Commutativity of the Valdivia–Vogt structure table

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Preliminaries: Topological Tensor Products

Let E and F be two separated locally convex spaces. We use the following two topologies on the tensor product $E \otimes F$.

$E \otimes_\pi F$ ... finest locally convex topology such that

$$\mathrm{can}\colon E \times F \to E \otimes F, (x, y) \mapsto x \otimes y$$

is continuous.

$E \otimes_\iota F$ ... finest locally convex topology such that

$$\mathrm{can}\colon E \times F \to E \otimes F, (x, y) \mapsto x \otimes y$$

is partially continuous.

$E \widehat{\otimes}_\pi F$ and $E \widehat{\otimes}_\iota F$ completion of $E \otimes_\pi F$ and $E \otimes_\iota F$, respectively.

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The Valdivia-Vogt structure table

$$\mathcal{D} \subset \mathcal{S} \subset \mathcal{D}_{L^p} \subset \dot{\mathcal{B}} \subset \mathcal{D}_{L^\infty} \subset \mathcal{O}_C \subset \mathcal{O}_M \subset \mathcal{E}$$

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$$\begin{aligned} \mathcal{D} = \mathcal{D}(\mathbb{R}^n) &= \{f \in \mathcal{C}^\infty(\mathbb{R}^n); \operatorname{supp} f \text{ compact}\} \\ s^{(\mathbb{N})} &= \left\{ (x(j))_{j\in\mathbb{N}} \in s^{\mathbb{N}}; \exists N\, \forall k \geq N \colon x(k) = 0 \right\} \\ &\cong \lim_{k\to} (s)^k = \lim_{k\to} (\mathbb{C}^k \widehat{\otimes} s) = \mathbb{C}^{(\mathbb{N})}\widehat{\otimes}_\iota s \end{aligned}$$

Representation: M. Valdivia (1978), D. Vogt (1983), B (2012)

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\wr\| & & \wr\| & & & & & & & & & & & & \\
\mathbb{C}^{(\mathbb{N})}\widehat{\otimes}_\iota s & \subset & s\widehat{\otimes} s & & & & & & & & & & & &
\end{array}$$

$$\mathcal{S} = \mathcal{S}(\mathbb{R}^n) = \left\{ f \in \mathcal{C}^\infty(\mathbb{R}^n); \forall k \in \mathbb{N}_0 \forall \alpha \in \mathbb{N}_0^n \colon (1+|x|^2)^{k/2}\partial^\alpha f(x) \in \mathcal{C}_0 \right\}$$

$$s\widehat{\otimes} s = \left\{ (x(i,j))_{(i,j)\in\mathbb{N}^2}; \forall k \in \mathbb{N}_0 \forall l \in \mathbb{N}_0 \colon \sup_{i,j\in\mathbb{N}} |i^k j^l x(i,j)| < \infty \right\}$$

Representation: A. Grothendieck (1955), L. Schwartz (1957), M. Valdivia (1982) and R. Meise and D. Vogt (1992)

The Valdivia-Vogt structure table

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$$\mathcal{D}_{L^p} = \mathcal{D}_{L^p}(\mathbb{R}^n) = \left\{ f \in \mathcal{C}^\infty(\mathbb{R}^n); \forall \alpha \in \mathbb{N}_0^n \colon \partial^\alpha f \in L^p(\mathbb{R}^n) \right\}$$

$$\ell^p\widehat{\otimes}s = \left\{ (x(i,j))_{(i,j)\in\mathbb{N}^2}; \forall k \in \mathbb{N}_0 \colon \sup_{j\in\mathbb{N}} \left(\sum_{i\in\mathbb{N}} j^{pk} |x(i,j)|^p \right)^{1/p} < \infty \right\}$$

Representation: M. Valdivia (1981), D. Vogt (1983), N. Ortner and P. Wagner (2013)

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\end{array}
$$

$$\dot{\mathcal{B}} = \dot{\mathcal{B}}(\mathbb{R}^n) = \left\{ f \in \mathcal{C}^\infty(\mathbb{R}^n); \forall \alpha \in \mathbb{N}_0^n \colon \partial^\alpha f \in \mathcal{C}_0(\mathbb{R}^n) \right\}$$

$$c_0\widehat{\otimes} s = \left\{ (x(i,j))_{(i,j)\in\mathbb{N}^2}; \forall k \in \mathbb{N} \colon \lim_{i\to\infty} \sup_{j\in\mathbb{N}} |j^k x(i,j)| = 0 \right\}$$

Representation: M. Valdivia (1982), D. Vogt (1983)

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$$\ell^\infty\widehat{\otimes}s = \left\{ (x(i,j))_{(i,j)\in\mathbb{N}^2}; \forall k \in \mathbb{N}_0 \colon \sup_{i,j\in\mathbb{N}} \{|j^k x(i,j)|\} < \infty \right\}$$

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\end{array}
$$

$$\mathcal{O}_C = \left\{ f \in \mathcal{C}^\infty(\mathbb{R}^n); \exists k \in \mathbb{N}_0 \, \forall \alpha \in \mathbb{N}_0^n \colon \sup_{x \in \mathbb{R}^n} (1 + |x|^2)^{-\frac{k}{2}} \, |\partial^\alpha f(x)| < \infty \right\}$$

$$s'\widehat{\otimes}_\iota s = \left\{ (x(i,j))_{i,j\in\mathbb{N}}; \exists k \in \mathbb{N}_0 \, \forall l \in \mathbb{N}_0 \colon \sup_{i,j\in\mathbb{N}} |i^{-k} j^l x(i,j)| < \infty \right\}.$$

Representation: B (2012)

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\end{array}$$

$$\mathcal{O}_M = \left\{ f \in \mathcal{C}^\infty(\mathbb{R}^n); \forall \alpha \in \mathbb{N}_0^n\, \exists k \in \mathbb{N}_0\colon \sup_{x\in\mathbb{R}^n} (1+|x|^2)^{-\frac{k}{2}} |\partial^\alpha f(x)| < \infty \right\}$$

$$s'\widehat{\otimes}_\pi s = \left\{ (x(i,j))_{(i,j)\in\mathbb{N}^2}; \forall l \in \mathbb{N}_0 \exists k \in \mathbb{N}_0\colon \sup_{i,j\in\mathbb{N}} |i^{-k} j^l x(i,j)| < \infty \right\}$$

Representation: M. Valdivia (1981)

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Question: Can the isomorphisms in this table be chosen in a way such that it becomes a commutative diagram?

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Question: Can the isomorphisms in this table be chosen in a way such that it becomes a commutative diagram?

Problem: The above isomorphisms (besides $\mathcal{D}_{L^p} \cong \ell^p\widehat{\otimes} s$ for $1 < p < \infty$) are not known explicitly.

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\end{array}
$$

Theorem

There is an isomorphism $\Phi\colon \mathcal{E}(\mathbb{R}^n) \to \mathbb{C}^{\mathbb{N}}\widehat{\otimes} s$ such that the isomorphisms above can be chosen as the restrictions of Φ, i.e., the Valdivia–Vogt structure table can be interpreted as a commutative diagram.

The main idea of the proof

Observations:

- Construction of an explicit isomorphism $\Phi\colon \mathcal{E}(\mathbb{R}^n) \to \mathbb{C}^{\mathbb{N}} \widehat{\otimes} s$ seems to be very hard.
- It holds $\mathbb{C}^{\mathbb{N}} \widehat{\otimes} s = s^{\mathbb{N}}$.

Idea:

- Find a space of functions $\mathcal{E}_0 \cong s$ and decompose functions in $\mathcal{E}$ (uniquely) into sequences of functions in $\mathcal{E}_0$.

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Whitney functions and extension operators

Let $A \subset \mathbb{R}^n$, then we denote by $\mathcal{E}(A)$ the space of Whitney jets on A, which are by Whitney's extension theorem the jets arising from restrictions of all derivatives of smooth functions to A.

If A is a convex compact set (or more general admits a fundamental system of compact sets consisting of convex sets), then $\mathcal{E}(A)$ carries the topology of uniform convergence of all partial derivatives (on compact subsets).

We will only need the cases where $A = [0, \infty)^n$ or $A = [0, 1]^n$.

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An extension operator $E\colon \mathcal{E}([0,\infty)^n) \to \mathcal{E}(\mathbb{R}^n)$

The case $n = 1$ (for a half-space R. T. Seeley 1964):

Take sequences $(a_k)_{k\in\mathbb{N}_0}$ and $(b_k)_{k\in\mathbb{N}_0}$ such that $b_k \le -1$, $b_k \to -\infty$ and for all $l \in \mathbb{N}_0$ it holds $\sum_{k=0}^{\infty} a_k (b_k)^l = 1$ and $\sum_{k=0}^{\infty} |a_k|\,|b_k|^l < \infty$.
Moreover take $\varphi \in \mathcal{E}(\mathbb{R})$ with $0 \le \varphi(x) \le 1$, $\varphi(x) = 0$ for $x > \frac{1}{2}$ and $\varphi(x) = 1$ for $0 \le x \le \frac{1}{4}$.

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For $f \in \mathcal{E}([0,\infty))$, we define the operator E by

$$(Ef)(x) = \begin{cases} f(x) & x \geq 0 \\ \sum_{k=0}^{\infty} a_k \varphi(b_k x) f(b_k x) & x < 0. \end{cases}$$

An extension operator $E\colon \mathcal{E}([0,\infty)^n) \to \mathcal{E}(\mathbb{R}^n)$

The case $n = 2$:
Take sequences $(a_k)_{k\in\mathbb{N}_0}$ and $(b_k)_{k\in\mathbb{N}_0}$ such that $b_k \leq -1$, $b_k \to -\infty$ and for all $l \in \mathbb{N}_0$ it holds $\sum_{k=0}^{\infty} a_k (b_k)^l = 1$ and $\sum_{k=0}^{\infty} |a_k|\,|b_k|^l < \infty$.
Moreover take $\varphi \in \mathcal{E}(\mathbb{R})$ with $0 \leq \varphi(x) \leq 1$, $\varphi(x) = 0$ for $x > \frac{1}{2}$ and $\varphi(x) = 1$ for $0 \leq x \leq \frac{1}{4}$.
For $f \in \mathcal{E}([0,\infty)^2)$, we define the operator E by

$$(Ef)(x,y) = \begin{cases} f(x,y) & \text{for } x, y \geq 0 \\ \sum_{k=0}^{\infty} a_k \varphi(b_k x) f(b_k x, y) & \text{for } x < 0, y \geq 0 \\ \sum_{k=0}^{\infty} a_k \varphi(b_k y) f(x, b_k y) & \text{for } x \geq 0, y < 0 \\ \sum_{k,l=0}^{\infty} a_k a_l \varphi(b_k x)\varphi(b_l y) f(b_k x, b_l y) & \text{for } x < 0, y < 0 \end{cases}.$$

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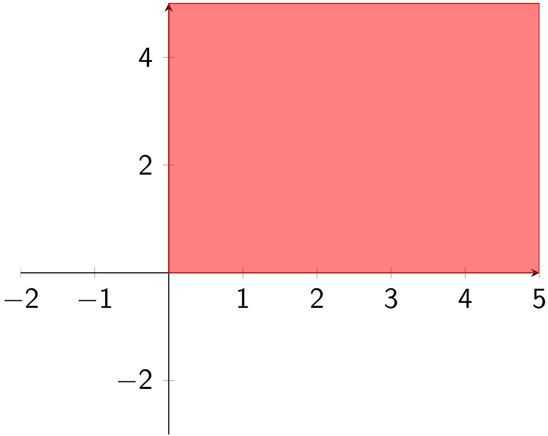
Moreover take $\varphi \in \mathcal{E}(\mathbb{R})$ with $0 \leq \varphi(x) \leq 1$, $\varphi(x) = 0$ for $x > \frac{1}{2}$ and $\varphi(x) = 1$ for $0 \leq x \leq \frac{1}{4}$.
For $f \in \mathcal{E}([0,\infty)^2)$, we define the operator E by

$$(Ef)(x,y) = \begin{cases} f(x,y) & \text{for } x, y \geq 0 \\ \sum_{k=0}^{\infty} a_k \varphi(b_k x) f(b_k x, y) & \text{for } x < 0, y \geq 0 \\ \sum_{k=0}^{\infty} a_k \varphi(b_k y) f(x, b_k y) & \text{for } x \geq 0, y < 0 \\ \sum_{k,l=0}^{\infty} a_k a_l \varphi(b_k x) \varphi(b_l y) f(b_k x, b_l y) & \text{for } x < 0, y < 0 \end{cases}.$$

An extension operator $E\colon \mathcal{E}([0,\infty)^n) \to \mathcal{E}(\mathbb{R}^n)$

The case $n = 2$:



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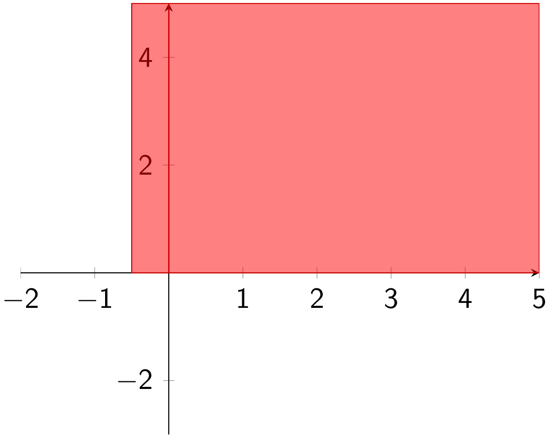

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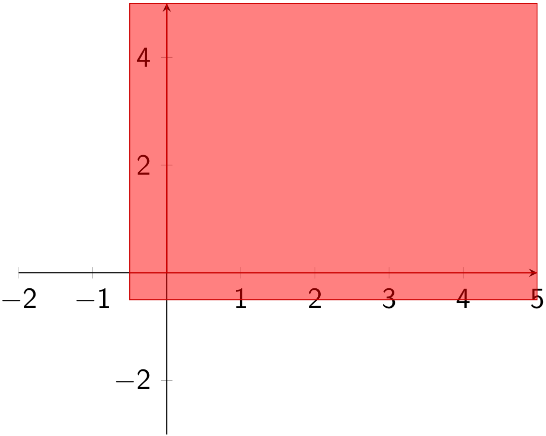

An extension operator $E\colon \mathcal{E}([0,\infty)^n) \to \mathcal{E}(\mathbb{R}^n)$

The case $n > 2$:
The operator is defined inductively by iterated extension (as indicated in the case $n = 2$).

An extension operator $E\colon \mathcal{E}([0,\infty)^n) \to \mathcal{E}(\mathbb{R}^n)$

We set

$$\mathbb{H}_{i,+} := \{x \in \mathbb{R}^n, x_i \geq 0\}$$

for $i = 1, \ldots, n$ and denote by

$$E_i\colon \mathcal{E}(\mathbb{H}_{i,+}) \to \mathcal{E}(\mathbb{R}^n)$$

the modified version of Seeley's extension operator.
Additionally, we define the operator

$$F_i\colon \mathcal{E}(\mathbb{R}^n) \to \mathcal{E}(\mathbb{R}^n), f \mapsto E_i\left[f|_{\mathbb{H}_{i,+}}\right].$$

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The isomorphism $\Phi\colon \mathcal{E}(\mathbb{R}^n) \to (\mathcal{E}_0)^{\mathbb{Z}^n}$

We define $B_1 = [0,1]^n \setminus [0,1)^n$ and

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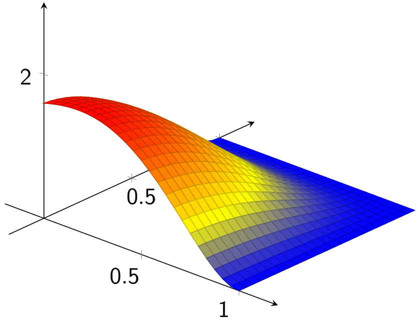
An example for $n = 2$



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Proposition

The space $\mathcal{E}_0$ is a nuclear Fréchet space, has the properties (Ω) and (DN) and is isomorphic to the space s of rapidly decreasing sequences, i.e., $\mathcal{E}_0 \cong s$.

The isomorphism $\Phi\colon \mathcal{E}(\mathbb{R}^n) \to (\mathcal{E}_0)^{\mathbb{Z}^n}$

Given $f \in \mathcal{E}(\mathbb{R}^n)$, we set $f_0 = f$ and

$$f_{(i+1)} = f_{(i)} - \tau_{e_{i+1}} F_{i+1} \left[\tau_{-e_{i+1}} f_{(i)}\right]$$

for $1 \leq i \leq n-1$. Let $j = (j_1, \ldots, j_n) \in \mathbb{Z}^n$, we define

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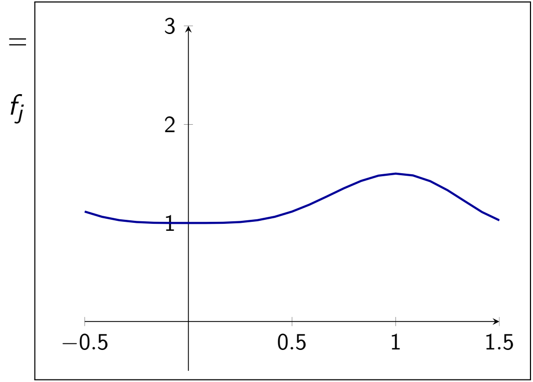

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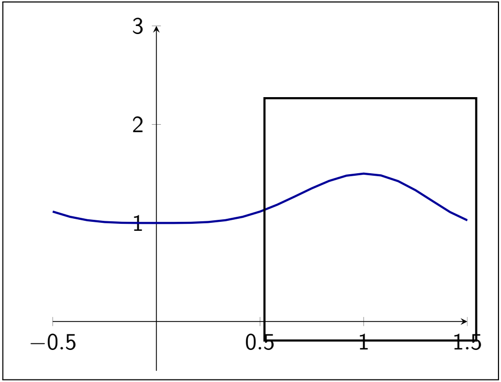

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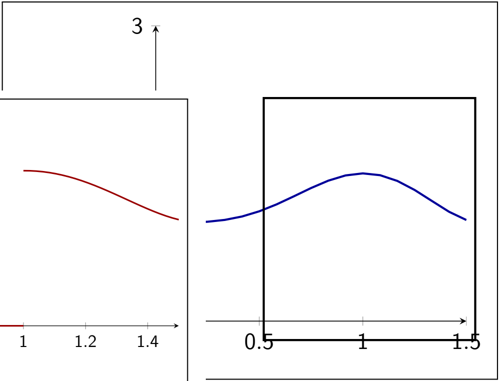

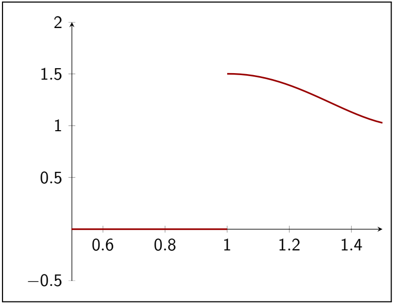

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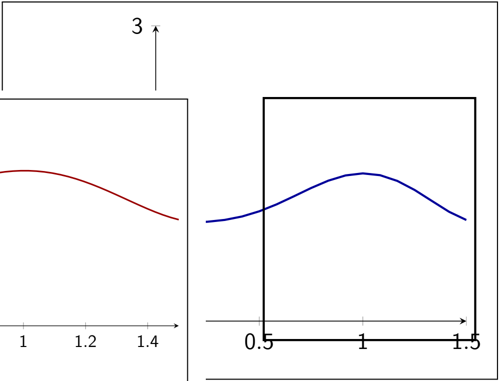

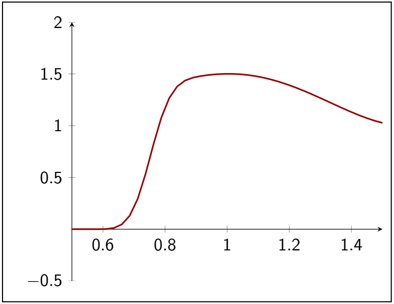

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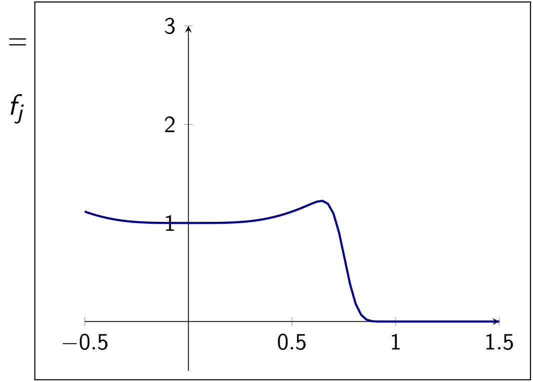

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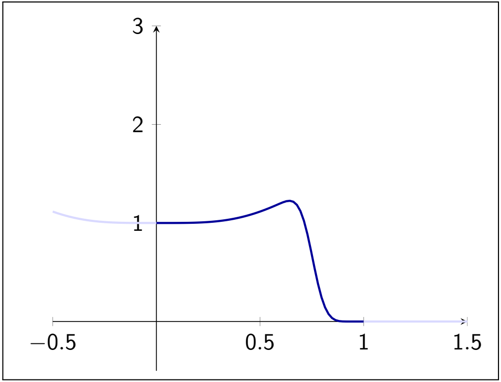

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$$\Phi^{-1}\colon (\mathcal{E}_0)^{\mathbb{Z}^n} \to \mathcal{E}(\mathbb{R}^n), (f_j)_{j\in\mathbb{Z}^n} \mapsto \sum_{j \in \mathbb{Z}^n} \tau_j E \tilde{f}_j.$$

A corollary

The dual Validivia–Vogt structure table

$$\begin{array}{ccccccccccccccc}
\mathcal{E}' & \subset & \mathcal{O}'_M & \subset & \mathcal{O}'_C & \subset & \mathcal{D}'_{L^1} & \subset & \mathcal{D}'_{L^p} & \subset & \mathcal{D}'_{L^\infty} & \subset & \mathcal{S}' & \subset & \mathcal{D}' \\
\wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| \\
\mathbb{C}^{(\mathbb{N})}\widehat{\otimes} s' & \subset & s\widehat{\otimes}_\iota s' & \subset & s\widehat{\otimes}_\pi s' & \subset & \ell^1\widehat{\otimes} s' & \subset & \ell^p\widehat{\otimes} s' & \subset & \ell^\infty\widehat{\otimes} s' & \subset & s'\widehat{\otimes} s' & \subset & \mathbb{C}^{\mathbb{N}}\widehat{\otimes} s'
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Corollary

There is an isomorphism $\Psi\colon \mathcal{D}'(\mathbb{R}^n) \to \mathbb{C}^{\mathbb{N}}\widehat{\otimes} s'$ such that the isomorphisms above can be chosen as the restrictions of Ψ, i.e., the dual Valdivia–Vogt structure table can be interpreted as a commutative diagram.

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The sequence $(e_i) \otimes (e_j)$ (with proper ordering) is a common Schauder basis for most of the sequence-spaces in the structure table.

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L. Schwartz defines the space $\dot{\mathcal{B}}'$ of "distributions vanishing at infinity" using the following analogy to the space $\dot{\mathcal{B}}$ of smooth functions vanishing at infinity:

$\dot{\mathcal{B}}$ is the closure of $\mathcal{D}$ in $\mathcal{D}_{L^\infty}$

Translate this situation the setting of distributions:

Define $\dot{\mathcal{B}}'$ as the closure of $\mathcal{E}'$ in $\mathcal{D}'_{L^\infty}$.

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