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Threshold cointegration and nonlinear adjustment between stock prices and dividends

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According to several empirical studies, the linear present-value model fails to explain the behaviour of stock prices in the long run. We analyse the possible presence of threshold cointegration between real stock prices and dividends for the US market during the period from 1871:1 to 2004:6. According to our results, the null hypothesis of linear cointegration between stock prices and dividends is rejected in favour of a two-regime threshold cointegration model. We find also that stock prices do not respond to equilibrium error, and dividends respond to the past divergence only if the deviation from the equilibrium error does not exceed the estimated threshold parameter. This in turn would support theoretical models assuming that the stock price–dividend relation is nonlinear.

I. Introduction

According to several empirical studies, the linear present-value (PV) model fails to explain the behaviour of stock prices in the long run. Due to adjustment costs, the conventional linear cointegration model and linear vector error correction model (VECM) might be inappropriate for testing the PV model of stock prices in the long run. To resolve this puzzle, several stock market models introduced nonlinearities in the relationship between stock prices and dividends [see, e.g. the works cited in Bohl and Siklos (2004) and Kanas (2005)]. Futhermore, some empirical studies that investigate the presence of nonlinearities in the stock price-dividend relation have recently appeared [see, Gallagher and Taylor (2001), Kanas (2003) and Kanas (2005)].

In this article we test for the presence of threshold cointegration between real stock prices and dividends for the US market during the period from 1871:1 to 2004:6. Two main research issues in this study concern the possibility of the presence of a threshold in the PV model of stock prices and the asymmetric movements between stock prices and dividends. As a extension of previous studies, we make use of the methodology developed by Hansen and Seo (2002), based on a threshold cointegration model. They propose an algorithm for estimating the complete threshold cointegration model and a sup*LM* test for the presence of a threshold. In particular, the threshold cointegration model allows for nonlinear adjustment to long run equilibrium.

The rest of the article is organized as follows. The linear PV model of stock prices is presented in Section II. The empirical methodology (threshold cointegration model) is briefly outlined in Section III. Section IV implements the tests LM for threshold

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cointegration for the US stock market data and describes the findings. Finally, Section V summarizes draws the conclusions.

II. The Linear PV Model of Stock Prices

Standard models of cointegrated variables assume linearity and symmetric adjustments. Let x_t be a p-dimensional $I(1)$ time series which is cointegrated with one $p \times 1$ vector β and $w_t(\beta) = \beta' x_t$ denotes the $I(0)$ error correction term. The cointegrated regression model can be approximated by the VECM of order $l+1$, such as:

$$\Delta x_t = A' X_{t-1}(\beta) + u_t \tag{1}$$

where

$$X_{t-1}(\beta) = \begin{pmatrix} 1 \\ w_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-2} \\ \vdots \\ \Delta x_{t-l} \end{pmatrix}$$

In order to test the PV model of stock prices in the context of the cointegration theory, the empirical studies on the expectations hypothesis have commonly used a linear model such as:

$$P_t = \alpha + \theta D_t + \varepsilon_t \tag{2}$$

where P_t is the real price of a share (or real stock price) and D_t is real dividend per share. Campbell and Shiller (1987) argued that a standard rational expectations model of asset market implies that P_t and D_t should be nonstationary and linked through a cointegration relationship $(1, -\theta)$ with $\theta = R^{-1}$, where R is a constant or a time-varying expected return (or discount rate). We use a logaritmic approximation that implicitly assumes that the logarithms of the price, p_t, and dividend indexes, d_t, are cointegrated with a cointegrating vector $(1, -1)$ and the log dividend price ratio is a stationary process.

Alternatively, we may write the log linear regression model (2) as a bivariate linear cointegrating VAR model (with one lag, $l = 1$) such as:

$$\begin{pmatrix} \Delta p_t \\ \Delta d_t \end{pmatrix} = \mu + \gamma w_{t-1} + \Gamma \begin{pmatrix} \Delta p_{t-1} \\ \Delta d_{t-1} \end{pmatrix} + \varepsilon_t \tag{3}$$

where the long run relationship is defined as $w_{t-1} = (1-\beta)x_t = p_{t-1} - \beta d_{t-1}$ with cointegrating vector $(1, -\beta)$. In this case, the error correction is the difference between the stock price and a multiple β of dividends. Setting $\beta = 1$, the log dividend price ratio would be a stationary process.

Equation 3 says that stock price changes as well as dividend changes (Δx_t) are simultaneously explained by deviations from the long-run equilibrium (error correction term, w_{t-1}), the constant terms, and lagged short-term reactions to previous stock prices changes and dividends payment changes (Δx_{t-i}).

III. Threshold Time-series Model of Stock Prices

The concept of threshold cointegration was first introduced by Balke and Fomby (1997) as a feasible way to combine nonlinearity and cointegration. Systems in which variables are cointegrated can be characterized by an error correction model (ECM), which describes how the variables respond to deviations from the equilibrium. Hence, the ECM can be characterized as the adjustment process along which the long run equilibrium is maintained. However, the traditional approach, assumes that such a tendency to move towards the long-run equilibrium is present every time period. Balke and Fomby (1997) point out the possibility that this movement towards the long run equilibrium might not occur in every time period, due to the presence of some adjustment costs on the side of economic agents. This type of discrete adjustment could be particularly useful to describe the nonlinear behaviour of the PV model of stock prices. Particularly, the model of threshold cointegration can be applied to stock market models which consider transaction costs and optimal adjustments.

More recently, Hansen and Seo (2002) contributed further to this literature by examining the case of an unknown cointegration vector. In particular, these authors propose a two-regime threshold VECM with one cointegrating vector and a threshold effect based on the error correction term, and develop a Lagrange multiplier (LM) test for the presence of a threshold effect. This will be the approach followed in this article.

As an extension of model (1), Hansen and Seo (2002) consider a nonlinear VECM of order $l+1$, such as:

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) \le \gamma \\ A_2' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) > \gamma \end{cases} \tag{4}$$

where γ is the threshold parameter.

The aim of this study is to test for asymmetric transmission between stock prices and dividends using the threshold cointegration. Unlike other

Fig. 1. US real stock prices and dividends, 1871:1–2004:6

methodologies that assume parameters are known *ex ante*, the methodology of Hansen and Seo (2002) assumes both parameters β and γ are unknown and estimated from data.

Futhermore, Hansen and Seo (2002) proposed a heteroskedastic consistent LM test statistics for the null hypothesis of linear cointegration [i.e. there is no threshold effect or model (1)], against the alternative of threshold cointegration [i.e. model (4)] when the true cointegrating vector is unknown, and is denoted by:

$$\sup LM = \sup_{\gamma L \leq \gamma \leq \gamma U} LM(\tilde{\beta}, \gamma) \quad (5)$$

where $\tilde{\beta}$ is the β estimated.

IV. Results

In this section, we re-examine the issue of the lineal PV model to explain the behaviour of stock prices. We explore the possibility that a threshold cointegration model as (4) provides a better empirical description to test the PV model of stock prices that a linear model as (1) or (3). We use the approach developed by Hansen and Seo (2002) to examine whether nonlinear cointegration exists between stock prices and dividends for the US market. The series on real stock prices and dividends are taken from Robert Shiller's website (http://www.econ.yale.edu/~shiller/data/). The stock price index is the January values of the Standard and Poor's 500 Composite Stock Price Index. The evolution of the two series, real stock prices, p_t, and real dividends, d_t, is shown in Fig. 1.[1,2]

Here we apply the test of threshold cointegration proposed by Hansen and Seo (2002), namely, supLM (estimated β) to our data. The supLM statistic has a nonstandard asymptotic distribution as shown by Hansen and Seo (2002). They proposed two bootstrapping techniques for calculating the p-values for supLM test: one is the fixed regressor bootstrap and the other is the residual bootstrap (both are calculated with 5000 simulation replications). We reject the null hypothesis of linear cointegration if the

[1] Real stock prices and dividends series were expressed in natural logaritms. The lowercase letters denote the logs of the variables.
[2] We found evidence that real stock prices and real dividends series are nonstationary variables. The results are available upon request.

Table 1. Tests for threshold cointegration

Estimates	supLM $l=4$
Cointegrating vector β	1.23
Threshold parameter γ	2.15
supLM test value	41.35
Fixed regressor critical value	38.58
$(p$- value)	(0.020)
Residual bootstrap critical value	40.26
$(p$- value)	(0.037)

bootstrapping p-values are smaller than the size chosen.

Before we implement the test of threshold cointegration, we estimate the threshold VECM. To select the lag length of the VAR, we have used the AIC and BIC criteria, both of them leading to $l=4$. The test statistics and p-values for model (4) are shown in Table 1. The evidence of bivariate threshold cointegration using both bootstrapping techniques clearly rejects the null hypothesis of linear cointegration at the 5% significance level. Consequently, the threshold cointegration model is more suitable for our data.

Table 2. Estimation of threshold VECM[a,b]

	Δp_t				Δd_t			
Dependent variable	Regime 1		Regime 2		Regime 1		Regime 2	
w_{t-1}	−0.080	(0.098)	−0.003	(0.003)	0.035*	(0.013)	0.001	(0.0007)
Intercept	0.16	(0.20)	0.009	(0.008)	−0.08*	(0.02)	−0.002	(0.002)
Δp_{t-1}	0.35*	(0.12)	0.28*	(0.03)	−0.04*	(0.02)	−0.04*	(0.009)
Δp_{t-2}	0.27	(0.41)	−0.07	(0.08)	0.44*	(0.10)	0.42*	(0.05)
Δp_{t-3}	−0.01	(0.16)	−0.05	(0.03)	−0.03	(0.02)	0.003	(0.009)
Δp_{t-4}	0.32	(0.42)	0.26*	(0.09)	−0.25	(0.14)	0.14*	(0.03)
Δd_{t-1}	−0.005	(0.13)	−0.04	(0.03)	−0.014	(0.016)	−0.003	(0.01)
Δd_{t-2}	−0.01	(0.55)	−0.02	(0.09)	0.63*	(0.16)	−0.012	(0.03)
Δd_{t-3}	−0.05	(0.19)	0.07*	(0.03)	−0.001	(0.01)	0.009	(0.009)
Δd_{t-4}	−0.64	(0.42)	−0.03	(0.09)	−0.02	(0.10)	0.05	(0.03)

Notes: *Indicates coefficient is significant at the 5% significance level.
[a]Eicker–White SE in parenthesis.
[b]Regime 1: $w_{t-1} \leq 2.15$; Regime 2: $w_{t-1} > 2.15$.

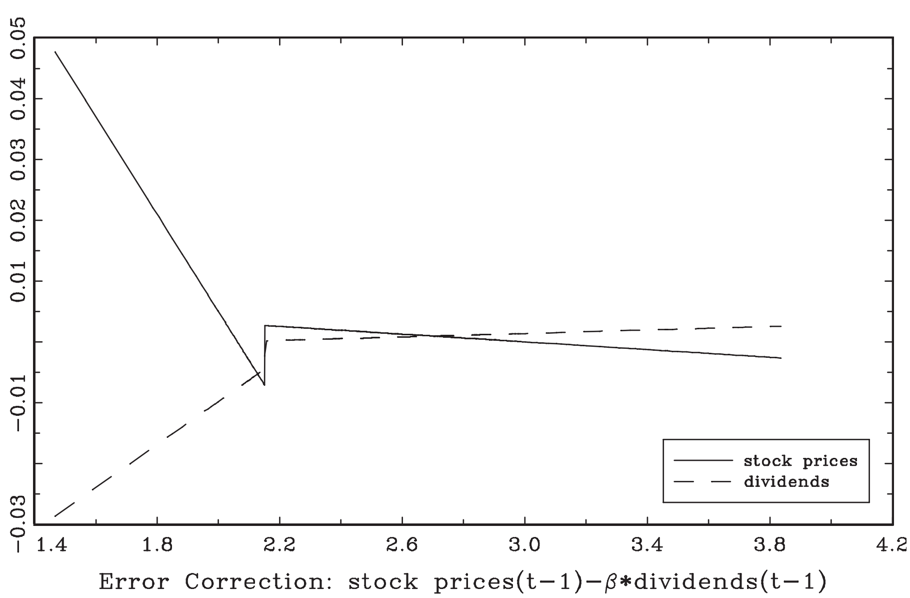

Fig. 2. Response of stock prices and dividends to error correction

The estimated cointegrated relationship is $(1, -1.23)$ and the estimated threshold is $\hat{\gamma} = 2.15$. Based on these parameters, the threshold VECM is partitioned into two regimes. The first regime would occur when the deviation from the long run equilibrium, $p_{t-1} - 1.23d_{t-1}$, is below 2.15. This would be the relatively unusual regime, including only 5% of the observations. In turn, the second or usual regime, with 95% of the observations, would occur when the divergence between stock prices and the adjustment for dividends is above 2.15. The results of the estimation of threshold VECM appear in the next section.

Table 2 shows the estimation result of the threshold VECM, which is estimated by maximum likelihood estimation at the VAR lag length 4. SE are calculated from the heteroskedasticity robust covariance estimator. The adjustment coefficient on stock prices is not significant in both regimes. The equilibrium error persists for stock prices because the adjustment coefficients are insignificant. Moreover, there is a significant error correction effect only in the unusual regime in the dividend equation, i.e. when the deviation from the long run equilibrium does not exceed the threshold parameter.

Figure 2 shows the response function of stock prices and dividends to the discrepancy between the former and the adjustment for the latter, in the previous period. The response function is based on the estimates of the intercept and the adjustment vector in each regime given the other short-run dynamics. It can be seen the flat, near zero, error correction effect on the right-hand side of the threshold parameter for both stock prices and dividends. This implies that the divergence between stock prices and dividends is persistent because stock prices and dividends do not respond to the error correction term. Moreover, on the left-hand side of the threshold parameter the response of stock prices and dividends to error correction is significant. There is a sharp negative relationship for stock prices (stock price decreases as the error correction term increases) and a sharp positive relationship for dividends (dividend increases as the error correction term increases).

V. Conclusions

In this article we test for the presence of threshold cointegration between real stock prices and dividends for the US market during the period from 1871:1 to 2004:6. Two main research issues in this study concern the possibility of the presence of a threshold in the PV model of stock prices and the asymmetric movements between stock prices and dividends. As a extension of previous studies, we make use of the methodology developed by Hansen and Seo (2002), based on a threshold cointegration model. This approach proposes an algorithm for estimating the complete threshold cointegration model and a sup*LM* test for the presence of a threshold. In particular, the threshold cointegration model allows for nonlinear adjustment to long-run equilibrium.

According to our results, the null hypothesis of linear cointegration between stock prices and dividends is rejected in favour of a two-regime threshold cointegration model, with the threshold parameter estimated at 2.15%. Futhermore, we find that stock prices do not respond to equilibrium error and dividends respond to the past divergence only if the deviation from the equilibrium error does not exceed the estimated threshold parameter.

These results would suggest the presence of a significant nonlinear behaviour in the US stock price–dividend relation. Specifically, our results are consistent with optimal adjustment models which consider the transaction costs in stock markets. This in turn would support the theoretical models that assume that the stock price–dividend relation is nonlinear.

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