Induced EMF in a solenoid: a simple quantitative verification of Faraday's law

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A previous paper (Bisquert *et al* 1990) proposed some laboratory experiments for studying the oscillations of a magnet in a magnetic field. The aim of the present note is to discuss a simple version of a classical experiment which allows a quantitative testing of Faraday's law of magnetic induction: the measurement of the electromotive force (EMF) induced in a solenoid by a time-dependent current carried by another solenoid. One of the main characteristics of the demonstration is that it requires very cheap, portable equipment: a variable AC source, two multimeters and two home-made solenoids. Because of its simplicity and economy of equipment, the experiment could be of use for school and first-year undergraduate courses.

We believe that the introduction of this demonstration is of interest for the following reasons: (i) Faraday's law of induction can present considerable *conceptual difficulty*, and a simple set of measurements (which can be made easily in the laboratory) will help the students' understanding of concepts like magnetic flux and induced EMF, and (ii) many of the examples of this law that appear in high school and first university course textbooks as well as in classroom and laboratory demonstrations are of a purely *qualitative* nature (e.g. a galvanometer deflects when a magnet is moving with respect to a coil connected to the galvanometer). On the other hand, with modest additional work the experiment discussed here may also be employed as a clear example of mutual induction between two electrical circuits.

Fundamentals

Let us consider a (primary) solenoid of N_i turns (circular loops) in two layers. The (average) radius R_i of the solenoid is very small compared with its longitudinal dimension L_i, so that the solenoid can be considered to be 'infinitely long' (see figure 1 for details). Another solenoid (the secondary) is wrapped just around the first one and contains N_e turns arranged also in two layers. The length of the secondary solenoid is approximately half the length of the primary one.

When the primary solenoid carries an electric current I, a magnetic field B appears. This field produces a magnetic flux through the cross section of the secondary solenoid. In order to evaluate this flux, we need to know the magnetic field over this cross section. Because of the geometry shown in figure 1, end-effects can be ignored and the magnetic field can be considered to be constant inside the centre of the long primary solenoid (Tipler 1991):

$$B=\mu_0\frac{N_i}{L_i}I \tag{1}$$

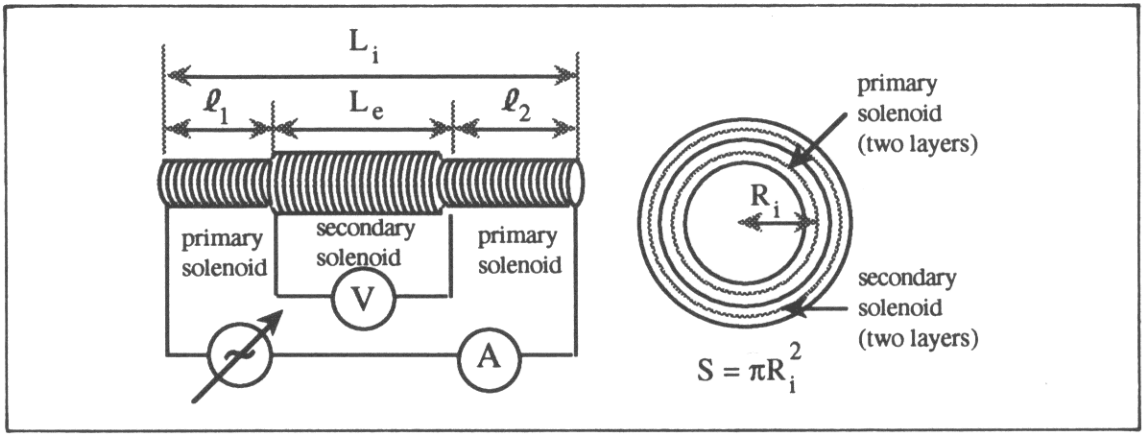

Figure 1. Schematic view of the experimental arrangement. A and V denote the multimeters employed as ammeter and voltmeter respectively. A variable voltage AC source is used. The solenoid dimensions are as follows: $L_i = 102.00 \pm 0.05$ mm, $L_e = 57.50 \pm 0.05$ mm, $l_1 = 19.45 \pm 0.05$ mm, $l_2 = 25.05 \pm 0.05$ mm and $R_i = 3.95 \pm 0.10$ mm.

where $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2} is the permeability of free space. The magnetic flux ϕ through the cross section of the secondary solenoid can then be evaluated as the product of the field in equation (1) and the surface area S over which this equation is approximately valid. Since the primary solenoid is arranged in two layers, we can take S to be πR_i^2 where R_i is the average radius of the primary solenoid (see figure 1). Thus the magnetic flux is (Roller and Blum 1986)

$$\phi = \boldsymbol{B} \cdot \boldsymbol{S} = \mu_0 \frac{N_i}{L_i} IS. \tag{2}$$

Now, if the current I is time-dependent, the change of ϕ with time will induce an EMF in the secondary solenoid. In particular, if the primary solenoid carries an alternating current of angular frequency ω,

$$I = I_0 \sin(\omega t) \tag{3}$$

then the induced EMF is

$$\varepsilon = \left| \frac{\mathrm{d}\phi}{\mathrm{d}t} \right| = \mu_0 N_e \frac{N_i}{L_i} S \frac{\mathrm{d}I}{\mathrm{d}t}$$

$$= \mu_0 N_e \frac{N_i}{L_i} S I_0 \omega \cos(\omega t) \tag{4}$$

or, in terms of root mean square (rms) values (Tipler 1991),

$$\varepsilon_{rms} = \mu_0 N_e \frac{N_i}{L_i} S \omega I_{rms}. \tag{5}$$

where ε_{rms} is the rms value of ε in equation (4) and I_{rms} is the rms value of the electric current in equation (3). A plot of ε_{rms} versus I_{rms} then gives a straight line whose slope depends on the frequency ω, the geometrical characteristics of the primary solenoid (S and L_i) and the number of turns in the two solenoids (N_i and N_e).

Since the multimeters measure rms values at the frequency (50 Hz) used in this experiment, it is clear that we can readily determine μ_0 from an experiment like that in figure 1. The comparison between this μ_0 and the exact value will give us a quantitative test of Faraday's law of induction. Thus, obtaining μ_0 is just an *intermediate step* in checking Faraday's law. (Let us note in passing that there are other methods of obtaining μ_0, e.g. by measuring the magnetic field within a solenoid or in the centre of a single circular loop as a function of the electric current (Nolan and Bigliani 1982) but the measurement of magnetic fields usually requires more sophisticated experimental equipment than that used in this experiment.)

Experimental arrangement

The experimental set-up is shown in figure 1. The primary solenoid is a home-made arrangement prepared by wrapping around a pencil $N_i = 610 \pm 1$ turns of winding wire of diameter $e = 0.30 \pm 0.05$ mm in two layers. A cylindrical solenoid of length $L_i = 102.00 \pm 0.05$ mm and average radius $R_i = 3.95 \pm 0.10$ mm is then obtained. The presence of the pencil contributes to the mechanical rigidity required by the set-up, and does not affect the measurements in a significant way (see Results and discussion below). The secondary solenoid is constructed by wrapping $N_e = 315 \pm 1$ turns of the same wire (also in two layers) immediately around the primary solenoid. The secondary solenoid begins at a distance of about 20 mm from one end of the primary solenoid and its length is $L_e = 57.50 \pm 0.05$ mm.

The induced EMF is measured as shown in the experimental arrangement of figure 1, which contains a variable 0–50 V, 10 A AC/DC Eimler-Basanta-Haase source and two student laboratory multimeters (Metex M-3620) employed as voltmeter and ammeter. Note that the ohmic drop in the secondary solenoid is negligible because of the high input impedance of the voltmeter, and thus the voltmeter readings correspond to the induced EMF in the secondary solenoid.

The experimental results are fitted to equation (5) expressed in the form

$$\varepsilon_{rms} = C\mu_0 I_{rms} \quad (6)$$

where $C = N_e N_i S\omega / L_i$. A simple treatment of errors (Squires 1985) leads in our case to the value

$$C = (290 \pm 5) \times 10^2 \text{ V A N}^{-1}. \quad (7)$$

Results and discussion

A typical plot of the induced EMF in the secondary solenoid as a function of the current imposed through the primary solenoid is shown in figure 2. (Currents greater than 1 A should not be employed in order to avoid overheating of the solenoid.) The experimental points are fitted to a straight line by standard least-squares fitting. A correlation coefficient $r = 0.999\,986$ is obtained for this case. From the resulting slope, $C\mu_0 = (36.23 \pm 0.03) \times 10^{-3}$ V A^{-1}, and the value of C given in equation (7), we obtain

$$\mu_0(\text{exp}) = (12.5 \pm 0.2) \times 10^{-7} \text{ N A}^{-2}. \quad (8)$$

After carrying out a series of experiments similar to that shown in figure 2, we observed that the result in equation (8) is reasonably reproducible (the slopes found agree within a few parts per thousand). The experimental value in equation (8) can now be compared with the exact value (Cohen and Taylor 1989):

$$\mu_0(\text{exact}) = 12.566\,370 \ldots \times 10^{-7} \text{ N A}^{-2}. \quad (9)$$

The difference observed between these two values lies within the range of the experimental uncertainty (about 2% for this experiment).

In the measurements described above, the ratio of solenoid lengths was $L_e/L_i = 0.56$. In an attempt to study the effect of this ratio on the measured value of μ_0, we have conducted an additional series of experiments with a (long) secondary solenoid

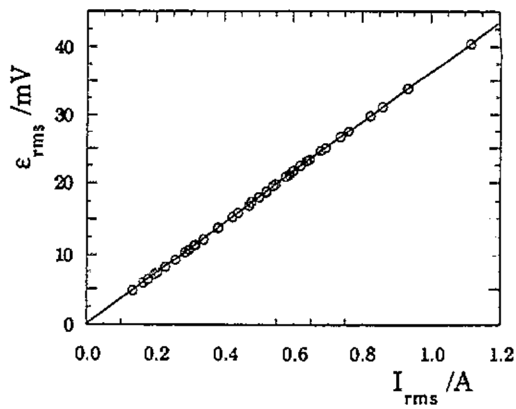

Figure 2. Induced EMF (ε_{rms}) in the secondary solenoid versus current (I_{rms}) through the primary solenoid. The circles correspond to experimental points.

for which $L_e/L_i = 0.86$, $L_i = 95.37 \pm 0.06$ mm and $R_i = 4.065 \pm 0.011$ mm. We obtained $\mu_0(\text{exp}) = (12.6 \pm 0.2) \times 10^{-7}$ N A^{-2} for this case, a value which can be compared with that given in equation (8). (Note that the assumption of 'long solenoid' continues to be valid as long as $L_e, L_i \gg R_i$.)

Therefore, we conclude that the demonstration discussed is, in spite of its simplicity, capable of giving a quantitative verification of Faraday's law of induction. The approximations introduced seem very reasonable. Indeed, a more rigorous calculation of the magnetic field within the solenoid (Tipler 1991) shows that, due to the small value attained by the quotient (radius/length) for the solenoid used, the introduction of the idealized model of an 'infinitely long' solenoid leads to errors not significantly higher than the uncertainty associated with the experimental value of μ_0. Also, the assumption that the entire variation of the flux in the primary solenoid contributes to the EMF induced in the secondary one seems to be confirmed by the experimental results.

Finally, let us mention that although the value μ_0 corresponds strictly to the permeability of free space, it is clear that none of the components of the pencil shows a magnetic permeability significantly different from μ_0 within the precision of the experiment (Tipler 1991). Other common materials were also used (e.g. we employed an aluminium bar in a series of experiments) and no significant differences were observed. The measurement of the magnetic permeabilities of such materials would require a more specific experimental set-up (e.g. a Rowland ring) as well as laboratory equipment allowing for more accurate electrical measurements (e.g. a ballistic galvanometer), questions that are far beyond the objectives of the elementary demonstration discussed here.

References

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