Linear dynamics: Somewhere dense orbits are everywhere dense!

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This course will deal with some surprising results on the dynamics of a linear and continuous map (from now on, *operator*) $T : X \to X$ on a general topological vector space X.

We recall that the *orbit* of $x \in X$ under T is

$$\mathrm{Orb}(x, T) = \{x, Tx, T^2x, \ldots\},$$

and x is a *hypercyclic vector* for T (in this case, T is called a *hypercyclic operator*) if $\mathrm{Orb}(x, T)$ is dense in X, i.e., $\overline{\mathrm{Orb}(x, T)} = X$.

We will consider the following problems, which, a priori, do not involve linearity.

- If T has a dense orbit, does then every power T^p also have a dense orbit?
- Suppose that the union of a finite collection of orbits is dense. Will then at least one of these orbits be actually dense?
- If an orbit is somewhere dense, is it (everywhere) dense? We recall that a set is called somewhere dense if its closure contains a nonempty open set.

Each of these questions has a negative answer for arbitrary, nonlinear maps. It is therefore even more surprising that they all have a positive answer for (linear) operators, and that without any restrictions. The proofs depend in a crucial way on connectedness arguments.

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References

[1] S.I. Ansari, Hypercyclic and cyclic vectors, J. Funct. Anal. **128** (1995), 374–383.

[2] J. Banks, Regular periodic decompositions for topologically transitive maps, Ergodic Theory Dynam. Systems **17** (1997), no. 3, 505–529.

[3] F. Bayart and É. Matheron, *Dynamics of linear operators*. Cambridge Tracts in Mathematics, Vol. 179, Cambridge University Press, Cambridge 2009.

[4] P.S. Bourdon and N.S. Feldman, Somewhere dense orbits are everywhere dense, Indiana Univ. Math. J. **52** (2003), no. 3, 811–819.

[5] G. Costakis, On a conjecture of D. Herrero concerning hypercyclic operators, C.R. Acad. Sci. Paris **330** (2000), 179–182.

[6] K.-G. Grosse-Erdmann and A. Peris Manguillot, *Linear Chaos*. Universitext. Springer, London, 2011.
`http://dx.doi.org/10.1007/978-1-4471-2170-1`

[7] A. Peris, Multihypercyclic operators are hypercyclic, Math. Z. **236** (2001), 779–786.

[8] J. Wengenroth, Hypercyclic operators on non-locally convex spaces, Proc. Amer. Math. Soc. **131** (2003), no. 6, 1759–1761.