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A Cointegration Analysis of Car Advertising and Sales Data in the Presence of Structural Change

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ABSTRACT *This paper examines whether there is a long-run stable equilibrium relationship between advertising and sales across the market segments of the UK car industry over the period 1971–2001. In order to achieve this goal, we allow for structural breaks in the series using cointegration techniques. The results show the existence of long-run equilibrium relationships in all six market segments, although in four of them the relationship is not stable. In general, one structural change is detected in the late 1970s and another in the early 1990s, coinciding with two economic recessions. When we do not account for structural changes, the estimated long-run elasticities of advertising on sales are seen to be substantially downwardly biased. Finally, a noticeable increase is observed in long-run elasticities in most car market segments during the nineties with respect to previous decades.*

Key Words: Advertising; Sales; Cointegration; Structural Break; Car Industry.

JEL Classifications: C12, C22, L10.

1. Introduction

The existence of a long-run equilibrium relationship (cointegration) between advertising and sales supports the hypothesis of an optimal strategic behaviour of firms setting advertising expenditure as a fixed percentage of sales revenues. However, as market circumstances change the long-run relationship may evolve over time. This paper investigates whether advertising and sales are cointegrated in the presence of structural breaks in the UK car industry.

Dorfman and Steiner (1954) and Schmalensee (1972) showed that, if advertising shifts the demand curve and a profit maximizing monopolist chooses prices

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and advertising simultaneously, there is an optimal advertising/sales ratio, which depends negatively on demand elasticity. That is, a profit maximizing firm will employ additional advertising rather than cut prices as a marketing strategy when demand becomes more price inelastic. For mature products, the advertising–sales ratio will be constant as the elasticity of demand may well be relatively constant. For new products, the optimal advertising/sales ratio will at first be relatively high, since a large portion of initial advertising is purely informative, but it will fall gradually as the elasticity of demand becomes greater. The existence of a long-run equilibrium relationship between advertising expenditure and sales revenues suggests that firms' decisions to fix advertising as a ratio of sales might represent optimal behaviour.[1] The lack of cointegration between the two variables indicates that firms are deviating from their optimal behaviour, suggesting the possibility of recognition of interdependence among firms in the industry (i.e., additional advertising to counterbalance other firms' advertising campaigns or suboptimal advertising expenditure to avoid retaliation). For that reason it is useful to have some knowledge of how advertising affects sales. Empirical research on advertising effectiveness generally finds a significant impact of advertising on sales, although differences are observed when it comes to determining whether that effect is longlasting or short-lived (Thomas, 1989; Kwoka, 1993; Landes and Rosenfield, 1994; Paton, 2002). For the UK car industry, Cowling and Cubbin (1971) find a positive elasticity of sales with respect to advertising expenditure among the four major domestic manufacturers between 1957 and 1968 but there is evidence of a suboptimal advertising/sales ratio, suggesting that some sort of non-competitive solution has been reached by the industry.

In line with our research, other empirical studies investigate the dynamics of the advertising-sales relationship using cointegration techniques. Baghestani (1991), Zanias (1994) and Leach and Reekie (1996) find that a cointegrated relationship existed between advertising and sales using firm-level data. Seldon and Jung (1995) find that advertising and sales are cointegrated using US macro-level data, while Chowdhury (1994) find no long run equilibrium relationship between UK aggregate advertising spending and consumption. Yet, cointegration between variables at firm level or at aggregate level cannot be used to establish conclusions about cointegration between industry variables. Using industry-level data, Elliott (2001) finds a long-run equilibrium relationship between advertising and sales for the UK food industry. However, Elliott (2001) find the long-run relationship disappears for the soft drink industry, suggesting intense rivalry between firms. A similar result is found by Cavaliere and Tassanari (2001) for the whiskey market in Italy. Franses (1994), O'Donovan *et al.* (2000), in contrast, find evidence of a long-run relationship between advertising and sales in various non-durable product industries. One of the limitations of previous research is that it does not allow for structural breaks in the data. When research covers a long period of time, market conditions change causing structural breaks in the series, which means that a unique long-run elasticity of sales on advertising is likely to be markedly biased.

This paper investigates whether advertisement expenditure and sales revenues in the UK car market are cointegrated over 30 years (1971–2001). We extend previous research in two ways. Past studies examined the dynamics of the advertising-sales relationships using industry level data, but no one has sought differences in effects by market segment. As the circumstances of each market segment are different, a cointegrating relationship is expected to occur between industry-level variables in some but not all market segments. Thus, we investigate six separate car market segments in order to identify possible differences in the long-run equilibrium relationship between advertising and sales. Secondly, we apply a time series approach based on structural change literature which allows for not only structural breaks in the data to be identified but also shows how the effects of advertising on sales evolve over time. Analysis is restricted to searching for cointegration in a bivariate regression context. This aids comparability with the analyses of the papers cited above, which only approach a bivariate model formulation.[2]

The rest of this paper is organized as follows. The data is described in Section 2 and the empirical results are shown in section 3. Section 4 presents the conclusions.

2. Data

The data set consists of annual data for UK sales revenues and advertising expenditure that is split into six car market segments between 1971 and 2001 (a total of 31 observations). Sales revenues come from unit sales and prices at model level. The unit sales for each car model and its variants come from the Society of Motor Manufactures and Traders (SMMT) and prices are taken from two trade publications, Motorists' Guide to New and Used Car Prices (1971–1980) and Parker's Guide to New and Used Car Prices (1981–2001) for all baseline variants. For each model, total revenues are calculated as the sum of the variants' unit sales multiplied by the price of the baseline variant. Advertising expenditure by car model comes from the UK Advertising Association and was obtained from ACNielsen Media. The values of each variable are in constant prices (1987 = 100), calculated by using the average annual Retail Price Index for all items. The car models were allocated by segment according to the classification used by the UK Government Department of Trade and Industry and completed using the aforementioned trade publications.[3] The segments are: Supermini/mini, small family, medium family, large family/executive, luxury, sport, 4b × 4 (or Jeep) and people carrier (or minivan).[4]

Advertising is relatively important in the car industry. The percentage of car advertising with respect to total UK advertising increased from 0.98% in 1970 to 4.49% in 2001. Advertising in the UK car industry rose by more than ten fold (in nominal terms) between the early 1970s and halfway through the 1990s. There has been an escalation of advertising in the UK car industry since the early 1970s – an increase of more than ten fold (in nominal terms) from the early 1970s through to the late middle 1990s.

Figure 1 ilustrates the evolution of sales and advertising together by segment over the period 1971–2001. All the series exhibited strong non-stationary behaviour. Both sales and advertising showed a pronounced fall during the period 1977–1979 and 1991–1993, coinciding with the two economic recessions. Table 1 shows the advertising/sales ratio by segment and by period. The advertising/sales ratio has increased from £5 per £1000 in sales revenues in 1971 to a maximum of £19 in 1999. The supermini/mini and small segments exhibited the largest advertising/sales ratio over the period as a whole. This coincides with the two segments that experienced the largest changes in the participation in the UK car market, as can be seen in Table 2. In 1971 the two largest segments were small

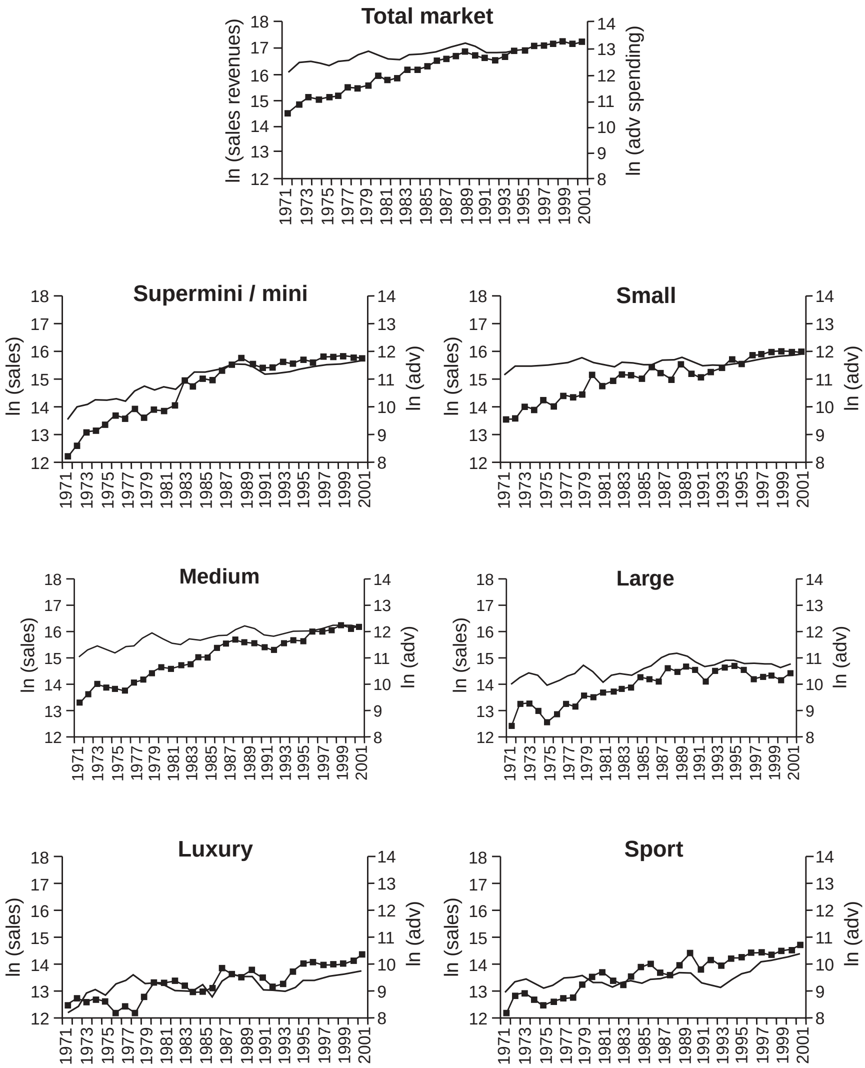

Figure 1. Evolution of the sales revenues and advertising expenditure in the car market by segments (1971–2001).
Note: Solid line for sales and dotted line for advertising

family and medium family cars, each representing more than 35% of the total market. In 2001 they were still the two largest segments, although the supermini/mini car segment had increased its market share (from 7.6% to 19%), mainly taken from the small family car segment. In 2001 the smallest car market segments were luxury cars (3%), sport cars (6%) and large/executive cars (8.8%). A similar picture is observed when advertising expenditure data is examined.

3. Empirical Results

We estimate the long-run relationship between advertising, ad_t, and sales, sa_t, both expressed in logarithms, using cointegration techniques. This methodology

Table 1. Average ratio advertising/sales (£ in advertising expenditure per £1000 revenue)

Period	Total market	Supermini and mini	Small	Medium	Large	Luxury	Sport
1971–1975	4.4	5.8	4.1	4.0	5.1	7.3	4.4
1976–1980	5.8	8.6	6.3	4.8	5.9	4.0	5.6
1981–1985	9.9	12.5	10.8	8.0	9.2	9.2	11.6
1986–1990	12.7	17.9	12.5	11.5	10.0	10.8	14.1
1991–1995	16.0	23.5	17.0	12.3	13.7	12.9	18.9
1996–2001	18.4	22.5	21.4	16.8	11.6	16.0	15.2

will allow us to avoid any spurious regressions while retaining the long-run information. Hence, we first test for unit roots in order to determine the order of integration of the series; secondly, we study the possible presence of structural changes in the series; thirdly, we estimate the cointegration relationship between the variables; finally, we discuss the possibility of structural changes or instabilities in this relationship.

3.1. Stationarity Analysis

We begin by examining the time-series properties of the series. We use a modified version of the Dickey and Fuller (1979, 1981) test (DF) and a modified version of the Phillips and Perron (1988) tests (PP) proposed by Ng and Perron (2001) for the null of a unit root.

Conventional unit root tests (DF and PP types) generally have from three problems. Firstly, many tests have low power when the root of the autoregressive

Table 2. Sales revenue and advertising expenditure share by segment (% of total UK car market*), 1971–2001

Sales revenues (% in all segment)						
Year	Mini	Small	Medium	Large	Luxury	Sport
1971	7.65	37.61	35.47	12.63	2.11	4.53
1981	14.87	33.68	35.57	8.53	3.42	3.94
1991	18.58	24.90	37.60	13.36	2.31	3.25
2001	19.02	25.36	37.76	8.78	3.01	6.08
Advertising expenditure (% in all segment)						
Year	Mini	Small	Medium	Large	Luxury	Sport
1971	9.87	38.05	30.26	12.70	5.14	3.97
1981	13.71	34.87	29.27	12.02	3.82	6.32
1991	28.22	21.25	30.85	14.35	2.10	3.24
2001	22.00	28.00	35.69	6.11	3.14	5.08

Notes:
* 4×4 and PCs excluded.

polynomial is close to but less than one unit (Dejong *et al.*, 1992). Secondly, the majority of the tests suffer from severe size distortions when the moving-average polynomial of the first differenced series has a large negative autoregressive root (Perron and Ng, 1996). Thirdly, the implementation of unit root tests often demands the *ad hoc* selection of an autoregressive truncation lag, *k*. Recently, Ng and Perron (2001) set out to resolve the three problems. They propose a class of modified tests, $\overline{M}^{GLS}_{MAIC}$, originally developed by Stock (1999) as *M* tests, with GLS detrending of the data as proposed in Elliot *et al*. (1996), and using the *MAIC* (Modified Akaike Information Criteria) to select the autoregressive truncation lag. They also propose a similar procedure to correct the problems of the standard Augmented Dickey-Fuller (ADF) test, ADF^{GLS}_{MAIC}.

Table 3 presents the results of the $\overline{M}Z^{GLS}_{\alpha}$, $\overline{M}Z^{GLS}_{t}$, $\overline{M}SB^{GLS}$ and ADF^{GLS} tests.[5] All test statistics formally examine the unit root null hypothesis against the alternative of stationary.[6] The null hypothesis of non-stationarity for series in level, $ad0_t$, $ad1_t$, $sa1_t$, $sa2_t$, $sa4_t$, and $sa6_t$ cannot be rejected, regardless of the test. Accordingly, this group of series would be I(1). In contrast, when the ADF^{GLS} statistic is applied, evidence of stationarity is found for $ad2_t$, $ad4_t$, and $ad6_t$ series at the 1% significance level, $ad3_t$, $ad5_t$ and $sa3_t$ series at the 5% significance level, and $sa0_t$

Table 3. Ng and Perron[a,b] tests for a unit root: I(1) vs. I(0) (1971–2001)

Case: $p = 1$, $\bar{c} = -13.5$

Variable	M^{GLS}_{MAIC} tests			
	$\overline{M}Z^{GLS}_{\alpha}$	$\overline{M}Z^{GLS}_{t}$	$\overline{M}SB^{GLS}$	ADF^{GLS}
$ad0_t$	−6.46	−1.66	0.257	−2.33
$ad1_t$	−4.41	−1.29	0.292	−1.85
$ad2_t$	−12.26	−2.43	0.198	−3.72***
$ad3_t$	−10.44	−2.22	0.213	−3.18**
$ad4_t$	−12.25	−2.42	0.197	−3.61***
$ad5_t$	−10.86	−2.31	0.212	−3.03**
$ad6_t$	−12.98	−2.54	0.196	−4.01***
$sa0_t$	−8.05	−2.00	0.248	−2.80*
$sa1_t$	−4.63	−1.45	0.313	−2.16
$sa2_t$	−7.17	−1.85	0.259	−2.61
$sa3_t$	−9.64	−2.19	0.227	−3.02**
$sa4_t$	−8.01	−1.95	0.243	−2.41
$sa5_t$	−7.51	−1.93	0.257	−2.71*
$sa6_t$	−6.41	−1.65	0.258	−1.89

Notes:
[a] A*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.
[b] The *MAIC* information criteria is used to select the autoregressive truncation lag, *k*, as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), table 1.

Critical values:	Case: $p = 1$, $\bar{c} = -13.5$		
	10%	5%	1%
$\overline{M}Z^{GLS}_{\alpha}$	−14.2	−17.3	−23.8
$\overline{M}SB^{GLS}$	0.185	0.168	0.143
$\overline{M}Z^{GLS}_{t}$, ADF^{GLS}	−2.62	−2.91	−3.42

and $sa5_t$ series at the 10% significance level. Consequently, this group of eight variables could be I(1) or I(0).[7]

A potential difficulty in assessing the time series properties of economic variables is the existence of structural breaks in the form of infrequent changes in the mean or the drift of the series, due to eventual exogenous structural or policy shocks. Most conventional unit root tests (DF and PP test types) and also the ADF_{MAIC}^{GLS} tests have low power when applied to series with structural changes.[8] This could be the case with $ad2_t$, $ad3_t$, $ad4_t$, $ad5_t$, $ad6_t$, $sa0_t$, $sa3_t$ and $sa5_t$ series.

In order to investigate the possibility that these series are best represented by a structural change, we also applied the unit root tests with unknown break points proposed by Perron (1997) and Vogelsang and Perron (1998). This procedure is based on simple autoregressions (estimated by OLS) appropriately augmented with trend and dummy components. The test statistics are based on the values of the *t*-statistics in order to test that the sum of the autoregressive coefficients is equal to one. Two approaches and three kinds of models, are considered. In the first approach, the 'Innovational Outlier Model' (IOM), the break occurs slowly over time and feeds dynamics back into the process as opposed to the second approach, the 'Additive Outlier Model' (AOM), where the change is assumed to occur instantly and has no further effects on future observations. Next, three kinds of models are considered according to their deterministic components: (i) a break in the intercept (Case A); (ii) a break in the slope (Case B); (iii) both changes are allowed (Case C). Finally, only five cases are tackled by the empirical application as Case B is only compatible with the AOM.[9]

The results from the unit root Perron-Vogelsang tests with structural breaks are reported in Table 4. Only the case where a rejection of the null hypothesis was obtained is presented. On analysing the results, the following conclusions are reached. First of all, the unit root is rejected at the 1% level for the $ad2_t$, $ad3_t$ and $ad6_t$ series, and at the 5% level for the $ad4_t$, $ad5_t$ and $sa5_t$ series. With a rejection of the unit root, the *t*-statistic on the change in slope or intercept is asymptotically normally distributed. For all these variables, this *t*-statistic is highly significant. Furthermore, if the unit root is rejected, the selected value of $\hat{T}_b$ for these variables is a consistent estimate of the break point. So, the estimates of the break points reveal important episodes which had permanent effects on the long term behaviour of the variables and that might have affected the cointegration relationship between them.

Secondly, the unit root is not rejected for the $sa0_t$ and $sa3_t$ series. When the unit root is not rejected the *t*-statistic on the change in slope or intercept is not asymptotically normally distributed and the selected values of $\hat{T}_b$ are not likely to give a consistent estimate of the break point.

After analysing the results from the stationarity analysis, the following conclusions are reached. Firstly, the series $ad0_t$, $ad1_t$, $sa0_t$, $sa1_t$ and $sa2_t$, $sa3_t$, $sa4_t$, and $sa6_t$ are I(1). Secondly, the series $ad2_t$, $ad3_t$, $ad4_t$, $ad5_t$, $ad6_t$ and $sa5_t$ are stationary and possibly have undergone a structural change.

3.2. Long-run Relationship

Once the order of integration of the series has been analysed, we will estimate the long-run or cointegration relationship between advertising, ad_t, and sales, sa_t. Given the relatively small sample size, we will estimate and test the coefficients of

Table 4. Perron and Vogelsang tests for unit roots with structural breaks (1971–2001)

Variable	Model	$\hat{T}_b$	κ	$\hat{\alpha}$	$t_{\hat{\alpha}}$
*ad*2	AOM-B	1981	0	–0.06	–5.73***
*ad*3	IOM-C	1985	5	0.72	–6.83***
*ad*4	AOM-B	1989	0	0.06	–5.06**
*ad*4	AOM-C	1989	0	0.02	–5.94**
*ad*5	IOM-A	1986	3	–0.19	–5.60**
*ad*6	IOM-A	1979	2	0.07	–6.96***
*ad*6	IOM-C	1978	2	–0.99	–7.29***
*ad*6	AOM-C	1978	2	–0.14	–7.58***
*sa*5	AOM-B	1978	1	0.25	–4.94**

Notes:
[a] A ** and *** denotes significance at the 5% and 1% levels, respectively.
[b] Criterion to select the truncation lag parameter κ: t-sig with κ max = 5, according to the recommendation of Ng and Perron (1995).
[c] $\hat{T}_b$ is breakpoint date.
[d] Tested models:
(i) IOM-C:

$$y_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{\kappa} c_i \Delta y_{t-i} + e_t,$$

where $DU_t = 1(t > T_b), DT_t = 1(t > T_b)t$ and $D(T_b)_t = 1(t = T_b + 1)$ with 1(.) the indicator function.
(ii) IOM-A:

$$y_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{\kappa} c_i \Delta y_{t-i} + e_t$$

(iii) AOM-C:

$$y_t = \mu + \beta t + \theta DU_t + \gamma DT_t^* + \bar{y}_t,$$

where and $DT_t^* = 1(t > T_b)(t - T_b)$ and

$$\bar{y}_t = \alpha \bar{y}_{t-1} + \sum_{i=0}^{k} w_j D(T_b)_{t-i} + \sum_{i=1}^{k} c_i \Delta \bar{y}_{t-1} + e_t.$$

(iv) AOM-B:

$$y_t = \mu + \beta t + \gamma DT_t^* + \bar{y}_t.$$

Critical values:

Model	10%	5%	1%	Source
IOM-C	–5.29	–5.59	–6.32	Perron (1997), table 1(d), T=70
IOM-A	–4.92	–5.23	–5.92	Perron (1997), table 1(a), T=60
AOM-C	–5.21	–5.56	–6.17	Vogelsang and Perron (1998), table 2(b), T=50
AOM-B	–4.48	–4.83	–5.45	Perron (1997), table 1(g), T=100

the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Also, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggests a new test where the null hypothesis is that of cointegration. In the first place, we estimate a long-run dynamic equation including the leads and lags of all the explanatory variables, the so-called DOLS regression; in our case:

$$s\alpha_t = \alpha_0 + \alpha_1 t + \beta ad_t + \sum_{j=-q}^{q} \gamma_j \Delta ad_{t-j} + \upsilon_t \quad (1)$$

Secondly, the Shin test is based on the calculation of two LM statistics from the DOLS residuals, C_μ and C_τ, to test for stochastic (when $\alpha_1 \neq 0$) and deterministic (when $\alpha_1 = 0$) cointegration, respectively. The parameter β is the long-run cointegrating coefficient estimated between advertising and sales (or long-run elasticity).

The coefficients from the DOLS regression and the results of the Shin test are reported in Table 5. As the concept of deterministic cointegration is stronger than the concept of stochastic cointegration, we first test for the presence of stochastic cointegration and then test for the presence of deterministic cointegration sequentially. Table 5A presents the results from stochastic cointegration. The null of stochastic cointegration is rejected at the 5% and 10% level of significance for all seven cases. Next, we check for the presence of deterministic cointegration using the demeaned specification. Table 5B presents the results from deterministic cointegration. Now, the null of deterministic cointegration is not rejected at the 1% level for all car market segments. We may conclude that there is strong evidence of deterministic cointegration between advertising and sales for the entire car market. However, for the luxury and small family car segments, the estimated long-run coefficients are very small, and in the case of the small family car segment the long-run parameter is estimated with a wrongly negative sign *a priori*.

3.3. Estimating the Model with Multiple Structural Breaks

Previous research has found evidence of a long-run relationship between sales and advertising at industry level. However, no research has tested whether this long-run relationship is stable over time. Our data covers 30 years of the history of the UK car market, during which time advertising and sales strategies have probably changed due to variations in macroeconomic and market forces, such as technical improvements, supply shocks, import competition, and financial and tax changes. Therefore, it is important to account for structural breaks in our data.

In this section, we estimate a model with endogenous breaks to check whether the long-run advertising-sales link is stable over time. In order to achieve this, we follow the methodology proposed by Bai and Perron (1998, 2003a, 2003b) so that different coefficients β are obtained for each regime.[10] A key feature of the Bai and Perron procedure is that it is possible to test for multiple breaks at unknown dates in such a way that it successively estimates each break point by using a specific-to-general strategy in order to consistently determine the number of breaks. In the case at hand, the procedure allows us to detect whether the long-run relationship between advertising and sales exhibits structural breaks allowing for the instability to occur at an unknown date.[11]

The results of applying the Bai-Perron tests to the relationship between sales and advertising are shown in Tables 6 and 7. We estimate a pure structural change model ($p = 0$) where all coefficients are subject to change, i.e., parameters that may have shifted over time include the coefficients of the constant and sales

Table 5. Estimation of long-run relationship between sales and advertising: Stock-Watson-Shin cointegration tests

(A) Stochastic cointegration ($\alpha_1 \neq 0$)							
Parameter estimates	Total market	Supermini and mini	Small	Medium	Large	Luxury	Sport
α_0	14.24	7.32	14.89	16.05	7.41	17.27	11.18
	(18.4)	(8.00)	(13.3)	(17.0)	(8.34)	(4.63)	(8.16)
α_1	–0.001	–0.03	–0.007	0.03	–0.03	0.04	–0.004
	(–0.22)	(–2.08)	(–0.87)	(3.79)	(–3.18)	(1.16)	(–0.28)
β	0.22	0.75	0.07	–0.06	0.77	–0.56	0.25
	(3.12)	(7.01)	(0.70)	(–0.69)	(7.42)	(–1.10)	(1.38)
Test:							
C_τ	0.142**	0.122**	0.103*	0.176**	0.142**	0.099*	0.098*
(B) Deterministic cointegration ($\alpha_1 = 0$)							
Parameter estimates	Total market	Supermini and mini	Small	Medium	Large	Luxury	Sport
α_0	14.41	9.11	15.83	12.58	10.10	13.02	11.56
	(64.2)	(23.3)	(48.7)	(32.9)	(24.1)	(15.1)	(34.8)
β	0.20	0.53	–0.01	0.29	0.45	0.01	0.19
	(12.2)	(15.9)	(–0.57)	(9.08)	(10.9)	(0.18)	(5.7)
Test							
C_μ	0.194	0.139	0.111	0.133	0.120	0.094	0.093

Notes:
[a] *t*-statistics in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $l = 5 \simeq INT(T^1/_2)$, as proposed in Newey and West (1987).
[b] We choose $q = 3 \simeq INT(T^1/_3)$, as proposed in Stock and Watson (1993).
[c] C_τ and C_μ are *LM* statistics for cointegration using the DOLS residuals from deterministic and stochastic cointegration, respectively, as proposed in Shin (1994). A*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.
[d] The critical values are taken from Shin (1994), table 1, from $m = 1$:

Critical values:	10%	5%	1%
C_τ	0.097	0.121	0.184
C_μ	0.231	0.314	0.533

variables. To estimate the partial structural change model we consider a particular specification with serially uncorrelated errors, different distributions for the data across segments and the same distribution for the errors across segments. We allowed up to 3 breaks and used a trimming of $\varepsilon = 0.20$, so each regime had at least 6 observations. We apply the procedure with a constant and sales as regressors (i.e., $z_t = \{1, sa_t\}$). The statistics $U\ D$ max and $W\ D$ max show (1% level of significance) that at least one break in the model exists in the market as a whole, supermini/mini, small, luxury, and sport segments. In contrast, the null hypothesis of no structural break is not rejected with both statistics for the medium and large segments. A similar conclusion is obtained when the statistics sup F_T is used. In this case, the null hypothesis that there is no break in the model is always

Table 6. Bai-Perron tests of multiple structural changes in the long-run relationship between sales and advertising, (1971–2001)

Tests[d]	Total market	Supermini and mini	Small	Medium	Large	Luxury	Sport
$U D$ max	30.70***	20.62***	23.78***	8.35	6.46	26.53***	37.26***
$W D$ max	40.54***	34.38***	39.64***	13.92	8.54	39.35***	37.26***
sup $F_T(1)$	14.67**	8.53	13.27**	5.38	4.32	7.21	37.26***
sup $F_T(2)$	30.70***	11.67***	17.35***	6.63	8.54	26.53***	25.02***
sup $F_T(3)$	17.64***	20.62***	23.78***	8.35**	4.32	23.60***	18.18***
sup $F_T(2 \mid 1)$	39.93***	12.34**	18.64***	5.81	6.46	9.13	9.91*
sup $F_T(3 \mid 2)$	9.23	26.37***	1.31	4.67	4.95	4.11	2.91
Number of breaks by:							
BIC	2	3	2	0	0	1	1
LWZ	2	0	0	0	0	0	1
SP	2	0	2	0	0	0	1

Notes:
[a] *yt* is the dependent variable, z_t represents the regressors allowed to change, x_t represents the regressors not allowed to change, q is the number of regressors subject to change, $p = 0$ means that the procedure considers only the pure structural change model where all coefficients are subject to change, $p > 0$ means that the procedure considers a partial structural change model where p is the number of regressors with fixed coefficients, h stands for the minimum number of observations in each segment, whereas M is the maximum number of breaks. Specifications: $y_t = \{sa_t\}$, $z_t = \{1, ad_t\}$, $q = 2$, $p = 0$, $h = 6$, and $M = 3$.
[b] A*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. The critical values are taken from Bai and Perron (1998, 2003b).
[c] The sup $F_T(k)$ tests and the reported standard errors allow for the possibility of serial correlation in the disturbances. The heteroskedasticity and autocorrelation consistent covariance matrix is constructed following Andrews and Monahan (1992) using a quadratic kernel with automatic bandwidth selection based on an AR(1) approximation. The residuals are pre-whitened using a VAR(1).
[d] Sequential procedure (SP) at the 5% size for the sequential test sup $F_T(l+ 1 \mid l)$.

rejected for the total market, supermini/mini, small, luxury, and sport segments, but is not rejected for the medium and large segments. The sup $F_T(l + 1 \mid l)$ test rejects the null hypothesis of one break in favour of two breaks for the total market (at the 1% level), supermini/mini (at the 5% level), small (at the 1% level), and sport segments (only at the 10% level). The next null hypothesis (of only two breaks) is also rejected in favour of three breaks only in the case of the supermini/mini segment using a 1% level of significance. However, for the case of medium and large segments, the sup F_T $(l + 1 \mid l)$ test is not significant for any l. Finally, the three methods to determine the number of breaks selects zero breaks for the case of the medium and large segments, one break for the case of the luxury (BIC) and sport segments (BIC, LWZ, and sequential), two breaks for the case of the total market (BIC, LWZ, and sequential) and the small segment (BIC and sequential), and three breaks for the case of the supermini/mini segment.

Combining the identified structural breaks with deterministic cointegration, Table 7 shows the results from the estimation of the model with three breaks for the supermini/mini segment, two breaks for the total market and the small segment, and one break model for the luxury and sport segments. Each model will allow for different coefficients β, which measure the long-run elasticity between sa_t and ad_t during each period.

Table 7. Estimation of deterministic cointegration model with multiple structural changes[a] (1971–2001)

Parameter estimates[b]	Total market	Supermini and mini	Small	Luxury	Sport
$\hat{\alpha}_{01}$	9.37	10.02	10.98	21.55	12.01
	(10.91)	(15.54)	(11.94)	(7.40)	(28.80)
$\hat{\alpha}_{02}$	9.99	13.21	12.70	9.82	2.76
	(13.42)	(14.79)	(8.70)	(10.45)	(0.55)
$\hat{\alpha}_{03}$	8.53	11.48	7.19	–	–
	(7.22)	(9.85)	(4.22)		
$\hat{\alpha}_{04}$	–	4.25	–	–	–
		(2.02)			
$\hat{\beta}_1$	0.64	0.44	0.44	–1.12	0.16
	(8.31)	(6.27)	(4.71)	(–2.94)	(3.43)
$\hat{\beta}_2$	0.55	0.14	0.25	0.39	1.14
	(9.26)	(3.65)	(1.98)	(3.76)	(2.30)
$\hat{\beta}_3$	0.65	0.34	0.71	–	–
	(7.23)	(3.32)	(5.00)		
$\hat{\beta}_4$	–	0.95	–	–	–
		(5.29)			
Estimated break point:[c]					
$\hat{T}_1$	1979	1977	1979	1978	1995
	(1977–1981)	(1974–1978)	(1977–1982)	(1976–1986)	(1993–1996)
$\hat{T}_2$	1990	1983	1992	–	–
	(1988–1991)	(1981–1985)	(1987–1993)		
$\hat{T}_3$	–	1990	–	–	–
		(1989–1991)			

Notes:
[a] From Table 6 we know that there is no structural change in the medium and large car segments (see Table 5 for their long-run elasticities).
[b] *t*-statistics in parentheses (robust to serial correlation).
[c] 95% confidence intervals for $\hat{T}_1$, $\hat{T}_2$ and $\hat{T}_3$ in brackets.

Firstly, most of the new estimates of the long-run elasticities for the different regimes are also greater than those coefficients estimated without structural breaks in Table 5. These results reveal a downward bias in the estimation of a unique long-run relationship in the presence of instability in the series. Therefore, it is necessary to control for important macroeconomic and industry episodes that alter the magnitude of the long-run equilibrium relationship between the industry variables.

Secondly, we found two structural breaks (three regimes) in the case of the total market. Before allowing for structural changes, the long-run elasticity of sales on advertising over the entire period (1971–2001) was 0.20; when we allow for structural changes, the new estimates of the coefficient β decrease after 1979 from 0.64 to 0.55, and increase again after 1990 from 0.55 to 0.65.

Thirdly, we observe important differences among the six market segments, suggesting the convenience of analysing each separately. The segments can be split into two groups. In the case of the supermini/mini segment and the small family segment the model detects more than one structural change. In the rest of segments the model detects one or none structural breaks. Table 2 in section 2 shows that the supermini/mini segment and the small family segment display the greatest changes in market participation, suggesting intensive competition between them.

In our analysis at segment level, we found three breaks (or the existence of four periods) in the supermini/mini segment. There is a substantial decrease in this coefficient after 1977 from 0.44 to 0.14, and it increases again in the third period (1983–1989) from 0.14 to 0.34, and finally, there is a substantial increase in the fourth period (1990–2001) from 0.34 to 0.95. In the case of the small segment the estimated model with two breaks implies the existence of three periods. The estimate of the coefficient β decreases after 1979 from 0.44 to 0.25, and increases again considerably after 1992 from 0.25 to 0.71. Finally, consider the case of the luxury and sports car segments where one structural break is identified. In the luxury segment the structural break is vaguely identified around 1978 and the coefficient β is negative (–1.12) in the first period (1971–1978) but turns positive (0.39) in the second period (1979–2001). In the case of the sports car segment, there is a substantial increase in this coefficient after 1995 from 0.16 to 1.14. In all the segments with structural changes there is a substantial increase in the long-run elasticity of sales with respect to advertising in the nineties compared to those in the 1970s and 80s.

3.4. An Economic Interpretation of the Results

In this section we investigate the determinants of the structural breaks in the UK car market and the strategic implications for the car industry of the estimated elasticities for each segment over three decades. First of all, from our analysis of the relationship between advertising and sales for the UK car market in a non-stationary environment we conclude that there is a deterministic cointegrating relationship for the total market and all market segment.[12] Therefore at market level an increase in advertising is associated with an increase in sales in the car market. At managerial level, this finding implies that advertising is effective in boosting total sales, i.e., not only by expanding market share at the expense of competing brands but by expanding overall consumption. This empirical result is similar to that obtained by Kwoka (1993) for the US car market but has been reached after recognizing the presence of non-stationarity in both sales and advertising.

Some authors have interpreted the lack of a long-term relationship between advertising and sales as evidence of intense rivalry within the industry (Elliott, 2001; Cavaliere and Tassanari, 2001).[13] Our findings do not support this hypothesis. However we do not reject the existence of intense rivalry within the car segments; our results indicate that advertising affects primary demand in all car segments but to what extent advertising is effective in expanding brand market shares at the expense of other competitive brands in the short run remains to be determined.

An interesting and novel result in the analysis of the long-term relationship between advertising and sales is that the presence of structural breaks alters the effectiveness of advertising on primary demand. More specifically, the long run relationship is not stable in four out of six car segments. The long-term relationship between advertising and sales has only remained stable over the last 30 years in the medium and large segments. The medium segment is the largest segment in the UK (above 35% of total sales revenues) and both sales and advertising expenditure have evolved smoothly over the entire period.

Over the last 30 years of UK car market history, technical changes, import competition, marketing and financial innovations, tax changes or shifts in demand tastes, among other shocks, may have altered the long term equilibrium relationship between sales and advertising in the car market. Our analysis has identified one structural change in the late 1970s and another in the early 1990s, coinciding with two economic recessions, the 1978–1981 crisis and the 1991–1993 crisis. Another episode that affected the UK car market in the seventies was the rapid penetration of Japanese competitors and the voluntary export restraints (VER) placed on Japanese exports of new cars since 1977. The competition of Japanese cars was particularly intensive in the mini and small segments. The advertising/sales ratio was relatively stable from 1971–1976 after which time it became sleeper. A break point was also estimated around 1978 (1976–1986) in the luxury segment, which is clearly associated to the first oil shock. The rise in fuel prices negatively affected the demand for executive and luxury cars. During the period from 1978–1982 the advertising expenditure in these two segments as a percentage of total advertising fell, being more intensive in the luxury segment.

After the 1978–1981 recession, advertising elasticities fell in all the segments due to the fact that macroeconomic and industry and conditions changed again. Credit restraints in the mid-1980s were removed and financial innovations were prolific, leading to an increase in new car purchases. On the supply side, new segments emerged, such as the jeeps in the early eighties and the MPVs in the late eighties. Favorable macroeconomic conditions and model proliferation may explain why the response of sales to advertising was weaker during the 80s.

The second structural break for the total car market occurred around 1990 (1988–1991) and was probably related to the short economic recession between 1990–1992. As was the case in the 70s, the negative economic shock affected both the mini and small segments at the same time. The sport segment also suffered a shock but later (around the period 1993–1996). Following the shock, advertising elasticities once again increased, making the primary demand for cars more sensitive to advertising expenditure in all car segments.

Another interesting finding is that advertising elasticities are quite different across segments, suggesting that it is necessary to split the total car market into segments to accurately identify the effectiveness of advertising on car sales. Advertising elasticities are positive in all segments over the entire period analysed except for luxury cars, which exhibited a negative elasticity during the seventies.[14] Sports car demand is also rather inelastic during the seventies but this changed drastically during the 90s. Since the mid-90s the sports car segments has displayed a very large advertising elasticity, followed by the supermini/mini segment. Therefore, advertising seems to have played a major role in the increase in demand in the UK car market during the nineties.

4. Conclusions

There are a limited number of papers on Industrial Organization using cointegration techniques to investigate the long-run relationship between industry variables. This paper contributes to this empirical literature by checking whether advertising and sales are cointegrated in six different car segments in the UK over the period 1971–2001. As a novelty we test for structural breaks in the series allowing the instability to occur at an unknown date. Moreover, we incorporate the important episodes detected in our cointegration estimations to check whether the long-run relationship between advertising and sales is stable over time.

Our results are consistent with the existence of deterministic cointegrating relationships between sales and advertising for the market as a whole and all segment markets considered. However, the estimated long-run elasticities are very small and in one case, surprisingly negative. When we test for structural changes in the series, only in the medium family and large family car segments do the series reject the presence of structural changes. For the rest of segments and for the overall market, there is evidence of structural instability. Therefore our analysis confirms the existence of a long-run equilibrium relationship between advertising and sales in the car industry, although the relationship is not stable, but rather evolves, thereby responding to important episodes that occur over the period analyzed. The number of estimated structural breaks is higher in the supermini/mini segment and small family segment than in the other segments. This result coincides with the fact that the greatest variation in the participation of sales revenues and advertising expenditure across segments in the UK car market occurred in the supermini/mini and small family car segments.

Finally, the long-run elasticities of advertising on sales were found to be substantially downwardly biased when structural breaks were not taken into account. Moreover, substantial shifts are broadly observed in the estimated coefficients of the long-run elasticities, once in the late 70s and again in the first half of the 90s. In the last period, there was a notable rise in the long-run elasticity of advertising on sales.

Nevertheless, our conclusions are only tentative. A bivariate analysis was employed to maintain comparability with previous work. However, an obvious potential orientation for future research is the expansion of the present analysis to a multivariate context since the omission of relevant explanatory variables such as the manufacturer price, quality and promotions may cause bias in the estimated cointegration relationship.

Notes

1. The optimal advertising/sales ratio can vary for other reasons. Different market structures may affect the optimal marketing mix strategy due to strategic interaction among players. For example, Lambin *et al.* (1975) extrapolate the Dorfman-Steiner condition to the case of an oligopoly with multiple competitive reactions, that is, a competitor may react to a change in price by changing both price and advertising. In all the cases under study they find an optimal advertising/sales ratio which depends on own price and advertising elasticities as well as on cross-price and cross-advertising elasticities.
2. Dekimpe and Hanssens (2000) provide a survey of the marketing literature in the nineties on the long run relationship between sales and advertising. Of the 21 papers reviewed, 16 used a bivariate approach.
3. The data can be obtained from the authors on request.
4. The following code is used to identify market segments: (0) total market (excluding 4b × 4 and PC); (1) supermini/mini; (2) small; (3) medium; (4) large; (5) luxury; and (6) sport. The two segments excluded (4 × 4 and PC) emerge in the eighties so the number of observations is considerably reduced for cointegration analysis. In 2001, the 4b × 4 and PC segments represented around 9% of sales revenues in the UK car market.
5. See Ng and Perron (2001) and Perron and Ng (1996) for a detailed description of these tests.
6. In the case of the $\overline{M}SB^{GLS}$ test, the unit root hypothesis is rejected in favour of stationarity when the estimated value is smaller than some appropiate critical value.
7. The null hypothesis of nonstationarity for the series in first differences [I(2) vs I(1)] was strongly rejected in all the cases.

8. See Monte Carlo simulations in Campos *et al*. (1996) and Perron (1997) for DF tests, and Perron and Rodríguez (2003) for $\overline{M}^{GLS}_{MAIC}$ and ADF^{GLS}_{MAIC} tests.
9. For more details, see Perron (1997).
10. Recent research has developed estimation techniques and tests of structural change at unknown break dates. See Andrews (1993) and Andrews and Ploberger (1994) for the case of a single structural change, and Andrews *et al*. (1996), Liu *et al*. (1997), and Bai and Perron (1998, 2003a, 2003b) for the case of multiple structural changes.
11. See the Appendix for technical details.
12. Notice that we found a deterministic rather than stochastic long term relationship, which implies a stronger cointegration between advertising and sales. Determistic cointegration implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of I(1) variables have nonzero linear trends this corresponds to stochastic cointegration. For definitions of deterministic and stochastic cointegration, see Ogaki and Park (1997).
13. Another line of work using firm-level data and unit root tests studies the volatility in temporal market shares as indicative of push-and-pull tacties of intense rivalry. See, for example, Gallet and List (2001).
14. Negative advertising elasticities are not uncommon in other studies. For example, Lariviere *et al.* (2000) report negative elasticities for alcoholic beverages.

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Appendix. The Bai and Perron Sequential Procedure for Estimating Breakpoints

Bai and Perron (1998, 2003a) considered theoretical issues related to the limiting distribution of estimators and the statistics in the lineal model with *m* multiple structural changes ($m + 1$ regimes):

$$\begin{aligned} y_t &= x_t'\beta + z_t'\delta_1 + u_t, \qquad t = 1,\ldots,T_1, \\ y_t &= x_t'\beta + z_t'\delta_2 + u_t, \qquad t = T_1+1,\ldots,T_2, \\ &\vdots \\ y_t &= x_t'\beta + z_t'\delta_{m+1} + u_t, \qquad t = T_m+1,\ldots,T. \end{aligned} \tag{2}$$

where y_t is the observed dependent variable at time t; $x_t(p \times 1)$ and $z_t(q \times 1)$ are vectors of covariates and β and $\delta_j(j = 1, \ldots, m + 1)$ are the corresponding vectors of coefficients; u_t is the disturbance at time t. The indices $(T_1, \ldots, T_m)$, or the break points, are explicitly treated as unknown. The purpose of this procedure is to estimate the unknown regression coefficients together with the break points when T observations on (y_t, x_t, z_t) are available. This model is a partial structural change model ($p > 0$) in the sense that the parameter vector β is not subject to shifts and where not all parameters are subject to shifts. When $p = 0$, the procedure considers only the pure structural change model where all coefficients are subject to change.

The issue of testing for structural changes is also considered under very general conditions on the data and the errors. The Bai and Perron tests are based upon an information criterion in the context of a sequential procedure, and makes it possible to find the number of breaks implied by the data, as well as estimating the timing and the confidence intervals of the breaks and the parameters of the processes between breaks.

One of the many important advantages offered by this procedure is that it is not computationally excessive. The Bai and Perron procedure permits the computation of the estimates using at most least-squares operations of order $O(T^2)$ for any number of structural changes m, unlike a standard grid search procedure which requires least squares operations of order $O(T^m)$.

To identify break points, Bai and Perron (1998, 2003a) propose four statistics to test for multiple breaks: (i) the sup $F_T(k; q)$ test, a sup F-type test of the null hypothesis of no structural break ($m = 0$) *versus* the alternative of a fixed (arbitrary) number of breaks ($m = k$); (ii) A double maximum tests of the null hypothesis of no structural break ($m = 0$) *versus* the alternative of an unknown number of breaks given some upper bound M ($1 \leq m \leq M$). (iii) The first test (UD max test) is an equal weighted version and the second test (WD max test) applied weights that depend on the number of regressors (q) and the significance level of the test (α); and (iv) the sup $F_T(l + 1 \mid l)$ test, a sequential test of the null hypothesis of l breaks *versus* the alternative of $l + 1$ breaks. Finally, Bai and Perron (1998, 2003a) proposed three methods to determine the number of breaks: Bayesian Information criterion (BIC); Schwarz modified criterion (LWZ); and, sequential procedure (SP).