

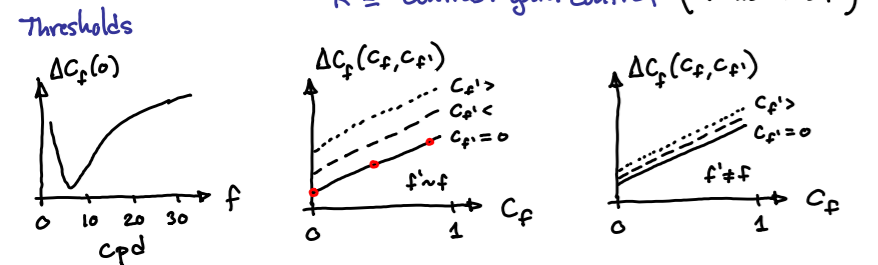
1 INTRODUCTION

PSYCHOPHYSICS

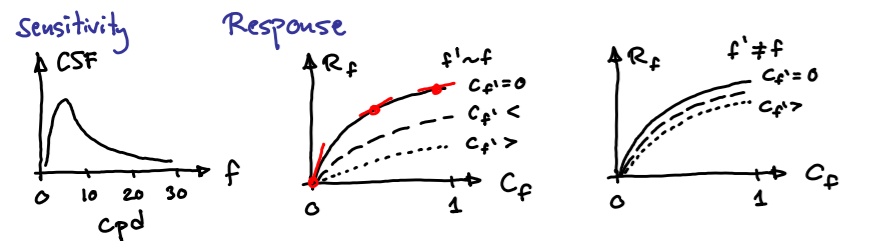
⊗ SPATIAL DIMENSIONS: NON-LINEAR Spatial frequency analyzers

$$x \xrightarrow{W} c \xrightarrow{F} c' \xrightarrow{R} r$$

$W \equiv$ Wavelet transform matrix (Watson 83)
 $F \equiv$ Diagonal CSF-based matrix (Campbell 68)
 $R \equiv$ Contrast gain control (Watson 87)

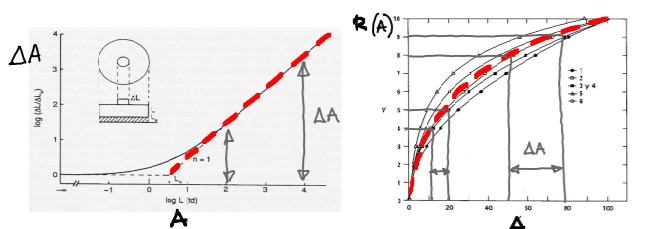


$$\frac{dR_f}{dc_f} \propto \frac{1}{\Delta C_f} \Rightarrow R_f = \int_0^c \frac{1}{\Delta C_f(c_f)} \cdot dc_f$$

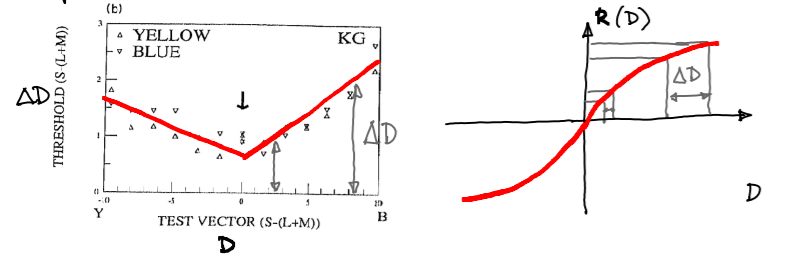


⊗ COLOR DIMENSIONS: Achromatic (A) and chromatic (T and D) mechanisms are NON-LINEAR

* Achromatic Channel: Weber law (e.g. Wyszecki & Stiles 82)



* Chromatic Channels: e.g. non-linearities in Yellow-Blue (D) channel (Krauskopf & Gegenfurtner 92, Romero et al. 93)



PREVIOUS (RESTRICTED) THEORETICAL DERIVATIONS

⊗ SPATIAL DIMENSIONS

- $W \equiv$ (Olshausen & Field 96, Bell & Sejnowski 97) → Global linear ICA
- $F \equiv$ (Atick 92) → Wiener Filter-like argument
- $R \equiv$ (Schwartz & Simoncelli 01) → Independence using divisive normalization (Malo & Gutierrez 06) → Non-linear (local-to-global) ICA

⊗ COLOR DIMENSIONS

- (Linear) opponent channels are similar to global PCA (e.g. Simoncelli & Olshausen 01)
- ATD non-linearities from (Mahalanobis) non-linear PCA (Laparra & Malo 08)

SPATIAL MASKING AND COLOR ADAPTATION FROM NON-LINEAR PCA



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Jesús Malo

2 OUR WORK:

- ⊗ Joint derivation of F and R using non-linear PCA
- ⊗ Derivation of ATD non-linearities using non-linear PCA
- ⊗ Better foundation of non-linear PCA

3 (LOCAL-TO-GLOBAL) NON-LINEAR PCA

- ⊗ General ideas (from local-to-global ICA Malo & Gutierrez 06)
 - + Describe manifolds with curved coordinates with metric related to the local density
 - + Integrate local behavior into global description

$$\text{Stimulus } x \xrightleftharpoons[T^{-1}]{T} \text{Response } r$$

(Lin 99)

$$\nabla T(x') = A_\ell(x')$$

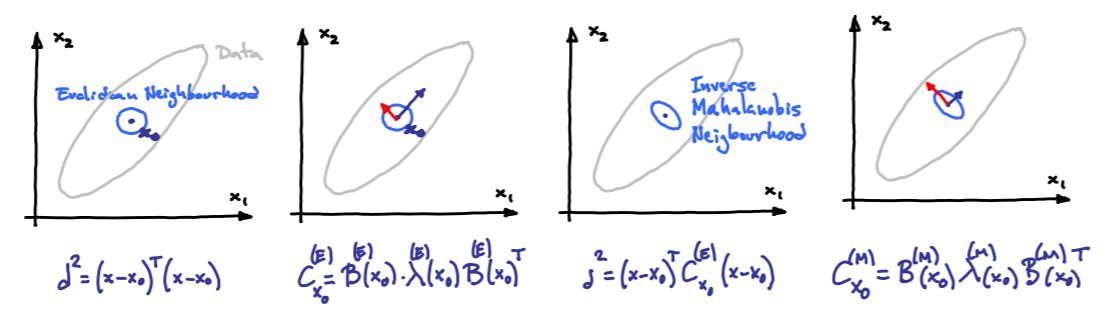
(Malo & Gutierrez)

$$T(x) \equiv r = r_0 + \int_{x_0}^x A_\ell(x') dx' \quad T^{-1}(r) \equiv x = x_0 + \int_{r_0}^r A_\ell(x'(r')) dr'$$

⊗ Assumptions:

- A.1 Local linear axes describe local structure of the PDF (local indep.)
- A.2 Integration of locally independent variables lead to global independence

⊗ Approach I: Local Mahalanobis



$$\nabla T(x') = B^{(m)}(x') \cdot \lambda^{(m)}(x')^{\frac{1}{2}}$$

- * Local axes from local PCA (on ellipsoids)
- * Local metric from inverse Mahalanobis
- * Weakness: variance is not necessarily correlated to data density

⊗ Approach II: line element for constant mass variation

$$\nabla T(x') = B^{(s)}(x') \cdot \Delta(x')$$

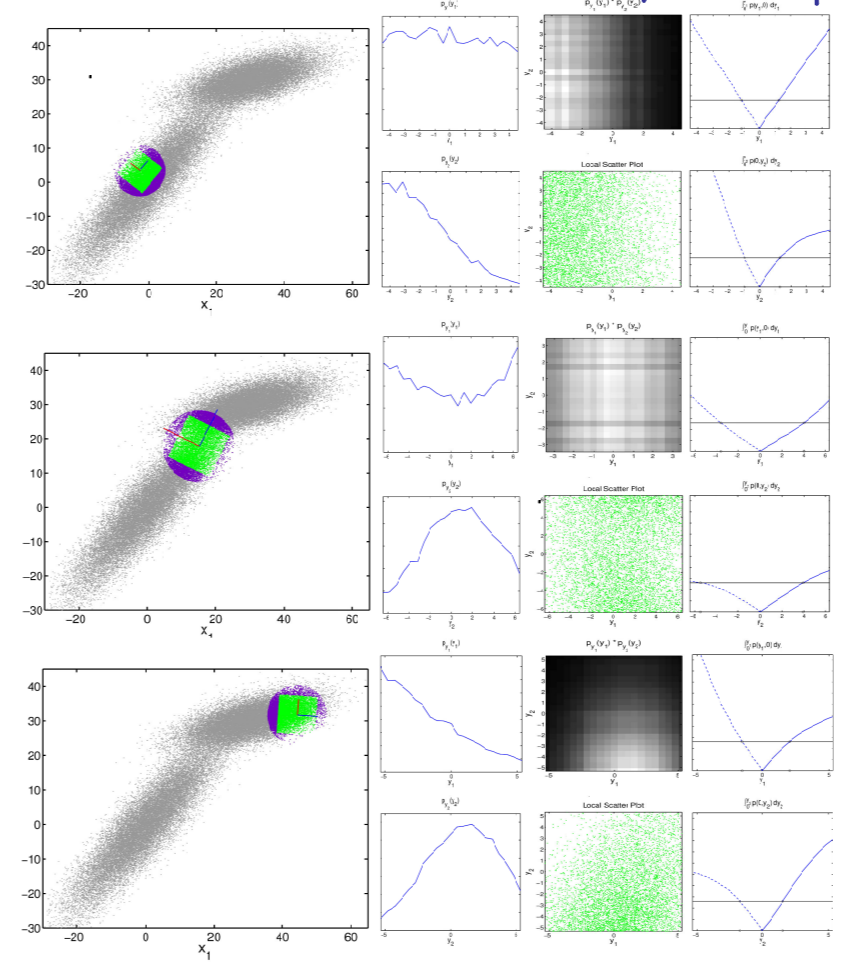
- * Local axes from local PCA (on spheres)
- * Local metric for equal increment of the CDF in each dimension

$$\Delta s_i = \Delta y_i / c = \int_0^c P(0, \dots, 0, y_i, 0, \dots, 0) dy_i$$

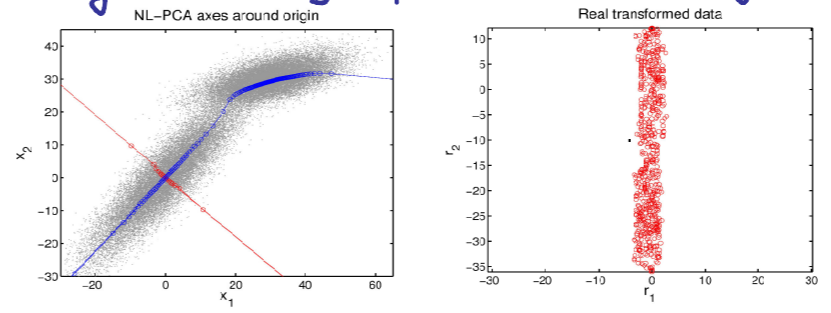
where $y \equiv$ Project. on local axis: $y = B^{(s)}(x')^T \cdot (x - x')$
and $P(y) = \prod_i P_i(y_i)$ (y_i are locally independent.)

4 DISCUSSION

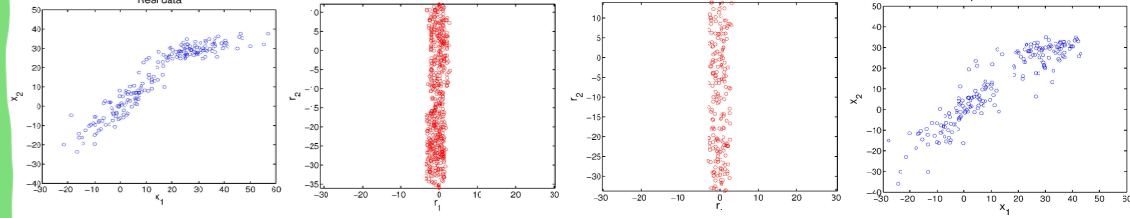
⊗ A.1 Local PCA is fine (enough) for local component independence



⊗ A.2 Integration of locally independent variables lead to global independence

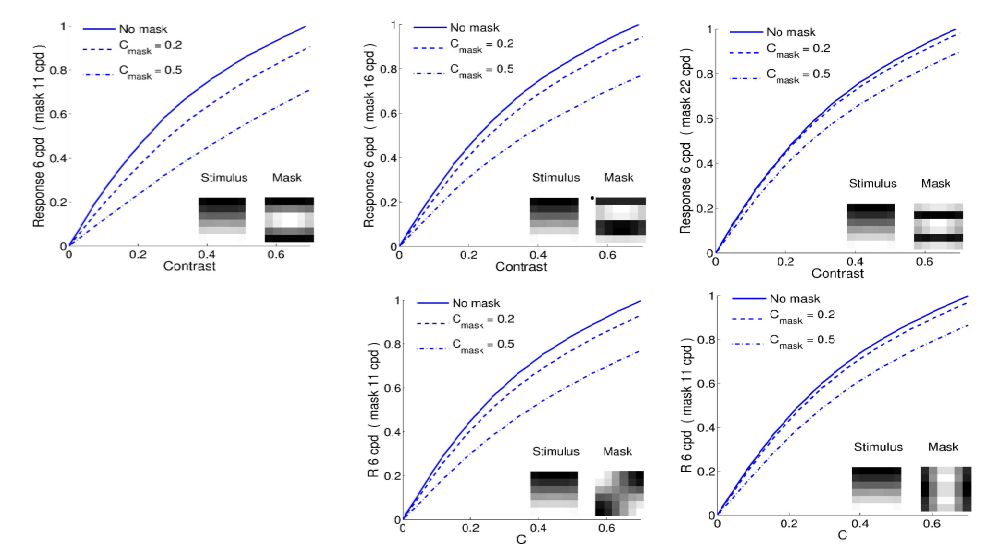
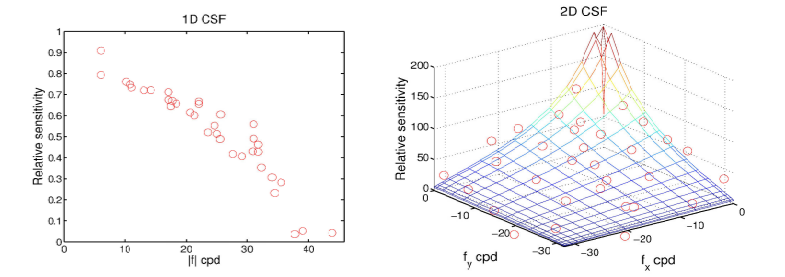


⊗ Transform and inverse: synthesis example

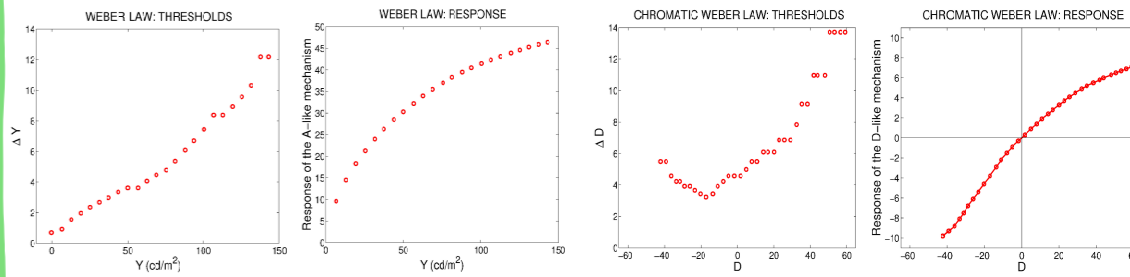


5 PERCEPTION RESULTS

⊗ SPATIAL DIMENSIONS



⊗ COLOR DIMENSIONS



6 CONCLUSIONS

- (1) Non-linear PCA on a representative database reproduces the spatial frequency sensitivity, F , the gain control, R , and classical nonlinearities in achromatic and chromatic channels.
- (2) Color results (Laparra & Malo 08) predict that specific environments or adaptation conditions may induce different non-linear behavior
- (3) Further work includes: (i) systematic testing of the assumptions and robustness of the method, (ii) experimental test of the predictions in (2), and (iii) exploring the applications (automatic contrast and color correct, non-linear classification, vector quantization...)

7 REFERENCES

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