\infty} x_i n_i}; \quad \bar{x}_z = \frac{\sum_{i=1}^{i=\infty} x_i^3 N_i}{\sum_{i=1}^{i=\infty} x_i^2 N_i} = \frac{\sum_{i=1}^{i=\infty} x_i^3 n_i}{\sum_{i=1}^{i=\infty} x_i^2 n_i}$$

$$\bar{x}_n = \frac{1}{\sum \frac{w_i}{x_i}}; \quad \bar{x}_w = \sum w_i x_i; \quad \bar{x}_z = \frac{\sum x_i^2 w_i}{\sum x_i w_i}$$

$$\bar{x}_v = \left( \frac{\sum x_i^{1+a} n_i}{\sum x_i n_i} \right)^{1/a} = \left( \sum x_i^a w_i \right)^{1/a}; \quad \Delta G_M = kT \chi_1 n_1 \phi_2 + kT (n_1 \ln \phi_1 + n_2 \ln \phi_2)$$