Algebraic Structures and Modes of Convergence

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Dpto. Análisis Matemático

09 de marzo de 2018

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Previous Concepts

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- *$\mathcal{B}$ is algebrable if $\exists \mathcal{C} \subset \mathcal{A}$ so that $\mathcal{C} \subset \mathcal{B} \cup \{0\}$ and the cardinality of any system of generators of $\mathcal{C}$ is infinite.*

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- *If in addition, $\mathcal{A}$ is a commutative algebra, we say that $\mathcal{B}$ is strongly algebrable if $\mathcal{B} \cup \{0\}$ contains generated algebra which is isomorphic to a free algebra.*

Everywhere Surjective Functions

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Example

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- Take any bijection $\Phi_n : C_n \longrightarrow \mathbb{R}$.
- Define $f : \mathbb{R} \longrightarrow \mathbb{R}$ by

$$f(x) := \begin{cases} \Phi_n(x) & \text{if } x \in C_n, \\ 0 & \text{in other case.} \end{cases}$$

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Theorem (Araújo, Bernal, Muñoz, Prado and Seoane, 2017)

The set of measureable everywhere surjective functions $\mathcal{MES}$ is $\mathfrak{c}$-lineable.

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Theorem (A, B, M, P and S, 2017)

The family of sequences $(f_n)_{n\in\mathbb{N}}$ of Lebesgue measurable functions such that $f_n \longrightarrow 0$ pointwise and $f_n \in \mathcal{MES}$ is $\mathfrak{c}$-lineable.

Measure versus Pointwise a.e. Convergence

Recall that $f_n \longrightarrow f$ in measure if $\forall\, \varepsilon > 0$ we have

$$\mu\left(\{x \in X \ : \ |f_n(x) - f(x)| \geq \varepsilon\}\right) \longrightarrow 0, \quad (n \to \infty).$$

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- Let (p_j) be the increasing sequence of prime numbers
- Consider the functions $f_{n,j}$ given by:

$$f_{n,j}(x) = x^{\log(p_j)} T_n(x),$$

where $T_n(x) = \chi_{\left[\frac{j}{2^k}, \frac{j+1}{2^k}\right]}$ with $n = 2^k + j$, $0 \leq j < 2^k$

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- Consider now the monic monomial m given by

$$m(x_1, x_2, \ldots, x_s) = \prod_{i=1}^{s} x_i^{\alpha_i}.$$

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- Define the index as $\operatorname{idx}(m) = \prod_{i=1}^{s} p_i^{\alpha_i}$.

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- Evaluating we get $m(f_{n,j_1}, f_{n,j_2}, \ldots, f_{n,j_s}) = x^{\log(\mathrm{idx}(m))}\chi_{[j/2^k,(j+1)/2^k]}(x)$

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- Take $X = L_0^{\mathbb{N}}$, $B = \widetilde{L} := \{\Phi = (f_n) \in L_0^{\mathbb{N}} \ : \ \exists N = N(\Phi) \in \mathbb{N} \mid f_n = 0 \ \forall n \geq N\}$

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$$f_n \not\longrightarrow 0 \text{ in } \|\cdot\|_{L^1} \text{ norm}$$

is $\mathfrak{c}$*-lineable.*

Uniformly versus L^1 Norm Convergence

Sketch of the Proof

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- Let $f_n = \frac{1}{n}\chi_{[n,2n]}$.

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Uniformly versus L^1 Norm Convergence

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Theorem (Calderón, G.M. and Prado)

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is strongly-algebrable.

Uniformly versus L^1 Norm Convergence

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Uniformly versus L^1 Norm Convergence

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- Consider the functions $f_{n,j}$ given by:

$$f_{n,j}(x) = \frac{1}{n^{\log(p_j)}}\chi_{[0,e^n]}(x),$$

Uniformly versus L^1 Norm Convergence

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- Evaluating we get

$$m(f_{n,j_1}, f_{n,j_2}, \ldots, f_{n,j_s}) = \frac{1}{x^{\log(\mathrm{idx}(m))}} \chi_{[0,e^n]}(x)$$

Uniformly versus L^1 Norm Convergence

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- Consider the algebraic combination p given by

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Uniformly versus L^1 Norm Convergence

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Uniformly versus L^1 Norm Convergence

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- $p(f_{n,j_1}, f_{n,j_2}, \ldots, f_{n,j_s}) = 0$ if and only if $\lambda_t = 0$.
- Thus, A is strongly-algebrable.

Thank you very much for your attention