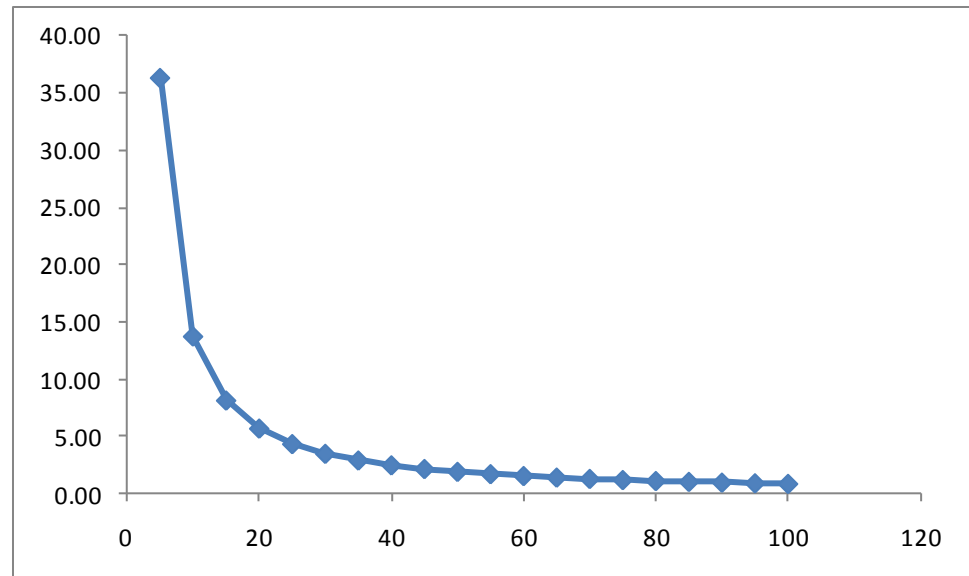


1.1 La aproximación de Stirling permite evaluar el logaritmo de factoriales de números grandes con un error pequeño. Calcula y representa el error relativo (en %) obtenido al utilizar la fórmula de Stirling para calcular $\ln x!$ Cuando $1 < x < 100$.

$$\begin{aligned}\ln x! &= \ln[x(x-1)(x-2)\dots 1] \\ &= \ln x + \ln(x-1) + \ln(x-2) + \dots + \ln 1 = \sum_{x=1}^x \ln x \approx \int_{x=1}^x \ln x dx \\ &= [x \ln x]_1^x - [x]_1^x = [x \ln x - \ln 1] - [x - 1] \approx x \ln x - x\end{aligned}$$

x	Ln x!	xLn x - x	Error(%)
5	4.78749174	3.04718956	36.35
10	15.1044126	13.0258509	13.76
15	27.8992714	25.620753	8.17
20	42.3356165	39.9146455	5.72
25	58.0036052	55.4718956	4.36
30	74.6582363	72.0359214	3.51
35	92.1361756	89.4371822	2.93
40	110.32064	107.555178	2.51
45	129.123934	126.299812	2.19
50	148.477767	145.60115	1.94
55	168.327445	165.403325	1.74
60	188.628173	185.660674	1.57
65	209.342587	206.335173	1.44
70	230.439044	227.394667	1.32
75	251.890402	248.811609	1.22
80	273.673124	270.562131	1.14
85	295.766601	292.625357	1.06
90	318.15264	314.98287	1.00
95	340.815059	337.618305	0.94
100	363.739376	360.517019	0.89



2. Realiza las siguientes derivadas:

$$\frac{d}{dx} [\ln(ax^n)] = \frac{\frac{d}{dx}(ax^n)}{ax^n} = \frac{a \cdot nx^{n-1}}{ax^n} = \frac{n}{x}$$

$$\begin{aligned} \frac{d}{dx} \left[\ln \frac{a}{x!} \right] &= \frac{d}{dx} [\ln a - \ln x!] = \frac{d}{dx} [-\ln x!] \approx \frac{d}{dx} [-x \ln x + x] \\ &= -\ln x - x \frac{1}{x} + 1 = -\ln x \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} \left[\ln \left(\frac{1}{1 - e^{-a/x}} \right) \right] &= \frac{d}{dx} [\ln 1 - \ln(1 - e^{-a/x})] = \frac{d}{dx} [-\ln(1 - e^{-a/x})] \\ &= -\frac{\frac{a}{x^2} e^{-a/x}}{1 - e^{-a/x}} = \frac{\frac{a}{x^2} e^{-a/x}}{1 - e^{-a/x}} = \frac{\frac{a}{x^2} e^{-a/x} e^{a/x}}{(1 - e^{-a/x}) e^{a/x}} = \frac{\frac{a}{x^2}}{(e^{a/x} - 1)} \end{aligned}$$

$$\frac{d}{dx} \left[\sum_i e^{-\frac{a_i}{x}} \right] = \sum_i \frac{d}{dx} e^{-\frac{a_i}{x}} = \sum_i \frac{a_i}{x^2} e^{-\frac{a_i}{x}}$$