ASSESSING NON-LINEAR STRUCTURES IN REAL EXCHANGE RATES USING RECURRENCE PLOT STRATEGIES*

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*The authors wish to thank Professor C.L. Webber for his helpful comments on the RQA methodology. Moreover, part of the computations in this paper were performed using his program RQA, publicly available at http:\\www.rushu.rush.edu/molbio/physiozbi.html.

Abstract: Purchasing Power Parity is an important theory at the basis of a large number of economic models. However, the implication derived form the theory that real exchange rates must follow stationary processes is not conclusively supported by empirical studies. In a recent paper, Serletis and Gogas (2000) show evidence of deterministic chaos in several OECD exchange rates. As a consequence, PPP rejections would be spurious. In this work, we follow a two-stage testing procedure to test for non-linearities and chaos in real exchange rates, using a new set of techniques designed by Webber and Zbilut (1994), under the denomination of Recurrence Quantification Analysis (RQA). Our conclusions differ slightly from Serletis and Gogas (2000), but they are also supportive of chaos for some exchange rates.

Key Words: PPP; non-linearity; chaos; recurrence analysis.

JEL Classification: C19; F31.

1. Introduction.

One of the most salient features of many open-economic theoretical models is that they require the fulfillment of the Purchasing Power Parity (PPP) theory. It is well known that it imposes (long run) equilibrium restrictions to the evolution of nominal exchange rates, and it is an unavoidable ingredient of the so called *real interest rate parity*.

As it is predicted by the PPP theory in its strong form, a good in a country should sell for the same price as in any other country. A weaker form of PPP states that the nominal exchange rates should reflect the movements in international inflation rates differentials. Any way, a direct consequence of both forms is that the *real* exchange rate must follow a stationary process.

However, though its theoretical and practical relevance, there is no conclusive evidence that supports the PPP theory among the large amount of empirical studies, in spite of the variety of techniques and approaches used in the analysis (see, e.g., Ardeni and Lubian (1991) , and Serletis and Zimonopoulos (1997)). Nonetheless, as it is stressed in a recent paper by Serletis and Gogas (2000), all the procedures applied have been based on the assumption that the process driving the dynamics of the exchange rates (and the economy as a whole) is inherently linear. But this can be a too restrictive assumption.

In recent years, many researchers have been worried about finding explanations to economic time series complex behaviour, which (Gaussian) linear stochastic models are not able to account for. Traditionally, the alternative hypothesis has been set as: 1) the underlying process is a non-linear stochastic one, or 2) the underlying process driving the variable is chaotic. In a nutshell, a variable is said to be chaotic if it "looks" like a random process when actually it is the realisation of a non-linear purely deterministic process (Hsieh (1991)).

We assume that economic systems can be thought as complex dynamical processes, maybe chaotic, that are continuously subjected to and updated by non-linear feedforward and feedback inputs. But chaotic systems are closed and noise-free, whereas economic systems are open and noisy. Thus, it is not surprising the controversy about the potential relevancy of chaos for economics and econometrics, despite its importance to other scientific disciplines (Barnett and He (2000)). However, since the early work of Brock (1986), such a controversy has not avoided a considerable effort in the search for deterministic chaos in economic and financial time series.

On the other hand, though it is very interesting to determine if a process is chaotic, it is also of interest to assess if there exist any kind of deterministic structures, chaotic or not. Even further, it is worth to know if there exist any non-linear structures at all, since non-linearity is a necessary condition for chaos and other forms of determinism. Thereafter, it seems “natural” to test for possible non-linearities as a previous step to check the possibility of chaos.

This has inspired a huge number of studies concerning the detection of non-linear dynamics in economic data, and the development of diverse testing procedures. As a test of general non-linearity, there seems to be an extended agreement on the use of the BDS test approach (see Brock et al. (1989)), by which, firstly, a linear model is fitted to data, and the residuals of the estimation process are tested for the iid hypothesis by means of the BDS statistics; if the linear model has been correctly specified, then a rejection of the null (iid) is a sign of further non-linear structure. However, this procedure is uninformative about the direction of the alternative process (non-linear stochastic or chaotic), although it is a very powerful test against a wide range of linear and non-linear models.

In order to discern the correct alternative, it has led to the extensive use of non-linear time series analysis tools, mainly inherited from Physics and not always used as accurately as it is needed (see Kantz and Schreiber (1997) for a guide of potential misunderstandings of techniques and results). Since small sample size and poor quality of data, relative to the amount and experimental character of data in other scientific disciplines, are serious drawbacks when facing the application of the existing procedures, this line of research has developed mainly in financial markets analysis (see Barnett and Serletis (2000) for a survey) rather than in macroeconomics research. To sum up, the studies show weak evidence of chaos for some financial variables, and a stronger evidence of non-linear mechanisms. But such non-linearities use to fade away when accounting for ARCH-type dynamics. It may be argued, however, that these results are dependent of the size properties and power of the tests involved in the research process.

In a recent paper, Barnett et al. (1997) design a single-blind controlled competition among five test for non-linearity or chaos with ten simulated data series. From that competition, they conclude that some of the tests could be viewed as complementary, rather than competing, given their different power against specific alternatives. Moreover, it stems from their research that a relatively new non-linearity test, the Kaplan’s test (1993), works very well in the detection of departures from linearity. In fact, it was the only inference method which got the right answer in every case analysed in the competition. Therefore, Barnett et al. (1997) suggest that the BDS or Kaplan’s test, or both, could be the first run to rule out the null of linearity.

Recently, a new set of techniques is available to the scientist community interested on the study of non-linear complex dynamics, based on the so called recurrence plot (RP, hereafter). The RP is a graphical method designed by Eckmann et al. (1987) to detect recurrence patterns and nonstationarities in experimental data, and it is part of the denominated *topological approach*. This approach is characterised by the study of the organisation of the strange attractor. Let us remind that a strange attractor is the set of points toward which a chaotic dynamical path will converge. The RP technique does not impose any kind of statistical distribution, minimum sample size or stationarity, and it can be quite powerful in the detection of chaos. Thus, it seems very suitable for financial data analysis, since it has been proven useful in the detection of hidden dynamical behaviours not detected by standard linear and non-linear tools.

The main aim of this paper is to extend the utility of this non-linear tool in the assessment of non-linear, maybe deterministic, structures in real exchange rates. The basis of such an extension is the work of Webber and Zbilut (1994), who introduced the concept of "quantification of recurrences" in various scientific fields as Physiology, Physics or Linguistics[1]. This will be the first step of a two-stage procedure in the detection of chaos in real exchange rates. We want to check the possibility of chaos, since a chaotic dynamic could explain why PPP (apparently) does not hold for some exchange rates when applying traditional tests.

The remainder of this paper is organised as follows. Next section discusses the RP strategies and the so called Recurrence Quantification Analysis (RQA), illustrated through a chaotic simulated example (the x component of the Rossler equations). Results with RQA analysis and the Kaplan's test for 16 OECD real exchange rates are presented in section three, and the chaos assumption is tested. Finally, fourth section concludes.

2. Recurrence Plots strategies.

2.1 Recurrence Analysis.

If we analyse a complex system characterised by the non-linear interaction of a set of variables, it is commonplace in economics that the complete description of such a set is unknown. However, Takens' (1981) theorem states that we can recreate a topologically equivalent picture of the original multidimensional system behaviour, using the time series of a single observable variable, by means of the method of time delays: for the scalar series $\{x_t\}_{t=1}^{T}$, we build up the (embedded) vectors

$$x_i^m = \left(x_i, x_{i+t}, x_{i+2t}, \ldots, x_{i+(m-1)t}\right)$$

[1] Including an interesting application to Dow Jones Industrial Average Index in Webber and Zbilut (1998).

The set of all embedded vectors, $i = 1,...,T\text{-}(m\text{-}1)t$, constitutes a trajectory in $\Re^m$, where m is the embedding dimension and t is the time delay. The sequence of embedded vectors recreates the original dynamics only if parameters m and t are properly chosen by using methods like *false nearest neighbours* (for m) and *mutual information* (for t; however, if the data are not continuous, usually t is selected as one). The choice of m must assure that $m > 2d + 1$, where d is the original (unknown) system's dimension. Next, (not necessarily Euclidean) distances D are computed between all pairs (I, J) of embedded vectors.

Essentially, an RP is a graphical representation of the distances matrix D_{IJ}, by darkening the pixel located at coordinates (I, J) that correspond to a distance value between I and J vectors lower than a predetermined cutoff (or radius) e . Note that the plot is symmetric ($D_{IJ} = D_{JI}$), and that the main diagonal is always darkened ($D_{IJ} = 0$, $I = J$).

If the time series is deterministic, the system's attractor will be revisited by the trajectory sometime in the future. Then, RP will show short line segments (parallel to the main upward darkened diagonal) which correspond to sequences $(i, j),(i+1, j+1),\ldots(i+k, j+k)$, such that $x_j^m, x_{j+1}^m, \ldots x_{j+k}^m$ are close to $x_i^m, x_{i+1}^m, \ldots x_{i+k}^m$. But if the series is purely random, then the recurrence plot will not present any structure at all.

As an example of application on non-linear deterministic simulated data, 1000 observations of the x-component of the Rossler's system,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -(y+z) \\ x+cy \\ a+xz-bz \end{bmatrix}$$

were generated using a fourth-order Runge-Kutta integration and an integrator timestep of 0.01, with $a = 0.2$, $b = 10$, and $c = 0.15$. Figure 1 shows the recurrence plot of the Rossler's series for a time delay $d = 14$ and an embedding dimension $m = 4$; note the recurrent (darkened) points forming distinct diagonals parallel to the main diagonal among scattered recurrent points. Figure 2 shows the recurrence plot of the randomly shuffled Rossler's series (shuffling destroys the phasic time-correlated information in the dynamics): as a result, the recurrent points are uniformly scattered, since shuffling has destroyed the deterministic structures in the Rossler's system[2].

[2] Computations were performed using programs written by the authors in Ox, v.2.20 (see Doornik, 1996).

INSERT FIGURES 1 AND 2

As Gilmore (1993) pointed out, the topological methods, as RP, have important advantages, among others:

1. They are applicable to relatively small data sets, such are typical in economics and finance;
2. They are robust against noise.

However, the set of lines parallel to the main diagonal might not appear so clearly to the human eye (i.e., the size of the lines being relatively short); to solve this problem, Zbilut and Webber (1992) propose the recurrence quantification analysis (RQA).

2.2. Recurrence Quantification Analysis.

Since recurrence plots can contain subtle patterns that are not easily ascertained by visual inspection, Zbilut and Webber (1992) and Webber and Zbilut (1994) introduced the concept of quantification of recurrences. The logic for this approach is based on the diagonal line structures found in recurrence plots, and it consists on computing six variables: % of Recurrences (%REC), % of Determinism (%DET), Entropy (ENT), Divergence (DIV), Ratio, and Trend. However, the variables which are focused on evaluation of the deterministic content of a time series are %REC, %DET, ENT and DIV.

Previously to computing these variables, the analyst must choose the following parameters: 1) the time delay t, 2) the embedding dimension m, 3) the radius e, and 4) the minimum number of consecutive darkened points which constitutes a diagonal line, L. In section 3 we discuss briefly some guidelines for an adequate selection.

The first variable, %REC, quantifies the percentage of the plot occupied by recurrent points, i.e., it is the number of recurrent points per total triangular area (by symmetry of the plot, we focus on just one of the two triangular areas), excluding the upward central diagonal (which represents the distance between each embedded vector and itself). Embedded processes which are periodic have higher percent recurrence values than processes which exhibit aperiodic dynamics. For the Rossler series, we select the parameters[3] $t = 14$, $m = 4$, $e = 20$ and $L = 2$; the original data has a percentage of recurrence of 5.5%, and the shuffled version has a %REC of just 1.03.

The second variable, %DET, quantifies the percentage of recurrent points that form upward diagonal line segments, and it allows to distinguish between dispersed recurrent points

[3] In this paper, we compute distances in reference to the overall mean distance, i.e., $e = 100$ means "radius equal to the mean distance", and $e = 20$ means "radius equal to 20% of the mean distance".

and those that are organised in diagonal patterns. Upward diagonals are the signature of determinism since they represent strings of vectors reoccurring (repeating themselves) in the future. For example, Rossler's data has 99.6% of its recurrent points in diagonal line structures of at least 2 points length, whereas the shuffled counterpart has 2.5% of recurrent points forming diagonal structures.

The third variable, ENT, is computed as the Shannon entropy, addressing the complexity of recurrence plots. Upward line segment lengths are counted and distributed over integer bins of a histogram. Shannon entropy is computed according to the formula:

$$ENT = -\sum P_i \log_2 (P_i)$$

where P_i is the individual probability of the nonzero bin greater than or equal to the shortest selected line segment length. As the logarithms are in base 2, the entropy can be interpreted as number of bits. The more complex the deterministic structure of the recurrence plot, the larger the value of the entropy. In our example, the original series has ENT = 5.078, and the shuffled data has ENT = 0.0 (degenerate case due to the lack of recurrent points in the shuffled series).

The variable DIV is equal to the reciprocal of the maximum line length (ML). As Eckmann et al. (1983) show, the Maximum Lyapunov Exponent of the system (MLE, hereafter; see below) is inversely related to the maximum line length. If the original series is deterministic, we must expect a lower Lyapunov exponent than the randomly shuffled version, i.e., a larger maximum line length. In our example, DIV (original series) = 1/957 and DIV (shuffled series) = ½.

The variable ratio is defined as the quotient of %DET divided by %REC. It is useful to detect transitions between states: this ratio increases during transitions but settles down when a new quasy-steady state is achieved.

The variable trend is computed as follows: 1) we compute the percentage of recurrent points in diagonals parallel to the central line, 2) fit by least squares the relationship

$$d_j = a + b h_j + u_j$$

where d_j is the percentage of recurrent points, and h_j is the distance away from the central diagonal. The trend is the value of $\hat{b}$. If there is not drift in a dynamical system, there is no paling of the recurrence plot away from the central diagonal, leading to low values (near zero) of $\hat{b}$; however, large values (positive or negative) of $\hat{b}$ is an evidence of a system exhibiting drift. Thereafter, ratio and trend are RQA variables specially suited for detection of nonstationarities.

As it has been shown, RQA variables for the original series can be compared with the values corresponding to the respective RQA variables for a surrogate series in order to assess deterministic structures. Nevertheless, we will follow the approach in Manetti et al. (1999), which consists on computing the RQA magnitudes for a *set* of randomly shuffled copies, say 400 replications. Then, we will check the position of the original RQA descriptors relative to the 95th percentile computed with the shuffled series, since our tests will be one sided (see next section for additional details).

We should note that determinism in RQA is *defined* as the percentage of recurrent points forming diagonal lines parallel to the central upward diagonal. But this is just a definition. It is indeed possible for a stochastic process to produce "deterministic" patterns in the RQA terminology, thus leading to higher %DET, ENT or ML[4] than the shuffled copies, and one should not conclude that the process is "deterministic" in the usual sense. This also applies when the data are not stationary. Moreover, RQA is also inconclusive respect to possible chaotic processes. However, it can be used as a powerful tool to detect hidden periodicities driven by non-linear mechanisms, and to test the stationarity hypothesis. Let us remind that the procedure is quite robust to noise, and it does not impose any kind of restrictions on the data.

The test procedure we will adopt is based on the BDS approach: firstly, we will render the data stationary; then, we will remove all linear-in-mean dependencies by fitting an ARMA model. Next, we will try to identify a non-linear model for the variance of the process (since we are analysing financial variables, the most natural alternatives will concern ARCH-type structures). Finally, we will compute RQA magnitudes (%REC, %DET, ENT and ML) on the (standardised) residuals to determine the existence of remaining non-linear structures. This procedure also follows Serletis and Gogas (2000), where the MLE is computed on the linearly and non-linearly filtered series.

Next section explains the Kaplan's test in detail. We will use this procedure in order to detect non-linear structures that could not be detected by the RQA variables. As it was mentioned, it showed a good behaviour in the work of Barnett et al. (1997). Moreover, there is a connection between this technique and the RQA methodology (see below).

[4] Consequently, a lower DIV value.

2.3. The Kaplan's test.

This test, proposed by Daniel Kaplan (1993), is based upon continuity in phase space; it uses the following fact: deterministic solution paths have the property that nearby points are also nearby under their image in phase space. Therefore, if x_i is close to x_j, then x_{i+1} and x_{j+1} are also close to each other. As it is clear, this is the logic beneath the RQA methodology, where the definition of "determinism" lays on the existence of nearby orbits that are also nearby in the future.

The test statistic, *K*, has a strictly positive lower bound for a stochastic process, but not for a deterministic solution path. Using this statistic, we can test the hypothesis that the data are deterministic versus the alternative that the data come from a *particular* stochastic process. The particular stochastic process is rejected if *K* is smaller for the data than for the stochastic process, by a statistically significant amount.

As stated in Barnett et al. (1997) and Barnett et al. (1995), by computing the test statistic from a large number of linear processes that might have generated the data, we can use this approach to test for noisy non-linear dynamics against the alternative of linear stochasticity. To construct the set of linear processes, we can use the method of surrogate data (Theiler et al. (1992)). This method basically consists on taking a Fourier transform of the raw data, keeping the original magnitudes, and randomising the phases: the resulting inverse Fourier transform contains the same linear correlations than the original data. If the value of *K* from 20 surrogates is never small enough relative to the value computed from the data, we can conclude a noisy continuous dynamical solution. If previously we filter out any linear dynamical structure in data, this is equivalent to test the iid hypothesis for the ARMA residuals of the fitted model, as in that case the hypothesised linear model for the residuals is simply a white noise process. In other words, we also could test for linearity of the original data with RQA via generating surrogates, which should be equivalent to test for iid behaviour of the ARMA residuals. In both cases, rejection of the null would be evidence of non-linear dynamics.

In a formal statement, given an embedding dimension *m*, one calculates the distances $d_{ij} = \left\| x_i^m - x_j^m \right\|$ and $e_{ij} = \left\| x_{i+1}^m - x_{j+1}^m \right\|$ for all pairs (i, j). Next, we define $E(r)$ as the average of the values e_{ij} over the pairs that satisfy $d_{ij} < r$. If the image of the points in the phase space are given by a continuous function f, such that $\mathbf{x}_{t+1} = f(\mathbf{x}_t)$, and the system is perfectly deterministic, one should expect that $E(r) \to 0$ as $r \to 0$. Kaplan's test statistic is defined as

$$K = \lim_{r \to 0} E(r)$$

However, the disposability of data does not allow to take the limit. Our solution resembles Barnett et al. (1995): we have implicitly selected a finite r by averaging e_{ij} over the 200 pairs of (i, j) that produce the smallest values of d_{ij}. Then, the value of the test statistic K can be compared to that generated from surrogate data, in two alternative ways. The simplest method is to compute the minimum value K from 20 surrogates, and impute that to the population of surrogates consistent with the procedure. As noted by Barnett et al. (1997), a more appealing approach is to compute the mean and standard deviation from the set of surrogates and then subtract a multiple (2 or 3) of the standard error to the mean, to get an estimate of the population minimum. In this work, we will show results with both procedures.

3. Real exchange rates analysis.

3.1. Purchasing Power Parity.

Purchasing Power Parity (PPP) is an equilibrium condition in the market for tradable goods, and it is a basic ingredient in a large number of models grounded on economic fundamentals. Briefly stated, if P is the domestic price, $P*$ is the foreign currency price, and S is the exchange rate measured as the domestic currency per unit of foreign currency, then PPP predicts that the following relationship holds,

$$P = SP*$$

in its strong form. In the weak form, the relationship is stated in terms of growth rates,

$$\dot{P} = \dot{S} + \dot{P}*$$

The real exchange rate is a measure of price competitiveness or the price of domestic relative to foreign goods, i.e.,

$$E = P * S / P$$

Then, if the PPP holds in any version, E must follow a stationary process. Conversely, if E contains a unit root it is sufficient for a violation of PPP.

In a recent work, Serletis and Gogas (2000) analyse the quarterly (US) dollar-based real exchange rates for 17 OECD countries, on the basis that the Purchasing Power Parity theory cannot hold if real exchange rates contain unit roots, i.e., if it is true that:

$$e_t = e_{t-1} + u_t$$

where e_t is the log-real exchange rate ($\log E_t$) at time t, and u_t is a purely stochastic process. However, if u_t behaves like a random process, an alternative explanation stems from chaos theory. As De Grauwe et al. (1993) show, a pure deterministic fundamentals model where, in the long run, the exchange rate is governed by PPP, can mimic a stochastic process with a unit root. Hence, deterministic non-linearities could lead to an erroneous rejection of PPP.

If a process is chaotic, it presents the so called "sensitive dependence upon initial conditions", i.e., two nearby trajectories diverge exponentially as they evolve in time. The rate of divergence is measured through the Maximum Lyapunov Exponent, and the clue of chaos is contained in the following assertion: if the system is chaotic, the MLE must be positive. Then, to test the hypothesis "the variable x evolves chaotically", some authors estimate the MLE for x. If the estimate is positive, the corresponding variable is labelled as "chaotic", otherwise, it is labelled as "random". Serletis and Gogas (2000) test for positive MLE using the Nychka et al. (1992) algorithm. Such a strategy shares a long tradition on detecting chaotic dynamics in exchange rates by estimating Lyapunov exponents: Bajo et al. (1992), Bask (1996, 2000), Dechert and Gençay (1992), Jonsson (1997), etc. All these papers analyse the *nominal* exchange rates, but they differ, among other details, in the method used to compute the Lyapunov exponents.

3.2. MLE estimation methods.

It is a well known fact that a positive MLE is a strong signature of chaos. Thus, given a time series, it is desirable to estimate that magnitude as accurately as possible. The first attempt for that purpose was designed by Wolf et al. (1985), but their algorithm is not very reliable (see Kantz and Schreiber (1997) for more details). Since then, a set of new procedures have been proposed (e.g., Sano and Sawada (1985), Eckmann et al. (1986)), but just a few have shown to be useful to test for positive MLE when analysing small and noisy data sets. We will refer to them as 1) Jacobian methods (Nychka et al. (1992), Gençay and Dechert (1992)), and 2) non-Jacobian methods (Rosenstein et al. (1993), Kantz (1994)).

The Jacobian-based methods firstly estimate the underlying dynamics by fitting a general non-linear function to data, more precisely, a multivariate feedforward neural network. Serletis and Gogas (2000) focuses on Nychka et al. (1992) procedure, whereas we will focus on Gençay and Dechert's approach (which is intimately related to the former).

Let the data $\{x_t\}_1^N$ be real-valued. Then, for a given embedding dimension m, we can build the sequence of embedding vectors $x_t^m = (x_{t-1}, x_{t-2}, \ldots, x_{t-m})$. As it was mentioned, according to Takens (1981) it is possible to reconstruct the dynamics of the original system, as x_t^m is the reconstructed state of the system at time t. Thus, the reconstructed trajectory is defined by

$$X = \{x_1^m, \ldots, x_M^m\}$$

where $M = N - m + 1$ is the number of embedded vectors in an m-dimensional space.

Initially, we use a single layer feedforward network,

$$v_{N,m}\left(x^m; \boldsymbol{b}, w, b\right) = \sum_{j=1}^{L} \boldsymbol{b}_j k\left(\sum_{i=1}^{m} w_{ij} x_i^m + b_j\right)$$

where $x^m \in \Re^m$ is the input, the parameters to be estimated are b, w, b, and k is a known hidden unit activation function. L is the number of hidden units, $b \in \Re^L$ represents hidden to output unit weights, and $w \in \Re^{L \times m}$ and b $\in \Re^L$ represent input to hidden unit weights. In our work we will use the logistic (sigmoid) function

$$k(y) = \frac{\boldsymbol{b}}{1 + \mathbf{exp}(-wy - b)}$$

as the hidden layer activation function. The estimation will be performed by Non-linear Least Squares for different values of m = 2,...,10, and different number of hidden units L = 1,...,10; we will base our choice of the final model on the Schwartz Information Criterion (SIC). Once estimated, we can use the partial derivatives of the fitted model (Gallant and White (1992)) to estimate the dominant Lyapunov exponent:

$$\hat{l}^{*} = \frac{1}{2N}\mathbf{log}\left|\hat{l}_1(N)\right|$$

where $\hat{l}_1(N)$ is the largest eigenvalue of the matrix $\hat{T}_N^T\hat{T}_N$, and where[5] $\hat{T}_N = \hat{J}_N\hat{J}_{N-1}\cdots\hat{J}_1$. Once it is estimated, one could use a formal test for positivity of the Lyapunov exponent, by using the procedure of Shintani and Linton (2000), if the estimate has been based on neural networks, or by the method of Whang and Linton (1999), if the estimate was performed by kernel-type non-parametric regression. However, we leave this subject for a further research, and we will focus just on the point estimate of the MLE.

3.3. Data and results.

The data used in this paper are the quarterly US dollar-based exchange rate series for 17 OECD countries spanning the period 1957:1-1998:4, thus including three years more than the work of Serletis and Gogas (2000) which covered until 1995:4. Real exchange rates have been constructed using the definition above, where *S* is the nominal exchange rate, and *P* and *P** are the domestic consumer's price index and the foreign consumer's price index, respectively. The source of all the data is the International Financial Statistics of the International Monetary Fund. The countries analysed are Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, The Netherlands, Norway, Spain, Switzerland, and the United Kingdom (though USA is also involved since the reference currency is the US dollar).

Previously, we transform the data to work with a stationary-in-mean series. Let us remind that nonstationarity is not a limitation for RQA, but we are trying to get results comparable with those in Serletis and Gogas (2000), which analyse linear transformations of the exchange rate series. As the unit root tests suggest, we take first differences of the logs of the series for all the currencies; thus, we will not work directly with the log-exchange rates in levels, but with the returns, *r*. Next, we try to filter out all the stochastic linear dependencies by fitting an AR model to each returns series

$$r_t = b_0 + \sum_{j=1}^{p} r_{t-j} + u_t$$

using for each series the number of lags, *p*, for which the Ljung-Box Q(36) statistic is not significant at the 5% level. Table I shows the selected order for each currency jointly with the

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[5] $\hat{J}$ denotes the estimate of the Jacobian matrix.

corresponding value of the Ljung-Box Q(36). Thus, we identify an order p = 1 for Canada, Finland, France, Italy, Japan, Spain and Switzerland, p = 2 for Denmark, p = 3 for Austria, Belgium, Germany, Ireland, Netherlands and Norway, p = 5 for Greece, and p = 6 for UK. These orders slightly differ from those in Serletis and Gogas (2000) in the case of Japan, Netherlands, and the United Kingdom.

INSERT TABLE I

After removing the linear dependencies in data, we check the existence of structure in the variance by means of the ARCH test: if the test rejects the null, an AR-ARCH-type model is fitted to the real exchange rate returns. As it can be seen in Table I, the ARCH test detects a time-varying variance only in the case of Italy (while Serletis and Gogas also find ARCH structure for UK). Then, we remove the non-linear stochastic dependence in variance for the Italian currency, by fitting an AR(1)-exponential GARH(1,1) (EGARCH(1,1)) model,

$$
\begin{aligned}
r_t &= b_0 + b_1 r_{t-1} + e_t \\
e_t &\sim N(0, s_t^2) \\
\log s_t^2 &= w_0 + b \log\left(s_{t-1}^2\right) + a\left|\frac{e_{t-1}}{s_{t-1}}\right| + g\,\frac{e_{t-1}}{s_{t-1}}
\end{aligned}
$$

basing our choice on comparing log likelihood values of alternative ARCH-type models.

As a way of determining the existence of neglected ARCH structures, we apply the ARCH test on the standardised residuals of the AR-EGARCH model, $\hat{e}_t / \hat{s}_t$ (fourth column in Table I): now, the null is accepted. The following step involves testing for general non-linearities of unknown form. The statistical device we will use is RQA, but, as it was remarked in the previous section, initially we must choose carefully the parameters t, m, e, and L for each real exchange rate returns series.

If the data are continuous, the delay parameter t needs to be selected so as to avoid false linearisations of the system. To that end, some authors propose to set the lag time equal to the first zero-autocorrelation lag, while others state that it is best to set it equal to the minimum time delayed mutual information. As it is stressed by Webber (1999), for discrete systems the delay is usually selected as one, but for continuous systems it usually exceeds one. Consequently, t = 1 is a reasonable choice, since we assume that quarterly real exchange rates are the result of discrete systems. Any way, there is not an “optimal” procedure to select the delay parameter; as Kantz and Schreiber (1997) assert, a good estimate of the delay is even more difficult to obtain than a good estimate of the embedding parameter, because we often analyse short and noisy data. Just with infinite precision, the embedding theorem would apply with no play to role for the lag time.

Concerning the embedding dimension, we need to select a value that is sufficiently high so as to capture major participant variables: the greater the complexity of the system, the larger this value should be. But we must be aware that an excessively large value would inflate the noise to the level of masking the true signal of the process. In this paper we assume that the process driving the real exchange rates is rather complex; thus, we set $m = 20$.

The value of the radius parameter, *e*, cannot be set too low as no recurrent point is found (%REC = 0), but it cannot be set so high that every point is a recurrent point (%REC = 100). As a way of determining an appropriate value, we can use the following procedure. Perform the calculations at the smallest *e*, say 0.1, and then increase the parameter in steps of, e.g., 0.1, repeating the calculations for every *e*. Then, plot the variable %REC against *e* and select that radius value where the %REC starts to rise off of the noise floor. In our case, each exchange rate corresponds a particular *e*, as it is shown in the first column of Table II.

Finally, the parameter L can be set in a slightly arbitrary manner. In our case, we have selected $L = 15$, which we regard as a rather conservative choice, since it is highly improbable that at least 15 consecutive diagonal points are recurrent, just by chance.

We have performed the calculation of the variables %REC, %DET, ENT and ML for each currency at a suitable value of the parameters. Next, we have produced 400 shuffled replications of each series, and we have repeated the calculation of the RQA variables at the same parameters values, leading to an empirical distribution of the variables under the null of independence and identical distribution.

At this point, however, we should emphasise the fact that the distribution of the AR-EGARCH residuals presents an abrupt change at some point near the Bretton-Woods crises, at the end of 1972. See figure 3 for an illustration in the case of Germany: the shaded area of the figure represents the period post Bretton-Woods.

INSERT FIGURE 3

If we produce a shuffled series for the whole period, we will get a clear rejection of the null due to the changing distribution, not for a true deviation of the independence hypothesis: we would conclude erroneously that the real exchange rate series is driven by non-linear dynamics. To avoid this problem, we split the original series before and after B-W, and we “locally” shuffle the segments. Then, we “join the pieces” leading to the finally analysed randomly shuffled series.

To test the null hypothesis of independence of the residuals, we compare the values of %REC, %DET, ENT and ML of the original series with the values of the 95th percentile of the empirical distribution of these variables under the null. Note that, in each case, the test is one-sided: if the series violates the assumption of independence, we expect larger % of recurrences, larger % of determinism, a larger entropy, and a larger maximum line (i.e., a lower divergence). In addition, we compute the number of “sigmas”,

$$s = \frac{x - \bar{x}_{surr}}{\hat{s}_{surr_x}}$$

for each RQA variable, where $\bar{x}_{surr}$ is the mean value of the corresponding RQA variable for the surrogates, and $\hat{s}_{surr}$ is the standard deviation of the RQA variable for the surrogates. If the variable would distribute asymptotically $N(\cdot)$, as the tests are one-sided a value above 1.65 would imply a rejection of the null. Nevertheless, the distribution of the RQA variables is unknown, though s still could be used as a rough measure of significance. Table II shows the results of the RQA analysis for the real exchange rates.

INSERT TABLE II

As it is seen, all the four original RQA variables are greater than the corresponding 95^{th} percentile in the case of Austria, Belgium, Canada, France, Germany, Ireland, Netherlands, Norway, and Switzerland: these real exchange rates clearly look like non-linear processes. The four original variables are lower than the resulting percentiles of the empirical distributions in the case of Finland, Spain and United Kingdom: these real exchange rates resemble linear stochastic processes. Note also, the surprisingly good agreement of the 95% critical value derived from a Normal distribution with the conclusion derived from comparing RQA original values with the corresponding 95^{th} percentiles.

Furthermore, if we consider that the variables which constitute the definition of “determinism” in the RQA terminology are DET, ENT and ML, we can se that there would remain some type of (non-linear) “determinism” in Greece (marginally), Italy, and Japan (marginally). Denmark would fall into the group of doubtful real exchange rates. However, we prefer to be cautious in the application of this new methodology, therefore we do not reject linearity if there is not a rejection of the null with *each* RQA variable, at the cost of a loss of power.

Next, we apply the Kaplan's test to the same time series. Now, however, we do not assess the statistical significance of the result, but we base the test on the comparison of the original test statistic with that computed on 20 "locally" shuffled replications. As it was remarked in section 2.3, we compute the statistics by averaging e_{ij} over a finite set of 200 pairs of (i, j) that produce the smallest values of d_{ij}. Next, we compare the statistic on the original series with the minimum on the 20 surrogates, and with the mean of the statistic less two times the standard error on the surrogates, alternately. We have computed the statistics for a range of embedding dimensions, as suggested by Kaplan (1993). Following Barnett et al. (1995), if the value of K for the original data is less than the minimum value for the surrogates at an embedding, non-linearity is deemed to be detected. The results are shown in Table III.

INSERT TABLE III

The conclusions from this table can be summarised as follows: linearity is rejected for Austria, Belgium (marginally), Canada, France, Germany, Netherlands, Norway, Switzerland, and the United Kingdom. In all the other cases, the linearity assumption is not rejected. These results are not sensitive to the embedding dimension, and they do not depend on the method to compute the minimum value of K on the surrogates.

Therefore, the set of exchange rates for which non-linearity is an acceptable hypothesis, is almost identical when we use the new RQA methodology. It reflects the connection between both procedures. The small differences (RQA rejects linearity for Ireland, but not for United Kingdom, whereas the converse is true when applying Kaplan's test) can be reflecting the ability of the frameworks for detecting specific kinds of subtle hidden patterns.

At the second stage of the methodology we follow, we estimate the MLE for each real exchange rate (Table IV), using the Gençay and Dechert (1992) procedure, as described in section 3.2. If we consider the preceding results, we should expect a negative estimate for Denmark, Finland, and Spain, since RQA analysis and Kaplan's test led to the acceptance of linearity for these exchange rates. For the other currencies, the sign of the estimate depends upon the kind of mechanism driving the dynamics of the real exchange rate (chaotic or not).

INSERT TABLE IV

It stems from Table IV, that the real exchange rates for Austria, Belgium, Germany, Greece, Japan, Netherlands, Norway, and Switzerland are chaotic, since the corresponding point estimates of the MLE are positive. The exchange rates for Denmark, Finland, and Spain present negative MLE's, as expected.

These results contrast with those in Serletis and Gogas (2000). In that paper, chaos was found for the exchange rates for Greece, Italy, Japan, Netherlands, Norway, Switzerland and UK. But we should keep in mind the following differences between our work and the work by Serletis and Gogas: 1) our sample spans a longer period, 2) we have assessed the existence of non-linear mechanisms, previously to compute the MLE, and 3) we have used a different algorithm to estimate the MLE.

4. Concluding comments.

When we try to determine if the Purchasing Power Parity is a theory sustained by the data, we must be aware of the possibility that real exchange rates can look like integrated processes when actually they are non-linear chaotic processes. Then, we could erroneously conclude that PPP does not hold. In this paper, we have analysed this phenomenon by means of a two-stage investigation, as Barnett et al. (1997) suggest.

At the first stage, we have checked the existence of non-linear mechanisms driving the real exchange rates, since non-linearity is a necessary, but not sufficient, condition for chaos. To that end, we have used a new powerful tool, the Recurrence Quantification Analysis (RQA) of Webber and Zbilut (1994), originally based on the computation of recurrence plots. In addition, we have computed the Kaplan`s (1993) test because of its good behaviour in the single-blind competition in Barnett et al. (1997), and because of its connection with RQA methodology: both techniques are based on the fact that, for a deterministic system, nearby points in an embedding have also nearby images. As a whole, we can claim that real exchange rates use to be driven by non-linear mechanisms of, in principle, unknown form.

Finally, at the second stage, we have estimated the MLE for each real exchange rate by means of the Jacobian-based algorithm of Gençay and Dechert (1992), though we have not assessed the statistical significance of the results. From the sign of the estimates, which slightly differ from Serletis and Gogas (2000), we can assert that chaos seems a reasonable hypothesis for a set of exchange rates, but we must assure that this result is not spurious; to that end, it is desirable to apply the methods available in order to compute a *t*-statistic to test the hypothesis of strict positivity. We leave this for a further research.

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TABLE I
AR-GARCH residuals diagnostics

Country	AR lag	Residuals diagnostics (p-values)		
		Q(36)-statistic (AR residuals)	ARCH-test (AR residuals)	ARCH-test (standardised AR-ARCH residuals)
Austria	3	0.207	0.154	-
Belgium	3	0.374	0.212	-
Canada	1	0.075	0.888	-
Denmark	2	0.100	0.236	-
Finland	1	0.285	0.519	-
France	1	0.718	0.398	-
Germany	3	0.279	0.184	-
Greece	5	0.143	0.412	-
Ireland	3	0.080	0.243	-
Italy	1	0.151	0.018	0.372
Japan	1	0.112	0.608	-
Netherlands	3	0.216	0.073	-
Norway	3	0.371	0.957	-
Spain	1	0.370	0.573	-
Switzerland	1	0.065	0.111	-
United Kingdom	6	0.069	0.062	-

TABLE II

RQA variables

Country	*e*	REC		DET		ENT		ML	
		Original series	Shuffled series	Original series	Shuffled series	Original series	Shuffled series	Original series	Shuffled series
Austria	30	6.17	4.07	65.37	64.36	2.55	1.97	37.00	34.00
		(3.87)		(1.88)		(2.79)		(1.80)	
Belgium	20	5.69	3.49	82.35	72.40	2.94	2.15	34.00	26.00
		(4.09)		(2.66)		(3.69)		(2.63)	
Canada	50	3.29	2.02	68.08	40.09	1.18	0.69	24.00	22.00
		(4.22)		(3.97)		(4.00)		(2.02)	
Denmark	30	1.38	4.49	34.25	71.43	0.69	2.04	19.00	31.00
		(-0.55)		(0.38)		(0.37)		(0.49)	
Finland	30	2.07	3.14	47.08	65.00	1.61	1.95	20.00	26.00
		(0.82)		(1.48)		(2.02)		(1.09)	
France	15	2.04	0.51	48.40	0.00	1.05	0.00	21.00	0.00
		(7.73)		(10.01)		(9.09)		(7.39)	
Germany	20	1.67	0.85	48.28	14.04	0.63	0.00	19.00	16.00
		(3.67)		(5.32)		(3.31)		(4.01)	
Greece	25	5.32	4.69	64.63	69.39	2.24	2.09	40.00	36.00
		(2.57)		(1.68)		(1.90)		(2.00)	
Ireland	30	1.81	0.97	44.97	11.77	1.39	0.00	19.00	15.00
		(4.34)		(6.15)		(11.62)		(4.34)	
Italy	60	1.32	1.76	51.41	27.10	0.69	0.00	26.00	20.00
		(1.10)		(5.12)		(4.87)		(3.17)	
Japan	30	6.5	6.61	85.82	84.64	2.59	2.56	37.00	38.00
		(1.90)		(1.55)		(1.47)		(1.24)	
Netherlands	30	2.27	1.61	41.35	36.74	1.38	0.00	23.00	20.00
		(3.50)		(2.56)		(5.39)		(2.68)	
Norway	30	5.25	3.76	72.81	61.54	2.22	1.79	37.00	29.00
		(3.78)		(2.61)		(3.13)		(2.31)	
Spain	60	4.47	4.67	18.96	69.03	0.64	2.04	22.00	30.00
		(1.51)		(-0.13)		(0.22)		(0.69)	
Switzerland	10	1.00	0.10	59.26	0.00	0.69	0.00	23.00	0.00
		(21.15)		(22.50)		(-)		(30.66)	
United Kingdom	25	1.36	0.49	22.79	0.00	0.00	0.00	16.00	0.00
		(5.93)		(5.59)		(-0.05)		(7.14)	

Note: The number of "sigmas" is in parenthesis.

TABLE III
Kaplan's Test Statistics

Country	Mean of K on surrogates	Minimum K on surrogates	Std. Dev. of K on surrogates	Embedding dimension	K on original data	Conclusion
Austria	0.032	0.026	0.003	5	0.023	Reject
	0.051	0.041	0.004	10	0.037	linearity
	0.069	0.061	0.005	15	0.052	
	0.087	0.068	0.010	20	0.066	
Belgium	0.023	0.016	0.004	5	0.015	(Marginally)
	0.034	0.022	0.006	10	0.024	reject
	0.052	0.029	0.009	15	0.032	linearity
	0.072	0.036	0.015	20	0.039	
Canada	0.023	0.018	0.002	5	0.016	Reject
	0.036	0.029	0.003	10	0.025	linearity
	0.050	0.041	0.005	15	0.034	
	0.063	0.051	0.007	20	0.044	
Denmark	0.029	0.021	0.004	5	0.024	Accept
	0.045	0.037	0.005	10	0.039	linearity
	0.063	0.048	0.008	15	0.064	
	0.080	0.060	0.010	20	0.090	
Finland	0.037	0.016	0.012	5	0.031	Accept
	0.057	0.026	0.019	10	0.049	linearity
	0.090	0.043	0.025	15	0.057	
	0.123	0.053	0.041	20	0.076	
France	0.036	0.022	0.009	5	0.023	Reject
	0.055	0.025	0.015	10	0.026	linearity
	0.094	0.042	0.024	15	0.033	
	0.138	0.067	0.029	20	0.040	
Germany	0.026	0.020	0.002	5	0.019	Reject
	0.042	0.031	0.004	10	0.030	linearity
	0.059	0.047	0.006	15	0.041	
	0.074	0.062	0.006	20	0.054	
Greece	0.022	0.020	0.002	5	0.019	Accept
	0.034	0.030	0.002	10	0.031	linearity
	0.045	0.041	0.002	15	0.042	
	0.055	0.049	0.003	20	0.051	

(Continued)

TABLE III
Kaplan's Test Statistics
(Continued)

Country	Mean of K on surrogates	Minimum K on surrogates	Std. Dev. of K on surrogates	Embedding dimension	K on original data	Conclusion
Ireland	0.040	0.029	0.007	5	0.036	Accept
	0.062	0.048	0.009	10	0.054	linearity
	0.083	0.062	0.011	15	0.069	
	0.106	0.077	0.015	20	0.090	
Italy	1.403	1.219	0.092	5	1.372	Accept
	2.269	1.983	0.149	10	2.175	linearity
	3.104	2.741	0.212	15	2.947	
	3.907	3.370	0.299	20	3.546	
Japan	0.028	0.023	0.002	5	0.025	Accept
	0.043	0.038	0.004	10	0.040	linearity
	0.059	0.050	0.006	15	0.053	
	0.074	0.059	0.008	20	0.065	
Netherlands	0.034	0.029	0.003	5	0.028	Reject
	0.055	0.044	0.004	10	0.043	linearity
	0.075	0.066	0.005	15	0.058	
	0.092	0.082	0.005	20	0.073	
Norway	0.024	0.022	0.002	5	0.018	Reject
	0.039	0.033	0.003	10	0.029	linearity
	0.054	0.045	0.004	15	0.040	
	0.067	0.056	0.005	20	0.052	
Spain	0.051	0.038	0.006	5	0.044	Accept
	0.078	0.061	0.010	10	0.062	linearity
	0.112	0.079	0.018	15	0.093	
	0.153	0.098	0.028	20	0.134	
Switzerland	0.021	0.016	0.004	5	0.014	Reject
	0.034	0.027	0.004	10	0.022	linearity
	0.046	0.036	0.005	15	0.028	
	0.057	0.044	0.006	20	0.034	
United Kingdom	0.04	0.031	0.005	5	0.025	Reject
	0.066	0.056	0.007	10	0.040	linearity
	0.096	0.076	0.013	15	0.052	
	0.127	0.095	0.016	20	0.091	

TABLE IV

Maximum Lyapunov Exponent

Country	**MLE estimate**	
Austria	0.28	*
Belgium	0.38	*
Canada	-0.17	
Denmark	-0.21	
Finland	-0.14	
France	-2.00	
Germany	0.37	*
Greece	0.31	*
Ireland	-0.21	
Italy	-0.28	
Japan	0.28	*
Netherlands	0.42	*
Norway	0.39	*
Spain	-0.35	
Switzerland	0.48	*
United Kingdom	-4.23	

Note: (*) Indicates chaotic behavior.

FIGURE 1.
Recurrence plot from Rossler series

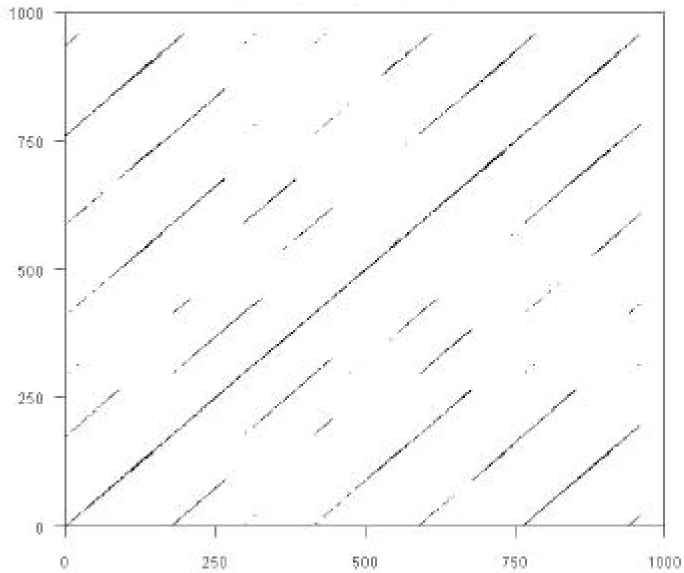

FIGURE 2.
Recurrence plot from Shuffled Rossler Series

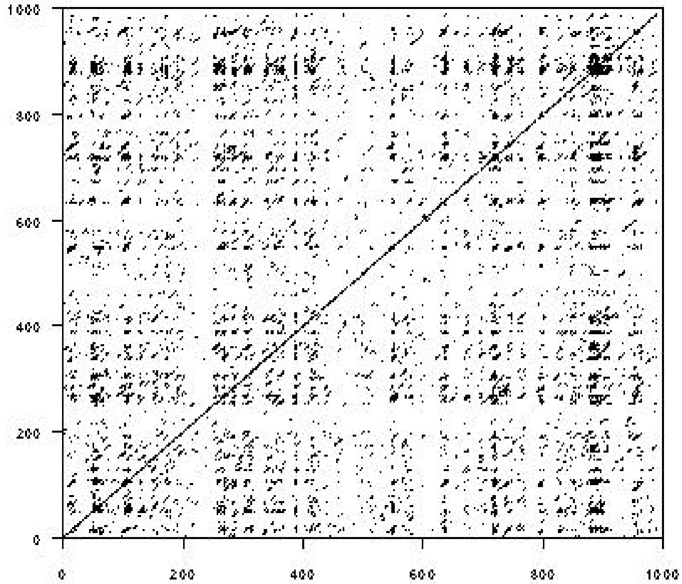

FIGURE 3
Real Exchange Rate Returns (AR Residuals)
Germany