Conjuntos super débilmente compactos y los espacios que generan

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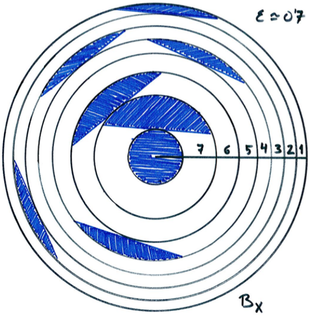

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It turns out that $\mathfrak{W}^{super}$ is a symmetric closed ideal such that

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Besides, the ideal $\mathfrak{W}^{super}$ lacks the factorisation property.

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The relation of equivalence in the ultrapower is $(x_i)_{i \in I} \sim (y_i)_{i \in I}$ if and only if $\lim_{i \in \mathfrak{U}} \|x_i - y_i\| = 0$. Note that it is enough to consider free ultrafilters on $\mathbb{N}$ since weakly compactness is separably determined after the Eberlein-Šmulyan theorem.

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(vi) there is an equivalent norm $|||\cdot|||$ on X which is uniformly convex on K.

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Among the interesting properties enjoyed by the class of super weakly compact convex sets we have:

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4. they have *normal structure* (and so the fixed point property for non-expansive maps) after a renorming of the ambient space;
5. of course, a Banach space is super-reflexive if and only if its unit ball is super weakly compact, and a operator $T : Z \to X$ is super weakly compact if and only if $\overline{T(B_Z)}$ is super weakly compact.

Spaces generated by ideals, sets or. . . other spaces

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The spaces enjoying those properties are calle *weakly compactly generated* and the class is denoted WCG.

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However, "space-generation" is more restrictive here as we will see.

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4. The norm is strongly UG smooth if it is H-UG smooth for some bounded and linearly dense subset $H \subset X$.

Generation by several kind of spaces

The relationships among different kinds of generated Banach spaces were clearly exposed by Fabian, Godefroy, Hájek and Zizler in the following result of 2003.

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Then $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv) \Rightarrow (v) \Rightarrow (vi)$. Moreover, no one of these implications can be reversed in general.

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Our proof strongly depends on the symmetry of the ideal $\mathfrak{W}^{super}$.

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Let us mention here that several good properties of the space $L_1(\mu)$ for μ finite can be understood from the fact that it is strongly Hilbert generated.

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On the other hand, there are S^2WCG spaces which are not super-reflexively generated.

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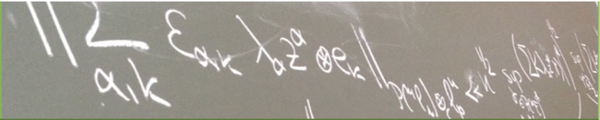
Announcement

NonLinear Functional Analysis



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Conference in Non Linear Functional Analysis

The Conference on Non-Linear Functional Analysis, will be held at the Universitat Politècnica de Valencia (Spain) on **17-20 October 2017**. This is a joint project of the Universitat de València, Universidad de Murcia and Universitat Politècnia de València that includes the 5th Workshop on Functional Analysis and the Workshop on Infinite Dimensional Analysis Valencia 2017. You can find information about previous editions at
http://www.um.es/beca/workshop2016/
and
http://mate.dm.uba.ar/~widaba14/
(the first edition of this one was held in Buenos Aires).

Each day we will have two or three plenary lectures in the morning and after them the two workshops will run in parallel sessions. We intend to have a wide audience with interest in any field in Functional Analysis. Main topics of the Meeting will be Banach space theory, polynomials and multilinear mappings, vector measures and complex analysis in finite or infinite dimensional spaces, but of course contributions in any other area of Functional Analysis will be very much welcome.

You can contact us by sending an e-mail to banach@um.es

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