Algebraic Structures and Modes of Convergence

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09 de marzo de 2018



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- \mathcal{B} is algebrable if $\exists \mathcal{C} \subset \mathcal{A}$ so that $\mathcal{C} \subset \mathcal{B} \cup \{0\}$ and the cardinality of any system of generators of \mathcal{C} is infinite.
- If in addition, \mathcal{A} is a commutative algebra, we say that \mathcal{B} is strongly algebrable if $\mathcal{B} \cup \{0\}$ contains generated algebra which is isomorphic to a free algebra.



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EXAMPLE

• Let $\{I_n\}_{n\in\mathbb{N}}=\{(a_n,b_n)\}_{n\in\mathbb{N}}$ where $a_n,b_n\in\mathbb{Q}\ \forall n\in\mathbb{N}.$

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The set of measureable everywhere surjective functions \mathcal{MES} is $\mathfrak{c}\text{-lineable}.$

THEOREM (A, B, M, P and S, 2017)

The family of sequences $(f_n)_{n\in\mathbb{N}}$ of Lebesgue measurable functions such that $f_n \longrightarrow 0$ pointwise and $f_n \in \mathcal{MES}$ is \mathfrak{c} -lineable.

Measure versus Pointwise a.e. Convergence

Recall that $f_n \longrightarrow f$ in measure if $\forall \varepsilon > 0$ we have

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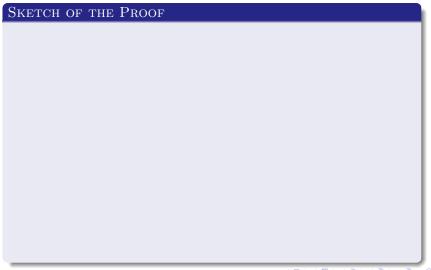
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where
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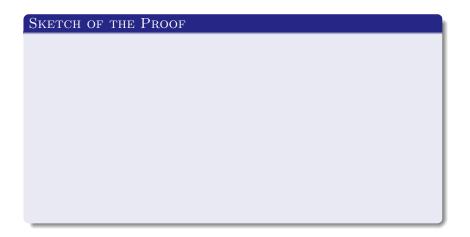
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• Define the index as $idx(m) = \prod_{i=1}^{s} p_i^{\alpha_i}$.





Measure versus Pointwise A.E. Convergence

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$$m(f_{n,j_1},f_{n,j_2},\ldots,f_{n,j_s})=x^{\log(\mathrm{idx}(m))}\chi_{[j/2^k,(j+1)/2^k]}(x)$$

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• Evaluating we get $m(f_{n,j_1},f_{n,j_2},\ldots,f_{n,j_s}) = \frac{1}{x^{\log(\operatorname{idx}(m))}} \chi_{[0,e^n]}(x)$

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- Thus, A is strongly-algebrable.

Thank you very much for your attention