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PRODUCTIVITY AND EFFICIENCY WITH DISCRETE VARIABLES AND QUADRATIC COST FUNCTION

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Abstract

We propose an index of productivity based on a quadratic cost function and developed for discrete data including technical and allocative inefficiency, jointly with technical change and returns to scale, as determinants of Total Factor Productivity. This new index is applied to Spanish stevedoring industry so as to identify the sources of change in the productivity of a multi-productive activity, where some companies do not produce some of the outputs and/or do not use some inputs. In this context, the functional quadratic form and the productivity index proposed prove particularly useful.

Keywords: productivity; quadratic cost function; inefficiency; discrete data

JEL Classification: D24

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1.- Introduction

Increased productivity is regarded as one of the most relevant factors to explain economic growth. Since the 1960s the amount of economic literature on the methods for measuring it has mushroomed. Likewise, there has been an interest by analysts and public officials alike in measuring outputs, in classifying the units analyzed as a function of the productivity achieved, and in assessing the suitability of economic policies through their effects on productivity. Due to the multitude of products and factors present in every production activity, index number theory has played a pivotal role in the aggregation of products and factors, since changes in productivity are calculated as the difference between the rates of change in an output and input indices[2]. Diewert (1976, 1978) gave a decisive impetus to the study of productivity with his theory on exact and superlative index numbers, which demonstrated the existence of a unique correspondence between the index number utilized and the underlying technology, hence the use of the term "exact". By accepting a specific functional form for the technology and assuming an optimal behavior for the agents in a competitive environment, productivity can be calculated without having to resort to estimating the parameters that characterize said technology. If the functional form chosen does not properly represent the actual technology, the resulting productivity measures will be skewed. To minimize these errors, Diewert proposed the use of flexible functional forms, resulting in the so-called exact and superlative index numbers.

In addition to measuring changes in productivity, many studies on productivity literature have focused on the decomposition of the determinants of it. With this objective have been used both nonparametric and econometric techniques for calculating or estimating the technology to measure the different components of productivity. The use of econometric techniques

[2] With the aim of offering a practical method that simplifies the measurement of productivity by managers of firms, Hannula (2002) presents a method to measure total productivity based on partial productivity ratios rather than index numbers.

raises the decomposition of productivity within a stochastic approach that takes into account the existence of statistical noise in the data. Moreover, the use of a flexible functional form to represent the underlying technology helps reduce the problems associated with the need to choose a specific functional form[3]. In this line, Denny, Fuss and Waverman (1981) developed an index that allows for the effects of a technical change and of economies of scale on productivity to be identified. This index was developed for continuous variations in data, whereas the information available on most economic variables only allows for calculations of its variation rates in discrete terms. To overcome this problem, they have generally used the flexible translog functional form that underlies the Törnqvist Index. This functional form, although it is consistent with a flexible representation of technology, has the disadvantage of not being well defined in a multioutput framework where some companies do not produce some output and/or use some input. Martinez-Budria, Jara-Díaz, and Ramos-Real (2003) extended the method of the Denny *et al* model to the quadratic functional form and discrete data. The quadratic functional form is flexible and it has the advantage that it is well defined for null outputs, which is relevant in multi-output contexts.

These studies assume efficient behavior by the decision maker, so only two sources of productivity gains, technical change and a better utilization of productive scale are considered. If the efficiency assumption is not appropriate, then decreasing inefficiency should be considered as a third source of productivity gain. Besides providing a better and more complete view of productivity *per se*, introducing inefficiency has three benefits: i) it avoids confusing technical change with inefficiency change, ii) production technology is correctly estimated and iii) it helps avoiding erroneous entrepreneurial decisions.

[3] There is a vast literature that has compared the measures of productive efficiency from nonparametric and econometric techniques. For instance, Murillo-Zamorano, L. and Vega-Cervera, J. (2001) shows empirical evidence that indicates that, in their study, the choice between both techniques could be irrelevant to rank firms according to their efficiency scores.

In order to incorporate changes in efficiency as an additional source of productivity, Bauer (1990) introduced inefficiency into the model of Denny *et al* (1981). This application has also been implemented using the translog functional form, without taking into account the existence of non-produced outputs or inputs not employed. This issue is addressed in Tsekouras *et al* (2004) where a dummy variable technique suggested by Battese (1997), is extended to a translog specification of the input distance function for the estimation of Malmquist productivity change index, when data contains observations with zero values.

In this work, we propose a model based on the quadratic cost function that is particularly adequate in multi-product activities where some companies do not produce some outputs and/or do not use some inputs. In addition, this new index includes technical and allocative inefficiency, and is developed for discrete data. In the next section, we build specific indexes for inputs and outputs for this functional form. Hence, the evolution of productivity is decomposed as a function of technical change, returns to scale, variations in technical and allocative efficiency, and the bias that stem from the manner by which inputs have been aggregated. In the third section we apply the model developed using data from the Spanish stevedoring industry to calculate the sources of productivity with firm and time variability. The final section contains a synthesis of the most relevant conclusions.

2.- The model

The rate of change in productivity is defined as

$$\overset{\circ}{TFP} = \overset{\circ}{Q} - \overset{\circ}{F} \qquad \textbf{[1]}$$

where *TFP* is the Total Factor Productivity Index, *Q* and *F* are aggregate indexes of inputs and outputs respectively, and dot variables indicate rate of change.

We now obtain and decompose $\overset{\circ}{F}$. Let us start by expressing the actual cost in period 1, C_1, as a function of prices, w_j, and quantities of inputs, x_j, used in periods 0 and 1, and their increments.

$$C_1 = \sum_j w_{j1} x_{j1} = \sum_j (w_{j0} + \Delta w_j)(x_{j0} + \Delta x_j) = \sum_j w_{j0} x_{j0} + \sum_j w_{j0} \Delta x_j + \sum_j \Delta w_j x_{j0} + \sum_j \Delta w_j \Delta x_j \qquad \textbf{[2]}$$

Taking common factor Δw_j of the last two terms of the previous expression, the rate of variation in the actual cost between two consecutive periods can be expressed in terms of discrete variations, as:

$$\overset{\circ}{C} = \frac{C_1 - C_0}{C_0} = \sum_j \frac{w_{j0} x_{j0}}{C_0} \frac{\Delta x_j}{x_{j0}} + \sum_j \frac{w_{j0} C_1}{w_{j1} C_0} \frac{w_{j1} x_{j1}}{C_{t1}} \frac{\Delta w_j}{w_{j0}} = \sum_j s_{j0} \overset{\circ}{x}_j + \sum_j G_{j1} \overset{\circ}{w}_j \qquad \textbf{[3]}$$

where: $G_{j1} = \frac{w_{j0} C_1}{w_{j1} C_0} s_{j1}$, and s_j is the share of input j in the cost.

Rearranging equation [3], we obtain the following expression:

$$\frac{1}{2} \sum_j s_{j0} \overset{\circ}{x}_j = \frac{1}{2} \overset{\circ}{C} - \frac{1}{2} \sum_j G_{j1} \overset{\circ}{w}_j \qquad \textbf{[4]}$$

In the same way, if we take the common factor Δx_j in [2] of the second and fourth term, we obtain:

$$\frac{1}{2} \sum_j \frac{C_1 x_{j0}}{C_0 x_{j1}} s_{j1} \overset{\circ}{x}_j = \frac{1}{2} \overset{\circ}{C} - \frac{1}{2} \sum_j s_{j0} \overset{\circ}{w}_j \qquad \textbf{[5]}$$

Adding [4] and [5] gives:

$$\overset{\circ}{F} = \frac{1}{2} \sum_j \left(\frac{C_1 x_{j0}}{C_0 x_{j1}} s_{j1} + s_{j0} \right) \overset{\circ}{x}_j = \overset{\circ}{C} - \frac{1}{2} \sum_j \left(G_{j1} + s_{j0} \right) \overset{\circ}{w}_j \qquad \textbf{[6]}$$

where $\overset{\circ}{F}$ is an implicit and aggregate index of inputs that can be interpreted in two ways. So, the first term of expression [6] indicates that $\overset{\circ}{F}$ is a weighted sum of the quantities of inputs used in both periods, whereas the second element highlights the fact that this implicit input index represents the change in the level of actual cost that is not explained by variations in the prices of inputs.

We now consider the determining factors of $\overset{\circ}{F}$ in greater depth. More specifically, our interest is focused in decomposing $\overset{\circ}{C}$ in expression [6]. To this end, actual cost, C, is expressed as:

$$C = \frac{C^*}{CE} \qquad [7]$$

where C^* is minimum cost and CE Farrell's cost efficiency index. Expressing [7] in variation rates,

$$\overset{\circ}{C} = \frac{C_1 - C_0}{C_0} = \frac{\frac{C_1^*}{CE_1} - \frac{C_0^*}{CE_0}}{\frac{C_0^*}{CE_0}} = \frac{\frac{C_0^* + \Delta C^*}{CE_1} - \frac{C_0^*}{CE_0}}{\frac{C_0^*}{CE_0}} = \frac{\frac{\left(C_0^* + \Delta C^*\right)CE_0 - C_0^* CE_1}{CE_1 CE_0}}{\frac{C_0^*}{CE_0}}$$

$$= \frac{\frac{\left(C_0^* + \Delta C^*\right)CE_0 - C_0^*\left(CE_0 + \Delta CE\right)}{CE_1 CE_0}}{\frac{C_0^*}{CE_0}} = \frac{\frac{\Delta C^* CE_0 - C_0^* \Delta CE}{CE_1 CE_0}}{\frac{C_0^*}{CE_0}} = \frac{\Delta C^*}{C_0^*}\frac{CE_0}{CE_1} - \frac{\Delta CE}{CE_1} \qquad [8]$$

Finally,

$$\overset{\circ}{C} = \frac{CE_0}{CE_1}\left[\overset{\circ}{C}* - \overset{\circ}{CE}\right] \qquad [9]$$

Hence, the actual cost variation rate, $\overset{\circ}{C}$, depends on the levels of cost efficiency in the two consecutive periods, on the rate of change of the optimum cost, $\overset{\circ}{C}*$, and on the efficiency index, $\overset{\circ}{CE}$. Expression [9] shows that either an increase (reduction) in optimum costs or a fall (increase) in the level of efficiency will lead to an increase (reduction) in actual cost.

We then decompose the rates of change of both optimum cost $\overset{\circ}{C}*$ and the efficiency index $\overset{\circ}{CE}$, as a function of the discrete variation rates of their respective determining factors. To this end, in the decomposition of $\overset{\circ}{C}{}^{*}$, the result obtained by Martinez-Budria *et al* (2003) is used, which gives the following expression:

$$\overset{\circ}{C}* = \frac{1}{2}\sum_{j}\left(G_{j1}^{*} + s_{j0}^{*}\right)\overset{\circ}{w}_{j} + \frac{1}{2}\left[\sum_{m} H_{m}\,\overset{\circ}{q}_{m}\right] + \frac{1}{2}\left[\frac{C_{1}^{*}}{C_{0}^{*}}\overset{\circ}{T}_{1} + \overset{\circ}{T}_{0}\right]\left(t_{1} - t_{0}\right) \qquad \textbf{[10]}$$

where: $G_{j1}^{*} = \frac{w_{j0}C_{1}}{w_{j1}C_{0}}s_{j1}^{*}$, and s_{j1}^{*} s_{j0}^{*} are the optimum share of input j in the periods 1 and 0

$H_{m} = \frac{C_{1}^{*}}{C_{0}^{*}}\frac{q_{m0}}{q_{m1}}\varepsilon_{c,q_{m1}} + \varepsilon_{c,q_{m0}}$, ε_{C,q_m} is the cost-product elasticity of output m, $\overset{\circ}{q}_{m}$ is its variation rate, and $\overset{\circ}{T} = \frac{1}{C^{*}}\frac{\partial C^{*}}{\partial t}$ is the rate of technical change.

We then decompose down the variation rate of Farrell's cost efficiency index, $\overset{\circ}{CE}$.

$$\overset{\circ}{CE} = \frac{CE_{1} - CE_{0}}{CE_{0}} = \frac{(ET_{1} \times EA_{1})}{ET_{0} \times EA_{0}} - 1 = \frac{\left[(ET_{0} + \Delta ET) \times (EA_{0} + \Delta EA)\right]}{ET_{0} \times EA_{0}} - 1 =$$

$$\frac{\Delta EA}{EA_{0}} + \frac{\Delta ET}{ET_{0}} + \frac{\Delta EA \Delta ET}{EAET} = \overset{\circ}{EA} + \overset{\circ}{ET} + \overset{\circ}{EA}\,\overset{\circ}{ET} \qquad \textbf{[11]}$$

This expression shows that changes in cost efficiency depend on the discrete variation rates of the technical efficiency index, $\overset{\circ}{ET}$, and allocative efficiency index, $\overset{\circ}{EA}$.

If we introduce the results found in [9], [10], and [11] in expression [6], we get an implicit and aggregate index of inputs, $\overset{\circ}{F}$, that contains four additive terms:

$$\begin{aligned}\overset{\circ}{F} &= \frac{1}{2}\frac{CE_0}{CE_1}\left[\sum_m H_m \overset{\circ}{q}_m\right] \\ &+ \frac{1}{2}\frac{CE_0}{CE_1}\left[\frac{C_1^*}{C_0^*}\overset{\circ}{T}_1 + \overset{\circ}{T}_0\right](t_1 - t_0) \\ &- \frac{CE_0}{CE_1}\left(\overset{\circ}{EA} + \overset{\circ}{ET} + \overset{\circ}{EA}\,\overset{\circ}{ET}\right) \\ &+ \frac{1}{2}\frac{CE_0}{CE_1}\left[\sum_j \left(G_{j1}^* - G_{j1}\right)\overset{\circ}{w}_j + \sum_j \left(s_{j0}^* - s_{j0}\right)\overset{\circ}{w}_j\right]\end{aligned} \qquad \textbf{[12]}$$

The first of these terms depends on the degree of economies of scale, the second is related to technical change, the third is a function of changes in the indexes of technical and allocative efficiency, while the final term is a function of the differences between optimum shares, s_j^*, and actual shares, to capture the biases in the implicit index of inputs caused by using observed shares in costs as a weighting when there is allocative inefficiency.

Martinez-Budria *et al* (2003) proposes an aggregate index of outputs which uses cost-product elasticities for weighting the rate of change of products, that we use below:

$$\overset{\circ}{M} = \frac{\frac{1}{2}\sum_m \left(\frac{C_1^* q_{m0}}{C_0^* q_{m1}}\varepsilon_{C,q_{m1}} + \varepsilon_{C,q_{m0}}\right)\overset{\circ}{q}_m}{\frac{1}{2}\sum_m \left(\frac{C_1^* q_{m0}}{C_0^* q_{m1}}\varepsilon_{C,q_{m1}} + \varepsilon_{C,q_{m0}}\right)} = \frac{\frac{1}{2}\sum_m H_m \overset{\circ}{q}_m}{\frac{1}{2}\sum_m H_m} \qquad \textbf{[13]}$$

Finally, by introducing [12] and [13] in expression [1], we get:

$$\overset{\circ}{TFP} = \overset{\circ}{M} - \overset{\circ}{F} =$$

$$\left[\frac{\sum_m H_m \overset{\circ}{q}_m}{\sum_m H_m}\right]\left[1 - \frac{1}{2}\frac{CE_0}{CE_1}\sum_m H_m\right]$$

$$-\frac{1}{2}\frac{CE_0}{CE_1}\left[\frac{C_1^*}{C_0^*}\overset{\circ}{T}_1 + \overset{\circ}{T}_0\right](t_1 - t_0)$$

$$+\frac{CE_0}{CE_1}\left(\overset{\circ}{EA} + \overset{\circ}{ET} + \overset{\circ}{EA}\,\overset{\circ}{ET}\right)$$

$$-\frac{1}{2}\frac{CE_0}{CE_1}\left[\sum_j \left(G_{j1}^* - G_{j1}\right)\overset{\circ}{w}_j + \sum_j \left(s_{j0}^* - s_{j0}\right)\overset{\circ}{w}_j\right] \qquad \textbf{[14]}$$

This form of decomposing productivity can be interpreted as follows:

1.- The first component accounts for the effect of the returns to scale on productivity and consists of two terms: an aggregate index of outputs ($\overset{\circ}{M}$) and the unit minus the weighted mean of the cost product elasticities in periods 0 and 1. If we assume that the indexes of efficiency are constant over the two consecutive periods and the cost function shows constant returns to scale, an equi-proportional increment in all outputs will not affect productivity. On the other hand, a radial increase in the production vector will lead to an increase (decrease) in productivity if there are increasing (decreasing) economies of scale. This result coincides with the results obtained by Denny *et al* (1981) for the case of continuous variations in variables.

2.- The second term is a weighted average of the technical change that has occurred in the two periods. The shift of the cost boundary caused by technical change shows that it is possible to obtain any level of production at a lesser (greater) cost in the case of technical progress (regression), which, in turn, will lead to an improvement (decline) in productivity.

3.- The third addend shows to what extent changes in productivity are caused by variations in technical and/or allocative efficiency. So, if a better (worse) use of resources and/or choice of productive process is achieved, increases (decreases) in productivity will be attained.

4.- The final component is different from those mentioned above and represents the biases in the measurement of the rates of change in productivity generated by choosing an input share other than the optimum share as a consequence of allocative inefficiency. It is not therefore a determinant of the productivity but a bias in the measurement of the same one.

5.- When there are changes in cost efficiency indexes between two consecutive periods ($\frac{CE_0}{CE_1} \neq 1$), expression [14] highlights the contribution of each of the four factors that determine productivity, which will be biased if said changes are ignored. In this sense, whether one erroneously admits efficiency in the two periods or whether one assumes that inefficiency is identical ($CE_0 = CE_1 \leq 1$), estimations of the effects of technical change, returns to scale, and bias related to input aggregation will be distorted.

Finally, note should be taken that in case that the cost inefficiency indexes are invariable between the two periods, ($CE_0 = CE_1 < 1$), and the indexes of technical and allocative inefficiencies do not change $\left(\overset{\circ}{ET} = \overset{\circ}{EA} = 0 \right)$, the contributions of technical change and of returns to scale to the productivity index will be also correctly calculated using [14].

3.- Application to Spanish stevedoring industry

The Spanish stevedoring industry was reformed in the late 1980s with the creation of State Stevedoring Companies (SEED, from its initials in Spanish). This reform entailed a reduction in the number of stevedores and in deregulating the composition of work teams. As in the rest of the world, the driving force behind this legislative reform was the profound technological change that the increasing use of the container represented for cargo handling operations (Talley, 2000). This resulted in significant investments in modern mechanical equipment capable of more rapidly transferring merchandise between ship and shore. Thus, the changes introduced by the legislative reforms hint at the existence of changes in productivity, whose measure and decomposition are the main objective of this application.

As shown in Jara-Díaz *et al* (2006, 2008), a proper knowledge of a port's production structure requires adopting a multi-productive approach. In the specific case of cargo handling services, this requires differentiating the services provided in order to package the cargo, since this is what determines the types of operations required and, thus, their costs. To that end, this study distinguishes among three different output types: containerized general cargo (CGC), non-containerized general cargo (NCGC) and solid bulk handled without any specialized facilities (SB). The Annual Statistics of the public agency Puertos del Estado (Ports of the State) have been used to obtain the information on the amount of merchandise handled annually at each port included in the sample, said output levels being expressed in tons/year.

Providing these stevedoring services requires the use of two production factors: labor and capital (cranes). Each port's Annual Reports, along with a questionnaire sent to the owners of the privately-held mechanical resources, allowed us to compile the information on the crane operating hours/year, as well as on the cost associated with this type of machinery. The other data source used was a survey sent to all of the SEEDs, which are responsible for organizing the work required to accomplish these tasks. This survey provided us with the relevant information on the labor factor as it pertains to labor costs and to the hours/year worked by the personnel involved in these port operations.

The costs analyzed were capital expenditures (cranes) and the work cost associated with handling the cargo traffic mentioned above. The cost is expressed in millions of 1998 pesetas, deflated using Spain's consumer price index (IPC) as published by the Spanish National

Statistics Institute. Indicators for the prices of factors were derived by calculating the average cost of each input. This was done by using the information on the total cost of each factor along with the number of hours/year as a measure of the amount of input utilized.

Using these data, we proceeded to construct unbalanced panel data comprising 19 ports in Spain's port system for the period from 1990 to 1998, inclusive. The ports included in this study were: Algeciras, Alicante, Bilbao, Cadiz, Cartagena, Castellon, Gijon, Huelva, La Coruña, Malaga, Mallorca, Alcudia, Motril, Pontevedra, Tenerife, Santander, Seville, Valencia and Vigo. This data sample is sufficiently representative of the Spanish port system allowing us to draw some conclusions about the evolution of the stevedoring industry as a whole and the changes brought by reform. Rodríguez-Álvarez *et al* (2007) studies cargo handling services, including infrastructure services, in a port terminal level using monthly data of one Spanish port. They estimate technical and allocative efficiency with firm and time-varying variability using a parametric distance function.

The econometric model estimated, which will permit us to identify the parameters required to decompose the TFP using equation [14], is presented in greater detail in Díaz-Hernández *et al* (2008a). The technical and allocative inefficiencies were incorporated into the model using the parametric approach developed by Atkinson and Cornwell (1994a,b). We also adapted the decomposition of both inefficiency types to the normalized quadratic function, as proposed by Kumbhakar (1997) using the translog functional form.

We now briefly describe the most relevant aspects of the econometric model used, which is constructed based on the normalized quadratic shadow cost function (NQSCF), expressed as

$$\frac{1}{b}C(P^*,Y,t)=\frac{1}{b}\begin{bmatrix}\sum_{i=1}^{m}\alpha_i P_i^* + \sum_{r=1}^{n}\alpha_r P_k^* Y_r + \frac{1}{2}\sum_{i\neq k}\sum_{j\neq k}\alpha_{ij}P_i^*\frac{P_j^*}{P_k^*} + \frac{1}{2}\sum_{r=1}^{n}\sum_{l=1}^{n}\alpha_{rl}P_k^* Y_r Y_l + \\ \sum_{i\neq k}\sum_{r=1}^{n}\alpha_{ir}P_i^* Y_r + \sum_{i\neq k}\alpha_{it}P_i^* t + \sum_{r=1}^{n}\alpha_{rt}P_k^* Y_r t + \alpha_t P_k^* t + \alpha_{tt}P_k^* t^2\end{bmatrix} \qquad [15]$$

where $Y=(Y_1,\dots, Y_n)$ be a vector of n outputs, $X=(X_1,\dots, X_m)$ a vector of m inputs and P the corresponding price vector, with $P^*=(k_1P_1,\dots, k_mP_m)$ as the shadow price vector for which the combination of actual inputs is allocatively efficient. $k_{ij}\geq 0$ is a parameter that indicates how the relative actual price ratio of input i to input j deviates from the relative shadow price ratio and defined using the $k_{ijt}=(1+\eta_{ij}+\eta_{ijt}t)^2$ time variability model, which will depend on the estimates for η_{ij} and η_{ijt}. Also, $0<b\leq 1$ is the parameter that corrects input oriented technical inefficiency[4]. Moreover, in (15) we have added a time trend as a proxy variable representing technical change that interacts with output levels and the prices of inputs.

A two-stage econometric estimation process based on the NQSCF was used. In the first stage, the equations for the input demand ratios were estimated, which allowed us to identify the parameters of the cost function that characterize the technology (α), as well as the parameters that account for the allocative inefficiency with its corresponding time variability (η_L,η_{LT}).

As we will consider only two inputs, the model to be estimated in the first stage is the ratio between capital (cranes) and labor demand equations, with the price of the former being used to normalize. Using capital and labor demand obtained trough the application of the Shephard´s lemma to the NCSCF, the econometric model to be estimated is

$$\frac{X^a_{Kft}}{X^a_{Lft}}=\frac{1/b\,(\alpha_k+\sum_{r=1}^{n}\alpha_r Y_{rft}-\frac{1}{2}\alpha_{LL}\left(1+\eta_L+\eta_{Lt}t\right)^2(\frac{P_{Lft}}{P_{Kft}})^2+\frac{1}{2}\sum_{r=1}^{n}\sum_{l=1}^{n}\alpha_{rl}Y_{rft}Y_{lft}+\sum_{r=1}^{n}\alpha_{rt}Y_{rft}t+\alpha_t t+\alpha_{tt}t^2)}{1/b\,(\alpha_L+\alpha_{LL}\left(1+\eta_L+\eta_{Lt}t\right)\frac{P_{Lft}}{P_{Kft}}+\sum_{r=1}^{n}\alpha_{Lr}Y_{rft}+\alpha_{Lt}t)}+\alpha_f D_f+\upsilon_{ft} \quad \textbf{(16)}$$

[4] We have modelled technical efficiency using a radial-approach, so these measures hold the relative proportions of inputs constant. An alternative non-radial technical efficiency measure has been considered (see Färe and Lovell, 1978). In this line, Chen (2003) decomposes the Malmquist productivity index using Data Envelopment Analysis. This non-radial efficiency measure is not invariant to the units of measurement and so a change in units affects the efficiency scores

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where D_f is a dummy variable for firm f introduced to control for unobserved port heterogeneity affecting demand ratios. Finally, υ_{ft} is a standard noise error term with zero mean. The results of the estimate are shown in Appendix 1[5].

In the second stage of the estimation process, actual cost is regressed versus the sum of the adjusted optimal cost level $(\hat{C}^{*})$ and the adjusted allocative inefficiency cost $(\hat{C}^{al})$. The right hand side is calculated from the results in the first step using the parameters reported in Appendix 1. Then the technical inefficiency parameters b_{it} are estimated from the following equation:

$$C_{ft}^{a} = \sum_{f} (1/b_{ft}) D_f \left[\hat{C}_{ft}^{*} + \hat{C}_{ft}^{al} \right] + \xi_{ft} \qquad [17]$$

where D_f is a f-port dummy variable, $(1/b)_{ft} = \beta_f + \beta_{ft} t$ in order to account for port and time variability, and ξ_{ft} is a standard noise error term with zero mean. The results of the estimate of this second stage are shown in Appendix 2. Note the accuracy of the fit and the significance of the estimated model.

3.1.- Decomposition of the TFP change

Based on the results of the two-stage estimate, the different components in expression [14] are calculated so as to measure the contribution of the technical change, of the scale effect, and of the technical and allocative efficiencies to the changes in the TFP with port and time variability. The aggregation biases were negligible, and thus not included. Likewise, the product of $\overset{\circ}{AE}$ and $\overset{\circ}{ET}$ was negligible, as a result of which it was possible to estimate the

[5] See Díaz-Hernández *et al* (2008a) for a more comprehensive discussion on the analysis of the results of this estimation. Of particular note is the good fit obtained and the statistical significance of the model. Also verified was the compliance with the monotonicity and curvature properties of the shadow cost function. The normalized quadratic shadow cost function corresponds to a well-behaved production function because it is monotonically increasing in shadow input prices and output quantities and concave in shadow input prices.

contribution of the allocative inefficiency separately from the technical inefficiency in the third addend of [14]. The results for this decomposition of TFP for the different years analyzed are shown in Table 1.

Table 1 about here

The accumulated productivity growth for the entire period was 42.8% at an average annual rate of 5.35%. The technical change accounts for 58.5% of this increase, while improved use of the production scale accounts for 10.1% of the TFP total. Improved efficiency represents the remaining 31.4% increase in productivity, with the technical efficiency providing approximately double the gains of the allocative efficiency.

An analysis of the evolution in time over the period in question shows a slight variability, given the small oscillation of the values around the average. The comparatively scarce contribution of returns to scale over a period with relatively high traffic fluctuations is due to the fact that the average port has a degree of economies of scale of 1.02 (Díaz-Hernández *et al*, 2008b), meaning it is in the area of constant returns. That is why this effect is not very pronounced except during large variations in traffic, which only occurred during the 1992-93 recession.

The improvement in technical efficiency contributed a significant part, 19%, of the changes in productivity. It was possible to identify two sub-periods. From 1990 to 1992, there was a noticeable deterioration in the use of production resources, evidence of the difficulty of adapting the labor force available during periods of decreasing production. Starting in 1993, when traffic started to rebound, we see a significant annual improvement in the technical efficiency, which proved better able to adapt to rising than to falling factors. A similar pattern is seen with the allocative efficiency, though with a reduced influence on TFP.

In summary, cargo handling operations in Spanish ports improved significantly in terms of productivity, attributable mainly to the effects of the technical change and, to a lesser extent, to a drop in inefficiency.

A detailed analysis of the productivity components for each of the ports analyzed offers the possibility of gaining a greater understanding of some of the factors influencing productivity in cargo handling operations. This raises the question regarding the existence of differences in productivity among Spanish ports. With this objective in mind, the decomposition of the rate of change of productivity for each of the ports analyzed is shown in Table 2, where we see the relatively low scattering in the TFP rate of change among Spanish ports for the time period in question.

The ports that exhibited significant economies of scale show a considerable contribution to productivity, resulting from increased activity during this period. The productivity values for the ports of Alcudia, Pontevedra, Mallorca and Castellon, in particular, showed considerable improvement due to better use of their production scale. The ports of Cadiz, Malaga, Motril and Santander exhibit diseconomies of scale (Díaz-Hernández *et al* 2008b), which explains why the increase in the amount of cargo handled negatively affected their productivity.

Table 2 about here

Secondly, we note the differences that exist between the ports in terms of contribution made by improved technical efficiency. It was the larger ports analyzed in this study (Algeciras, Valencia and Bilbao) that experienced a lower contribution from advances in productivity associated with advances in technical efficiency. This seems to be because these ports, over the course of the period in question, enjoyed elevated levels of technical efficiency that were maintained over time. We should also note that these ports have a greater presence in handling international traffic, and it is possible that the competitive pressure in these markets induces greater incentives for the efficient use of the factors utilized in these tasks. We also note the advances achieved by ports such as Malaga and Alicante that, despite having started with low technical efficiency indices, achieved significant improvements in this area. With regard to improvements in allocative efficiency, of note is its inability to explain the change observed in productivity. Furthermore, no significant differences were observed on a port-by-port basis.

4.- Conclusions

We have built a specific index of productivity for a quadratic cost function which has been developed for discrete data through the introduction in the model of the allocative and technical inefficiencies as components of the index, besides the contribution of technical change and economies of scale. This flexible functional form allows a correct treatment of the output and/or inputs with zero values. Moreover, taking into account the inefficiency in the model, we avoid distortions on production and we can properly calculate the contribution to productivity growth of different sources.

We also applied this index to Spanish stevedoring industry for the period from 1990 to 1998, immediately following the legislative overhaul of the stevedoring industry in the late 1980s. The average port saw a 42.8% increase in productivity over the period analyzed. The first explanation for this was the technological change, which contributed 25 percentage points to this increase. The second source behind the improved productivity was better efficiency, both technical and allocative, in the use of production factors. This accounted for 31.4%. In this sense, both the drop in output factors as well as the improved long-term adaptation by the companies to the prices of the factors contributed to this increase in productivity. Finally, there was the more efficient use of the production scale, which accounted for 10.1% of the changes in productivity. A port-by-port analysis shows a change structure for productivity similar to that found for the average port, that is, most of the improvement was due to the technical change and, to a lesser extent, to changes in efficiency.

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Appendix 1. Estimates of demand ratio equation parameters

Parameter	Estimate	t ratio	Port Dummy	Estimate	t ratio
α_K	0.011	4.521	**DAlgeciras**	0.0027	5.459
α_L	0.152	2.326	**DAlicante**	0.0031	4.748
α_{LL}	-0.110	-2.543	**DBilbao**	0.0129	5.644
α_{CGC}	0.720	2.503	**DCádiz**	0.0038	4.299
α_{NCGC}	2.366	2.640	**DCartagena**	0.0101	15.443
α_{SB}	0.327	2.515	**DCastellón**	0.0074	5.673
α_T	-0.016	-3.047	**DGijón**	0.0062	8.406
α_{CGCCGC}	-0.047	-1.985	**DHuelva**	0.0070	6.022
$\alpha_{NCGCNCGC}$	-0.027	-2.593	**DLa Coruña**	0.0093	5.351
α_{SBSB}	0.101	2.219	**DMálaga**	0.0073	9.001
$\alpha_{CGCNCGC}$	-0.047	-0.989	**DMallorca**	0.0141	12.215
α_{NCGCSB}	-0.606	-2.723	**DAlcudia**	0.0198	21.208
α_{CGCSB}	0.065	1.923	**DMotril**	0.0024	3.586
α_{CGCPL}	0.092	2.221	**DPontevedra**	0.0060	6.455
α_{NCGCPL}	0.109	3.740	**DTenerife**	0.0085	3.079
α_{SBPL}	0.063	3.023	**DSantander**	0.0108	10.587
α_{TT}	-0.003	-19.32	**DSevilla**	0.0160	13.679
α_{TPL}	-0.054	-5.210	**DValencia**	0.0041	5.021
α_{TCGC}	-0.006	-2.843	**DVigo**	0.0077	3.755
α_{TNCGC}	0.002	1.454			
α_{TSB}	0.007	2.213			
η_L	-0.134	-2.976			
η_{Lt}	0.004	2.483			

Source: Díaz-Hernández et al (2008a)

Appendix 2. Estimates of technical efficiency parameters

Port	β_i Estimate	t ratio	β_{it} Estimate	t ratio
Algeciras	1.059	11.612	-0.0067	-4.290
Alicante	1.154	4.105	-0.0115	-5.478
Bilbao	1.107	7.735	-0.0076	-6.832
Cádiz	1.160	7.833	0.0097	2.598
Cartagena	1.100	9.272	-0.0092	-8.629
Castellón	1.106	8.809	-0.0058	-6.808
Gijón	1.110	8.479	0.0018	2.661
Huelva	1.123	9.627	-0.0013	-2.612
La Coruña	1.170	7.216	0.0020	3.445
Málaga	1.091	9.365	0.0023	3.686
Mallorca	1.082	6.931	-0.0023	-3.890
Alcudia	1.114	4.988	-0.0009	-4.945
Motril	1.153	5.301	-0.0028	-4.503
Pontevedra	1.095	4.608	-0.0017	-2.897
Tenerife	1.172	9.848	-0.0074	-2.304
Santander	1.180	8.632	0.0009	2.373
Sevilla	1.093	9.392	-0.0025	-7.267
Valencia	1.096	9.509	-0.0117	-2.424
Vigo	1.122	8.311	-0.0079	-6.001

Source: Díaz-Hernández et al (2008a)

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Table 1.- Decomposition of the TFP change

Period	Technical Change	Scale effect	Technical efficiency effect	Allocative efficiency effect	TFP change
1991-1990	3,27	2,49	-1,08	1,12	5,80
1992-1991	3,80	-1,66	-0,83	-1,04	0,27
1993-1992	3,26	-0,75	2,36	1,56	6,44
1994-1993	2,53	1,32	2,02	0,79	6,66
1995-1994	2,95	1,09	1,96	0,44	6,44
1996-1995	2,92	0,81	0,97	0,76	5,47
1997-1996	2,98	0,74	1,70	0,54	5,96
1998-1997	3,30	0,27	1,59	0,64	5,81
Mean	**3,13**	**0,54**	**1,09**	**0,60**	**5,35**

Table 2.- Decomposition of the rate of change of productivity by port

Port	Technical Change	Scale effect	Technical efficiency effect	Allocative efficiency effect	TFP change
Algeciras	2,87	0,85	0,59	0,29	4,60
Alicante	3,20	1,08	1,47	0,20	5,96
Bilbao	2,94	0,46	0,99	0,90	5,30
Cádiz	3,16	-0,59	1,02	0,67	4,27
Cartagena	3,23	0,15	1,55	0,39	5,33
Castellón	3,17	1,24	1,01	0,47	5,89
Gijón	3,32	0,80	1,16	0,50	5,78
Huelva	3,11	1,13	0,82	0,79	5,85
Coruña	3,27	0,47	1,14	0,90	5,78
Málaga	3,63	-1,03	1,69	0,72	5,01
Mallorca	3,18	1,21	1,02	0,44	5,84
Alcudia	3,38	1,26	1,30	0,11	6,05
Motril	3,16	-0,24	1,34	0,83	5,09
Pontevedra	2,75	1,03	1,05	1,08	5,90
S/C Tenerife	2,60	1,25	0,82	0,32	4,99
Santander	3,20	-0,37	0,90	0,92	4,65
Sevilla	3,29	0,40	0,87	0,47	5,03
Valencia	3,05	0,63	0,64	0,39	4,71
Vigo	3,02	0,53	1,24	1,01	5,81
Mean	**3,13**	**0,54**	**1,09**	**0,60**	**5,35**