Applied Economics, 2008, **40**, 1609–1613

Routledge
Taylor & Francis Group

US deficit sustainability revisited: a multiple structural change approach

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In this article we re-examine the long-run sustainability of US budget deficits, using Bai and Perron's multiple structural change approach. While the deficit would have been weakly sustainable over the full sample (1947:1–2005:3), strong sustainability would appear only between January 1982 and February 1996.

I. Introduction

US government finances have experienced a remarkable turnaround in recent years. Large budget deficits in the 1980s and early 1990s led to a substantial amount of empirical work aimed to examine their long-run sustainability. However, later on, the record surpluses in the late 1990s and early 2000s turned into record deficits after 2002, with budget projections showing large federal deficits over the next decade. As a result, the US general government deficit is now among the highest in the OECD, and its sustainability has become again a highly relevant issue.

When analyzing the sustainability of budget deficits, the traditional approach consisted of testing whether the government's intertemporal budget constraint (IBC) holds, that is, whether the current market value of debt equals the discounted sum of expected future surpluses. However, empirical tests on sustainability are largely inconclusive due to differences in the econometric methodology, the particular specification of the transversality condition and the sample period used.

Several procedures to test for the IBC have been proposed in the literature, focusing either on the univariate properties of the government's deficit and debt (e.g. Hamilton and Flavin, 1986; Wilcox, 1989), or on the presence of a long-run cointegration relationship between government revenues and expenditures (e.g. Trehan and Walsh, 1988, 1991; Haug, 1991). Furthermore, the eventual occurrence of a structural break in the cointegrating relationship has been examined in Quintos (1995) and Martin (2000). Overall, the results of these and other studies suggest that the US deficit would have undergone a shift in recent times, with the deficit being either unsustainable or only weakly sustainable in the post-break period.

In this article we re-examine the sustainability of US budget deficits, using a new approach developed by Bai and Perron (1998, 2003a). This procedure allows to test endogenously for the presence of multiple structural changes in an estimated relationship, and has a number of advantages over previous approaches. In particular, the underlying assumptions are less restrictive, confidence intervals for the break dates can be calculated, the data and errors are allowed to follow different distributions across segments and the sequential method used in the application can allow for the presence of serial correlation in the errors and heterogeneous variances across segments (see Bai and Perron (2006) for details). As a further contribution, as compared with previous studies where the sample ends at the early 1990s, our period of analysis extends to 2005,

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Applied Economics ISSN 0003–6846 print/ISSN 1466–4283 online © 2008 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/00036840600843996

including the most recent developments in the evolution of the US budget deficit.

The rest of this article is organized as follows. A brief description of the underlying theoretical framework is provided in Section II, the methodology and empirical results are presented in Section III, and the main conclusions are summarized in Section IV.

II. Theoretical framework

Assuming that budget deficits are financed using bonds of one-period maturity, in any single period, a government faces the following budget constraint:

$$\Delta B_t = G_t - R_t \tag{1}$$

where B_t, G_t, and R_t denote, respectively, the real market value of government debt, real government expenditure inclusive of interest payments and real tax revenues. The real interest rate is assumed to be stationary around a mean r so that, defining EXP_t as $G_t - rB_{t-1}$, the constraint (1) can be rewritten as:

$$B_t = EXP_t - R_t + (1+r)B_{t-1} \tag{2}$$

Since (2) holds every period, solving for B_t and iterating forward over an infinite horizon yields the IBC:

$$B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1}(R_{t+j+1} - EXP_{t+j+1}) + \lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1} B_{t+j+1} \tag{3}$$

If we denote E_t as the expectations operator, conditional on information at time t, fiscal sustainability involves:

$$\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1} E_t B_{t+j+1} = 0 \tag{4}$$

i.e. the government must run future budget surpluses equal, in present-value terms, to the current value of its outstanding debt; in other words, the budget deficit would be sustainable if and only if the stock of debt is expected to grow no faster on average than r (taken as a proxy of the growth rate of the economy).

The cointegration framework for testing the IBC would follow once first differences are taken in (3):

$$\Delta B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1}(\Delta R_{t+j+1} - \Delta EXP_{t+j+1}) + \lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1} \Delta B_{t+j+1} \tag{5}$$

so that sustainability would require:

$$\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1} E_t \Delta B_{t+j+1} = 0 \tag{6}$$

Under a no-Ponzi scheme rule, the right-hand side of Equation 5 will be stationary as long as government revenues and expenditures, and the stock of public debt, are all stationary in first differences. In order to test for condition (6), the usual procedure consists of testing for the stationarity of $\Delta B_t = G_t - R_t$, provided that both G_t and R_t are $I(1)$, with a cointegration relationship $(1, -1)$, in a regression model of the form:

$$R_t = \alpha + \beta G_t + \varepsilon_t \tag{7}$$

and then testing the linear restriction $\beta = 1$. In particular, Quintos (1995) shows that:

(i) The fiscal deficit would be strongly sustainable if and only if R_t and G_t are cointegrated and $\beta = 1$.
(ii) The fiscal deficit would be only weakly sustainable if R_t and G_t are cointegrated and $0 < \beta < 1$.
(iii) The fiscal deficit would be unsustainable if $\beta \leq 0$.

III. Methodology and Empirical Results

In this section we provide a test of the sustainability of the US budget deficit, over the period January 1947 to March 2005.[1] The data on federal government revenues and expenditures, inclusive of interest paid on debt, are taken from the National Income Product Accounts (NIPA, Table 3.1), and real values are calculated using the GDP deflator (NIPA, Table 1.1.4).

As a first step of the analysis, we test for the order of integration of the series using the tests of

[1] An alternative, though indirect, way of analyzing the sustainability of fiscal policy (used, e.g. in the context of testing for the fiscal theory of the price level) involves the estimation of a long-run (cointegration) relationship between budget surplus and (lagged) government debt, so that a positive and significant estimate of the regression coefficient would be a sufficient condition for solvency. Then, when a government is solvent (i.e. satisfies the IBC), its fiscal policy is sustainable; this is the approach followed in Bohn (1998), Tanner and Ramos (2003) or Bajo-Rubio, *et al.* (2007).

Ng and Perron (2001). The results are shown in Table 1, and the null hypothesis of nonstationarity cannot be rejected, independently of the test, for the two series in levels; at the same time that the presence of two unit roots is clearly rejected at the 1% significance level. Accordingly, both series would be concluded to be $I(1)$.

Next, we perform a cointegration analysis of Equation 7 over the whole sample, with no breaks included. The estimation is made using the method of Dynamic Ordinary Least Squares (DOLS) of Stock and Watson (1993). So, we first estimate a long-run dynamic equation including leads and lags of the explanatory variables in Equation 7:

$$R_t = \alpha + \beta G_t + \sum_{j=-q}^{q} \gamma_j \Delta G_{t-j} + \upsilon_t \qquad (8)$$

where υ_t is an error term, and then perform Shin's (1994) test from the calculation of $C\mu$, a LM statistic from the DOLS residuals, which tests for deterministic cointegration (i.e. when no trend is present in the regression).

Table 1. Ng–Perron tests of unit roots

$I(2)$ vs. $I(1)$	Case: $p=0$, $\bar{c}=-7.0$		
Variable	MZ_α^{GLS}	MZ_t^{GLS}	$\mathrm{ADF}^{\mathrm{GLS}}$
ΔR_t	−42.30*	−4.58*	−6.03*
ΔG_t	−63.41*	−5.61*	−7.40*
I(1) vs. I(0)	Case: $p=1$, $\bar{c}=-13.5$		
Variable	MZ_α^{GLS}	MZ_t^{GLS}	$\mathrm{ADF}^{\mathrm{GLS}}$
R_t	−1.54	−0.77	−0.78
G_t	0.99	0.70	0.66

Notes: *Denotes significance at the 1% level. The critical values are taken from Ng and Perron (2001, Table 1).
The autoregressive truncation lag has been selected using the modified Akaike information criterion, as proposed by Perron and Ng (1996).

The results in the first column of Table 2 show that the null of deterministic cointegration between R_t and G_t is not rejected at the 1% level of significance, and the estimated value for β is 0.93, significantly different from zero at the 1% level. But this estimate would be significantly different from one at the 1% level, according to a Wald test on the null hypothesis $\hat{\beta} < 1$ against the alternative $\hat{\beta} < 1$, distributed as a χ_1^2 and denoted by W_{DOLS} in Table 2. Accordingly, since R_t and G_t would be cointegrated and $0 < \hat{\beta} < 1$, the US fiscal deficit would have been only weakly sustainable over the full sample 1947:1–2005:3. This would confirm, over a more extended period, previous results by Quintos (1995) and Martin (2000) for the same sample ending at 1992:3.

But the main objective of this section is estimating Equation 7 through a multiple endogenous break model. Hence, we now proceed to test for multiple breaks at unknown dates in Equation 7, making use of the approach of Bai and Perron (1998, 2003a), who suggest several statistics in order to identify the break points:

- The sup $F_T\ (k)$ test, i.e. a sup F-type test of the null hypothesis of no structural break vs. the alternative of a fixed (arbitrary) number of breaks k.
- Two maximum tests of the null hypothesis of no structural break vs. the alternative of an unknown number of breaks given some upper bound, i.e. $UD_{\max}$ test, an equal weighted version, and $WD_{\max}$ test, with weights that depend on the number of regressors and the significance level of the test.
- The sup $F_T(l+1|l)$ test, i.e. a sequential test of the null hypothesis of l breaks vs. the alternative of $l+1$ breaks.

Table 2. Estimation of long-run relationships: Stock–Watson–Shin cointegration tests

Parameter estimates	Full sample 1947:1–2005:3	First regime 1947:1–1955:2	Second regime 1955:3–1981:4	Third regime 1982:1–1996:2	Fourth regime 1996:3–2005:3
α	0.36 (2.42)	0.99 (2.47)	0.31 (2.14)	−0.65 (−0.59)	−26.5 (−3.48)
β	0.93 (47.4)	0.83 (12.76)	0.94 (46.24)	1.06 (7.59)	4.35 (4.51)
R^2	0.99	0.91	0.99	0.97	0.98
$\hat{\sigma}^2$	0.045	0.056	0.035	0.026	0.013
C_μ	0.087	0.079	0.061	0.093	0.310***
W_{DOLS}	10.15*	6.62*	6.22**	0.24	12.08*

Notes: *, ** and *** denote significance at the 1, 5 and 10% levels, respectively. The critical values for the Shin test are taken from Shin (1994 Table 1), for $m=1$.
t-Statistics in parentheses.The number of leads and lags selected was $q = 3 \simeq INT(T^{1/3})$, as proposed in Stock and Watson (1993). The long-run variance of the cointegrating regression residuals was estimated using the Bartlett window with $l = 5 \simeq INT(T^{1/2})$, as proposed in Newey and West (1987).

The results of applying the Bai–Perron tests to the relationship between R_t and G_t, allowing up to five breaks, are shown in Table 3. Both the $UD_{\max}$ and $WD_{\max}$ tests are highly significant, which implies that at least one break is present. Next, all the sup $F_T(k)$ tests are significant, with k running between 1 and 5, so that at least one break would be present in this relationship. In turn, the sup $F_T(l+1|l)$ test is not significant for any $l \geq 3$, so the sequential procedure selects three breaks. Hence, the results of the Bai–Perron tests would suggest a model of four regimes, with the dates of the breaks estimated at March 1955, January 1982 and March 1996; their confidence intervals are shown in Table 3.

Finally, we proceed to estimate the cointegration of Equation 8 for the four sub-samples, and the results are shown in the last four columns of Table 2. As can be seen, in the first and second regimes (1947:1–1955:2 and 1955:3–1981:4) the null of deterministic cointegration is not rejected at the 1% level, and the restriction on the estimate of β being equal to 1 is clearly rejected, which implies that the US budget deficit would have been only weakly sustainable as in the whole sample. In turn, in the third regime (1982:1–1996:2), the null of deterministic cointegration is again not rejected at the 1% level, but now the estimate of β would not be significantly different from one according to the Wald test, so that the US budget deficit would have been strongly sustainable during this period. However, in the fourth regime (1996:3–2005:3) no long-run relationship between public revenues and expenditures would appear, since the null of deterministic cointegration is now rejected at the 10% level, and the estimate of β would be above one; hence, no clear conclusions can be drawn for this period, characterized by decreasing deficits at the start, which became surpluses from 1998:1 onwards (and reached record figures in 2000), followed again by large deficits after 2001:3. The earlier results are summarized in Table 4.

IV. Conclusions

In this article we have re-examined the long-run sustainability of US budget deficits, using the multiple structural change approach of Bai and Perron (1998, 2003a). We found evidence of weak sustainability of the deficit over the full sample 1947:1–2005:3, extending previous results obtained for the period ending at the early 1990s. In addition, we have detected up to three breaks (estimated at 1955:3, 1982:1 and 1996:3) along the whole sample period, so that the US budget deficit would have been strongly sustainable only in the third regime (1982:1–1996:2), weakly sustainable in

Table 3. Bai–Perron tests of multiple structural changes in the long-run relationship

Tests statistics				
$UD_{\max}$ 138.67*	$WD_{\max}$ 136.45*			
sup $F_T(1)$ 136.45*	sup $F_T(2)$ 92.41*	sup $F_T(3)$ 92.66*	sup $F_T(4)$ 70.48*	sup $F_T(5)$ 57.63*
sup $F_T(2\|1)$ 38.27*	sup $F_T(3\|2)$ 64.22*	sup $F_T(4\|3)$ 4.76	sup $F_T(5\|4)$ 0.0	
Break dates estimates				
T_1	1955:3	[1955:1–1957:3]		
T_2	1982:1	[1980:4–1982:2]		
T_3	1996:3	[1996:2–1996:4]		

Notes: * Denotes significance at the 1% level. The critical values are taken from Bai and Perron (1998, tables I and II); and from Bai and Perron (2003b, tables 1and 2).The number of breaks (in our case, three) has been determined according to the sequential procedure of Bai and Perron (1998), at the 5% size for the sequential test sup $F_T(l+1|l)$.95% Confidence intervals in brackets.

Table 4. Sustainability of the US public deficit: summary results

	Full sample 1947:1–2005:3	First regime 1947:1–1955:2	Second regime 1955:3–1981:4	Third regime 1982:1–1996:2	Fourth regime 1996:3–2005:3
Cointegration	Yes	Yes	Yes	Yes	No
Estimate of β	0.93	0.83	0.94	1.06	4.35
Null $\hat{\beta}=1$	No	No	No	Yes	No
Sustainability	Yes (weak)	Yes (weak)	Yes (weak)	Yes (strong)	–

the first and second regimes (1947:1–1955:2 and 1955:3–1981:4, respectively), and no clear conclusions emerge for the final regime (1996:3–2005:3), where both record surpluses and large deficits would have coexisted.

Acknowledgements

The authors acknowledge financial support from the regional government of Castilla-La Mancha, through the project PBI-05-008; and from the Spanish Ministry of Education and Science, through the projects SEJ2005-08738-C02-01 (O. Bajo-Rubio and C. Díaz-Roldán) and SEJ2005-01163 (V. Esteve).

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