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A note on nonlinear dynamics in the Spanish term structure of interest rates[☆]

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Abstract

This note applies the methodology to test for threshold cointegration recently proposed by Hansen and Seo (2002) [Hansen, B. E. & Seo, B., (2002). Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics*, 110, 293–318] to the Spanish term structure of interest rates during the period 1980:1–2002:12. The evidence suggests that nonlinear cointegration between long and short interest rates is clearly rejected, so that a linear cointegration model would provide an adequate empirical description for the Spanish term structure of interest rate.

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1. Introduction

The expectations hypothesis of the term structure of interest rates is one of the oldest and simplest analytical framework that simplifies the rational behaviour in the financial markets. According to the expectations hypothesis, the long-term interest rates should reflect future short-term

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changes. Specifically, long-term interest rates would be the average of future expected short rates. Hence, the expectations hypothesis in the context of the cointegration theory suggests that the long and short interest rates are linked through a long-run relationship with parameters $(1, -1)$, i.e., that the interest rate spread is mean-reverting. Following the early work by [Campbell and Shiller \(1987\)](#), a number of further contributions have arisen. These works have been devoted to test the expectations hypothesis of the term structure of interest rates applying cointegration techniques on a linear model, leading sometimes to contradictory results. A non-exhaustive list of them would include [Cuthbertson \(1996\)](#), [Hall, Anderson, and Granger \(1992\)](#), [Engsted and Tanggaard \(1994\)](#), [Stock and Watson \(1988\)](#).

A recent empirical study of [Camarero and Tamarit \(2002\)](#) extends the previous analysis on the expectations model of the term structure of interest rates addressing the question of whether the relationship is stable over time, or exhibit a structural break allowing the instability to occur at an unknown point. For the Spanish economy, they found evidence of linear cointegration between long and short interest rates for the period 1980:1–1996:4, with a vector $(1, -1)$ as predicted by the theory. However, there is also evidence of structural instability, attributed by the authors to the financial changes that occurred in Spain as a result of its external commitments in the context of the process towards European Monetary Union.

The specification used in [Camarero and Tamarit \(2002\)](#) study is based on the linear model of the expectations hypothesis of the term structure of interest rates as proposed by [Shiller \(1979\)](#). Nevertheless, linearity is not implied by the theory of the term structure. In this paper, we investigate whether a nonlinear model might provide a better empirical description. In particular, we use a new approach developed by [Hansen and Seo \(2002\)](#). It is based on a threshold cointegration model that considers the possibility of a nonlinear long-run relationship between long and short interest rates. This method is applied to test the Spanish term structure of interest rates in the presence of nonlinearity during the period 1980:1–2002:12. The concept of threshold cointegration would capture the possibility of a nonlinear relationship between long and short interest rates, so that a mean-reverting dynamic behaviour of the interest rate spread (or a cointegrating relationship between long and short interest rates) should be expected only when a certain threshold is reached.

Two factors may provoke nonlinearities in this relationship in the term structure of interest rates through their effects on the rational-expectations. Firstly, in the context of regime changes, [Drifill, Psaradakis, and Zola \(1997\)](#), [Hamilton \(1988\)](#) and [Simon \(1990\)](#) find evidence of nonlinearities in this relationship when there is a change in the instrumentation of monetary policy. Secondly, [Seo's \(2003\)](#) empirical results support the nonlinear mean reversion in the U.S. term structure of interest rates when there are transaction costs in the financial market.¹

The rest of the paper is organized as follows. The empirical methodology (threshold cointegration model) is briefly outlined in Section 2. Section 3 implements the tests for threshold cointegration between long and short Spanish interest rates and describes the findings. Finally, Section 4 summarizes draws the conclusions.

¹ [Seo \(2003\)](#) used a three-regime threshold VECM. In our application, we use the two-regime threshold VECM previously proposed by from [Hansen and Seo \(2002\)](#).

2. Methodology

In order to test the term structure of interest rates in the context of the cointegration theory, the empirical studies on the expectations hypothesis have commonly used a linear model such as:

$$\text{bonds}_t = \alpha + \beta \text{cmr}_t + \varepsilon_t \quad (1)$$

where bonds_t is the interest rate of the long-term bonds and cmr_t the short-term interest rate. According to [Campbell and Shiller \(1987\)](#), bonds_t and cmr_t should be non-stationary and linked through a cointegration relationship with parameters (1, -1).

Alternatively, we may write the linear regression model (1) as a bivariate linear cointegrating VAR model with one lag, $l=1$, such as:

$$\begin{pmatrix} \Delta \text{bonds}_t \\ \Delta \text{cmr}_t \end{pmatrix} = \mu + \alpha w_{t-1} + \Gamma \begin{pmatrix} \Delta \text{bonds}_{t-1} \\ \Delta \text{cmr}_{t-1} \end{pmatrix} + \varepsilon_t \quad (2)$$

where the long-run relationship is defined as $w_{t-1} = \text{bonds}_{t-1} - \beta \text{cmr}_{t-1}$. Setting $\beta=1$, the long-run relationship would be the same as the interest rate spread.

Nevertheless, linearity, is not implied by the theory of the term structure. The concept of threshold cointegration was first introduced by [Balke and Fomby \(1997\)](#) as a feasible way to combine nonlinearity and cointegration. Systems in which variables are cointegrated can be characterized by an error correction model (ECM), which describes how the variables respond to deviations from the equilibrium. Hence, the ECM can be characterized as the adjustment process along which the long-run equilibrium is maintained. However, the traditional approach assumes that such a tendency to move towards the long-run equilibrium is present every time period.

[Balke and Fomby \(1997\)](#) point out the possibility that this movement towards the long-run equilibrium might not occur in every time period, due to the presence of some adjustment costs on the side of economic agents. In other words, there could be a discontinuous adjustment to equilibrium so that, only when the deviation from the equilibrium exceeds a critical threshold, the benefits of adjustment are higher than the costs, and economic agents move the system back to equilibrium. Threshold cointegration characterizes this discrete adjustment as follows: the cointegrating relationship does not hold inside a certain range, but holds if the system gets ‘too far’ from the equilibrium; i.e., cointegration would hold only if the system exceeds a certain threshold.

This type of discrete adjustment could be particularly useful to describe the behaviour of the term structure of interest rates. The concept of threshold cointegration would capture the possibility of a nonlinear relationship between long and short interest rates, so that a mean-reverting dynamic behaviour of the interest rate spread (or a cointegrating relationship between long and short interest rates) should be expected only when a certain threshold is reached.

When testing for threshold cointegration, [Balke and Fomby \(1997\)](#) propose applying several univariate tests previously developed in the literature, to the known cointegrating residual (i.e., the error-correction term). [Lo and Zivot \(2001\)](#) extend their approach to a multivariate threshold cointegration model with a known cointegrating vector, using [Tsay's \(1998\)](#) and multivariate extensions of [Hansen's \(1996\)](#) tests. More recently, [Hansen and Seo \(2002\)](#) contribute further to this literature by examining the case of an unknown cointegration vector. In particular, these authors propose a vector error-correction model (VECM) with one cointegrating vector and a threshold effect based on the error-correction term,

and develop a Lagrange multiplier (LM) test for the presence of a threshold effect. This will be the approach followed in this paper.

Hansen and Seo (2002) consider a two-regime threshold cointegration model, or a nonlinear VECM of order $l + 1$, such as:

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) \leq \gamma \\ A_2' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) > \gamma \end{cases} \quad (3)$$

with

$$X_{t-1}(\beta) = \begin{pmatrix} 1 \\ w_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-2} \\ \vdots \\ \Delta x_{t-l} \end{pmatrix}$$

where x_t is a p -dimensional $I(1)$ time series which is cointegrated with one $p \times 1$ cointegrating vector β , $w_t(\beta) = \beta' x_t$ is the $I(0)$ error-correction term, u_t is an error term, A_1 and A_2 are coefficient matrices that describe the dynamics in each of the regimes, and γ is the threshold parameter.

As can be seen, the threshold model (3) has two regimes, defined by the value of the error-correction term. As long as deviations from the equilibrium are lower or equal than the threshold, there is no tendency for the variables x_t to revert to an equilibrium (i.e., the variables would not be cointegrated); on the contrary, if deviations from the equilibrium are greater than the threshold, there is a tendency for the variables x_t to move towards some equilibrium (i.e., the variables would be cointegrated).

Next, Hansen and Seo (2002) propose two heteroskedastic-consistent LM test statistics for the null hypothesis of linear cointegration (i.e., there is no threshold effect), against the alternative of threshold cointegration (i.e., model (3)). The first test would be used when the true cointegrating vector is known a priori, and is denoted by:

$$\text{supLM}^0 = \sup_{\gamma_L \leq \gamma \leq \gamma_U} \text{LM}(\beta_0, \gamma) \quad (4)$$

where β_0 is the known value of β (in the case analyzed below, $\beta_0 = 1$); whereas the second test would be used when the true cointegrating vector is unknown, and is denoted by:

$$\text{supLM} = \sup_{\gamma_L \leq \gamma \leq \gamma_U} \text{LM}(\tilde{\beta}, \gamma) \quad (5)$$

where $\tilde{\beta}$ is the null hypothesis estimate of β . In both tests, $[\gamma_L, \gamma_U]$ is the search region set so that γ_L is the π_0 percentile of \tilde{w}_{t-1} , and γ_U is the $(1 - \pi_0)$ percentile; Andrews (1993) suggests setting π_0 between 0.05 and 0.15. Finally, Hansen and Seo (2002) develop two bootstrap methods to calculate asymptotic critical values and p -values.

3. Results

In this section, we re-examine the issue of the expectations hypothesis of the Spanish term structure of interest rates. To this end, we use the new approach of Hansen and Seo (2002) described in the last section.

The data are monthly for Spain and over the period 1980:1 to 2002:12. The variables utilized in the empirical application are the nominal long-term interest rate, $bonds_t$ (from January 1980 to December 1992, central government bonds at more than 2 years; and, from January 1993, central government benchmark bond of 10 years), and the nominal short-term interest rate, cmr_t (3-month interbank market rates or call money rate). Both series have been obtained from [Bank of Spain \(2003\)](#) and the IMF International Financial Statistics magnetic tapes.

As a first step of the analysis, we have tested for the order of integration of the two series. To this end, we have used a modified version of the Dickey–Fuller and Phillips–Perron tests proposed by [Ng and Perron \(2001\)](#), which try to solve the main problems present in these conventional tests for unit roots.

In general, most of the conventional unit root tests suffer from three problems. First, they have low power when the root of the autoregressive polynomial is close to, but less than unit ([DeJong, Nankervis, Savin, & Whiteman, 1992](#)). Second, most of the tests suffer from severe size distortions when the moving-average polynomial of the first differences series has a large negative autoregressive root ([Schwert, 1989](#)). Third, implementing the unit root tests often implies the selection of an autoregressive truncation lag, k , which is strongly associated with size distortions and/or the extent of power loss ([Ng and Perron, 1995](#)).

Trying to address these critiques, [Ng and Perron \(2001\)](#) have proposed a methodology that would be robust against the three problems quoted above. This consists of a class of modified tests, $\bar{M}Z_{\alpha}^{GLS}$, $\bar{M}SB^{GLS}$ and $\bar{M}Z_t^{GLS}$, originally developed in [Stock \(1999\)](#) as M tests, with GLS detrending of the data as proposed in [Elliot, Rothenberg, and Stock \(1996\)](#). In addition, [Ng and Perron \(2001\)](#) have proposed a similar procedure that corrects the problems associated with the standard Augmented Dickey–Fuller test, ADF GLS. In all cases, a Modified Akaike Information Criteria (MAIC) is used to select the autoregressive truncation lag, k , as proposed in [Perron and Ng \(1996\)](#).²

[Table 1](#) shows the results of Ng and Perron tests. As shown in the table, the null hypothesis of non-stationarity for the two series in levels cannot be rejected, independently of the test, whereas the existence of two unit roots is clearly rejected at the usual significance levels for the nominal long-term interest rate, but it cannot be rejected for the nominal short-term interest rate (except for the ADF^{GLS} at the 10% significance level). Therefore, according to the results of these tests, $bonds_t$ would be I(1), but cmr_t could be I(2) or I(1).

A potential difficulty in assessing the time series properties of monetary and financial variables is that they can be subject to potential structural breaks in the form of infrequent changes in the mean or the drift of the series, due to exogenous shocks or changes in the policy regime. Hence, in order to provide further evidence on the degree of integration of cmr_t , we have also applied the Perron–Rodríguez test ([Perron and Rodríguez, 2003](#)) for a unit root in the presence of a one time change in the trend function.

[Perron and Rodríguez \(2003\)](#) extend the tests for a unit root analyzed by [Ng and Perron \(2001\)](#) to the case where a change in the trend function is allowed to occur at an unknown time, T_B . The results are presented in [Table 2](#) for the case where the break date is selected minimizing the tests, as suggested by [Perron and Rodríguez \(2003\)](#). The tests show a rejection of the null hypothesis of unit root for Δcmr_t (the two unit roots is rejected), but there is no evidence against the unit root for the variable in levels, cmr_t .

Next, we have applied the tests of threshold cointegration proposed by [Hansen and Seo \(2002\)](#), namely, $\sup LM^0$ (for a given $l=1$) and $\sup LM$ (for an estimated β). In both cases, the p -values are

² See [Ng and Perron \(2001\)](#) and [Perron and Ng \(1996\)](#) for a detailed description of these tests and the MAIC information criteria.

Table 1
Ng–Perron tests for a unit root

I(2) vs. I(1)		Case: $p=0; \bar{c}=-7.0$				
Variable	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	ADF^{GLS}		
$\Delta bonds_t$	-67.76***	-5.88***	0.08***	-6.24***		
Δcmr_t	-3.97	-1.40	0.353	-1.73*		
I(1) vs. I(0)		Case: $p=1; \bar{c}=-13.5$				
Variable	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	ADF^{GLS}		
$bonds_t$	-6.85	-1.79	0.261	-1.76		
cmr_t	-7.05	-1.79	0.254	-1.80		
Critical values:	Case: $p=0, \bar{c}=-7.0$			Case: $p=1, \bar{c}=-13.5$		
	10%	5%	1%	10%	5%	1%
$\bar{M}Z_{\alpha}^{GLS}$	-5.7	-8.1	-13.8	-14.2	-17.3	-23.8
$\bar{M}SB^{GLS}$	0.275	0.233	0.174	0.185	0.168	0.143
$\bar{M}Z_t^{GLS}; ADF^{GLS}$	-1.62	-1.98	-2.58	-2.62	-2.91	-3.42

*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

The MAIC information criteria are used to select the autoregressive truncation lag, k , as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), Table 1.

calculated using a parametric bootstrap method (with 5000 simulation replications), as proposed by Hansen and Seo (2002). To select the lag length of the VAR, we have used the AIC and BIC criteria, both of them leading to $l=2$. We also report the results for $l=1$ for the sake of comparison. The results of the tests are reported in Table 3.

According to the supLM⁰ test (both for $l=1$ and $l=2$), when β is known, the null hypothesis of linear cointegration is not rejected. When β is estimated rather than fixed, the supLM test statistic also does not

Table 2
Perron–Rodríguez tests for a unit root in the presence of a one time change in the trend function

Variable	$\bar{M}Z_{\alpha}^{GLS}$	k	T_B	$\bar{M}Z_t^{GLS}$	k	T_B	$\bar{M}SB^{GLS}$	k	T_B	ADF^{GLS}	k
Δcmr_t	-73.79***	3	83:9	-6.06***	3	83:9	0.082***	14	83:9	-6.92***	3
cmr_t	-23.00	1	83:9	-3.38	1	83:9	0.147	14	83:9	-3.45	1
Critical values:	10%			5%			1%				
	$\bar{M}Z_{\alpha}^{GLS}$	23.7			-27.1			-33.5			
	$\bar{M}SB^{GLS}$	0.143			0.135			0.121			
	$\bar{M}Z_t^{GLS}$	-3.42			-3.64			-4.07			
ADF^{GLS}	-3.62			-3.90			-4.48				

*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

The MAIC information criteria are used to select the autoregressive truncation lag, k , as proposed in Perron and Rodríguez (2003), Table 1(a), Model II, $T=200$, MAIC criteria.

For the applications, we impose a minimal value $k=1$, and $c=-22.5$.

Table 3
Tests for threshold cointegration

	supLM ⁰		supLM	
	$\beta = 1$		$\hat{\beta}$ estimated	
	$l=1$	$l=2$	$l=1$	$l=2$
Test statistic value	18.56	20.69	14.10	14.95
Bootstrap 5% critical value	21.29	26.99	20.60	26.40
Bootstrap p -values	0.143	0.335	0.441	0.803
Threshold parameter ($\hat{\gamma}$)	0.41	0.41	0.59	0.59
Cointegrating vector estimate ($\hat{\beta}$)	–	–	0.85	0.85

point to the rejection of the null. Consequently, the evidence presented in Table 3 is not supportive of threshold cointegration.³

4. Conclusions

In this paper, we have investigated the possibility that a nonlinear model might provide a better empirical description of the term structure model of interest rates. Specifically, we use a new approach developed by Hansen and Seo (2002), based on a threshold cointegration model that considers the possibility of a nonlinear long-run relationship between long and short interest rates. This method is applied to test the Spanish term structure of interest rates during the period 1980:1–2002:12.

According to the results, when the long-run parameter is known ($\beta = 1$), the null hypothesis of linear cointegration is not rejected. If instead of fixing β we estimate it freely, the results are similar. Therefore, the evidence presented does not support threshold cointegration, so that a linear cointegration model would provide an adequate empirical description for the Spanish term structure of interest rate.

References

- Andrews, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point. *Econometrica*, 61, 821–856.
- Balke, N. S., & Fomby, T. B. (1997). Threshold cointegration. *International Economic Review*, 38, 627–645.
- Bank of Spain, 2003, Statistical Bulletin, February, historical series on internet.
- Camarero, M., & Tamarit, C. (2002). Instability tests in cointegration relationships. An application to the term structure of interest rates. *Economic Modelling*, 19, 783–799.
- Campbell, J. Y., & Shiller, R. (1987). Cointegration and tests of present value models. *Journal of Political Economy*, 95, 1062–1088.
- Cuthbertson, K. (1996). The expectations hypothesis of the term structure: the UK interbank market. *Economic Journal*, 106, 578–592.
- DeJong, D. N. J., Nankervis, J. C., Savin, N. E., & Whiteman, C. H. (1992). Integration versus trend stationary in time series. *Econometrica*, 60, 423–433.

³ Since the definition of the long-term interest rate changes after 1993, an anonymous referee suggested performing separate tests, before and after that date. Even though they should be taken with care, given the small number of observations available, the results (available from the author upon request) are similar, both for the whole sample, and the two subsamples.

- Drifill, J., Psaradakis, Z., & Zola, M. (1997). A reconciliation of some paradoxical empirical results on the expectations model of the term structure. *Oxford Bulletin of Economics and Statistics*, 59, 29–42.
- Elliot, G., Rothenberg, T. J., & Stock, J. H. (1996). Efficient test for an autoregressive unit root. *Econometrica*, 64, 813–836.
- Engsted, T., & Tanggaard, C. (1994). A cointegration analysis of Danish zero-coupon bond yields. *Applied Financial Economics*, 4, 265–278.
- Hall, S. G., Anderson, H. M., & Granger, C. W. J. (1992). A cointegration analysis of treasury bill yields. *Review of Economics and Statistics*, 74, 116–126.
- Hamilton, J. D. (1988). Rational-expectations econometric analysis of changes in regime. An investigation of the terms structure of interest rates. *Journal of Economic Dynamics and Control*, 12, 385–423.
- Hansen, B. E. (1996). Inference when a Nuisance Parameter is not identified under the null hypothesis. *Econometrica*, 64, 413–430.
- Hansen, B. E., & Seo, B. (2002). Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics*, 110, 293–318.
- Lo, M., & Zivot, E. (2001). Threshold cointegration and nonlinear adjustment to the law of one price. *Macroeconomic Dynamics*, 5, 533–576.
- Ng, S., & Perron, P. (1995). Unit root tests in ARMA models with data dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association*, 90, 268–281.
- Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69, 1529–1554.
- Perron, P., & Ng, S. (1996). Useful modifications to some unit root tests with dependent errors and their local asymptotic properties. *Review of Economic Studies*, 63, 435–465.
- Perron, P., & Rodriguez, G. (2003). GLS detrending, efficient unit root tests and structural change. *Journal of Econometrics*, 115, 1–27.
- Schwert, G. W. (1989). Tests for unit roots: a Monte Carlo investigation. *Journal of Business and Economic Statistics*, 7, 147–159.
- Seo, B. (2003). Nonlinear mean reversion in the term structure of interest rates. *Journal of Economic Dynamics and Control*, 27, 2243–2265.
- Shiller, R. J. (1979). The volatility of long-term interest rates and expectations models of the term structure. *Journal of Political Economy*, 87, 1190–1219.
- Simon, D. (1990). Expectations and the treasury bill-Fed funds rate spread over recent monetary policy regimes. *Journal of Finance*, 65, 567–577.
- Stock, J. H. (1999). A class of tests for integration and cointegration. In R. F. Engle, & H. White (Eds.), *Cointegration, Causality and Forecasting. A festschrift in Honour of Clive W.J. Granger* (pp. 37–167). Oxford University Press.
- Stock, J. H., & Watson, M. W. (1988). Testing for common trends. *Journal of the American Statistical Association*, 83, 1097–1107.
- Tsay, R. S. (1998). Testing and modeling multivariate threshold models. *Journal of the American Statistical Association*, 93, 1188–1198.