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Momentum Strategies using Risk-adjusted Stock Selection Criteria

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Agenda

- **Introduction**
- Stable distribution approach for treatment of non-normal data
- Risk-adjusted criteria for stock selection
- Construction of momentum portfolios
- Results and performance evaluation
- Formulation of the zero-value optimal portfolio problem
- Conclusions

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Momentum profitability poses a strong challenge to the theory of asset pricing – momentum effect is the most challenging asset pricing anomaly

The current research is unable to provide a consistent risk-premium based explanation*

- A momentum effect captures the short-term (6 to 12 months) return continuation effect that stocks with high returns over the past three to 12 months tend to outperform in the future (Jegadeesh & Titman, 1993).
- Very simple trading strategy – portfolio is constructed based on cumulative return criterion over certain time-horizon
- Historically momentum strategy earned profits of about 1% per month over the following 12 months.
- The profitability cannot be explained with the existing multi-factor models and macroeconomic-based risk explanations

*) The finding that returns exhibit momentum behavior at intermediate horizons is at odds with market efficiency.

Our approach enables the modification of the decision criteria for portfolio construction and allows treatment of non-Gaussian returns

We extend the momentum strategy methodology in several ways

- To reflect risk-return trade-off in portfolio selection, we use of risk-adjusted criterion instead of return only criterion for portfolio construction
- Use of daily data rather than monthly data, facilitating better capture of distributional properties of the data
- Risk-return criteria have form of risk-return ratios compliant with coherent risk measures
- Risk-return criteria are applicable when stock returns are not normally distributed

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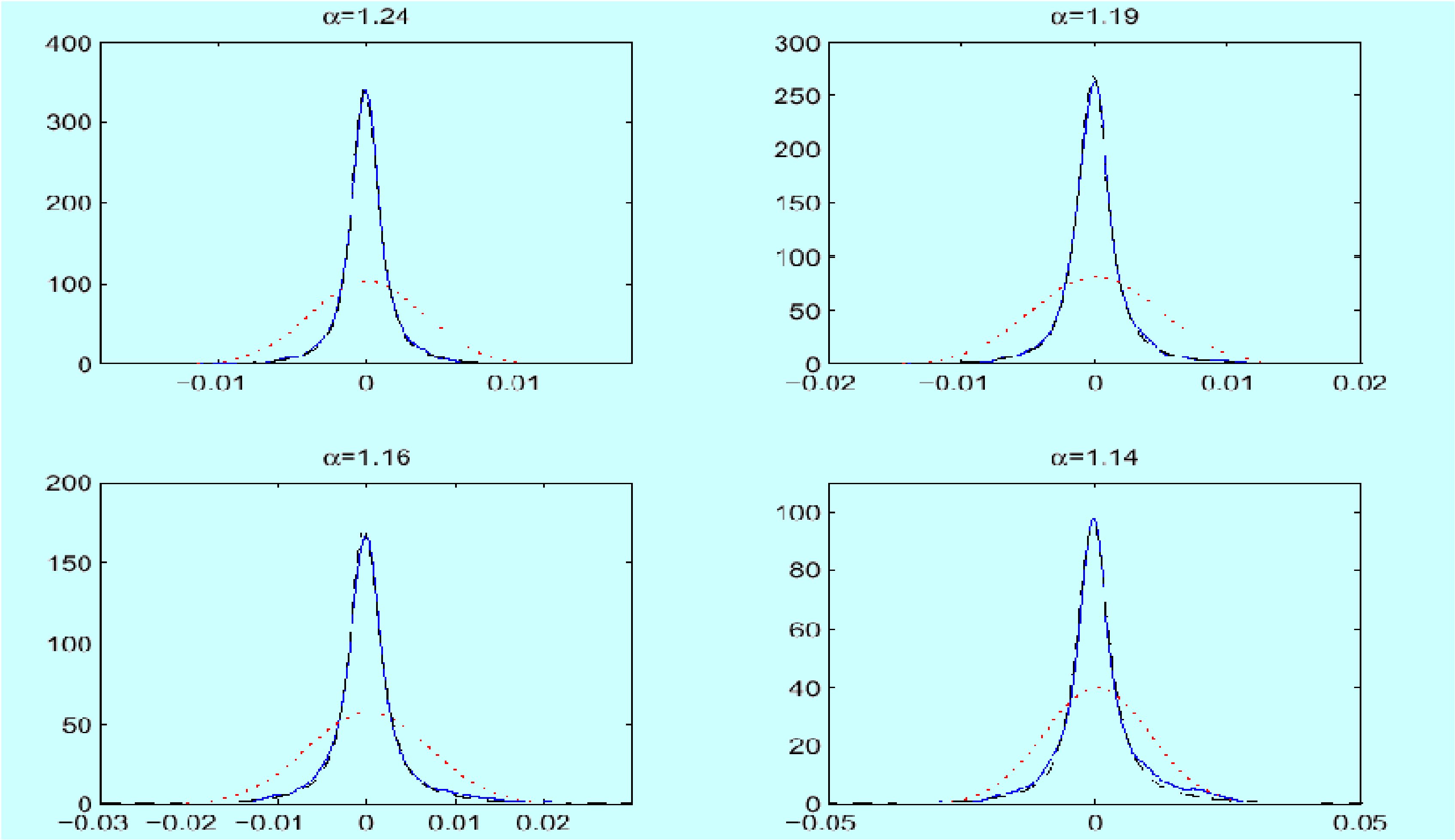
Empirical properties of financial time series show stylized facts that deviate from normal Gaussian assumption

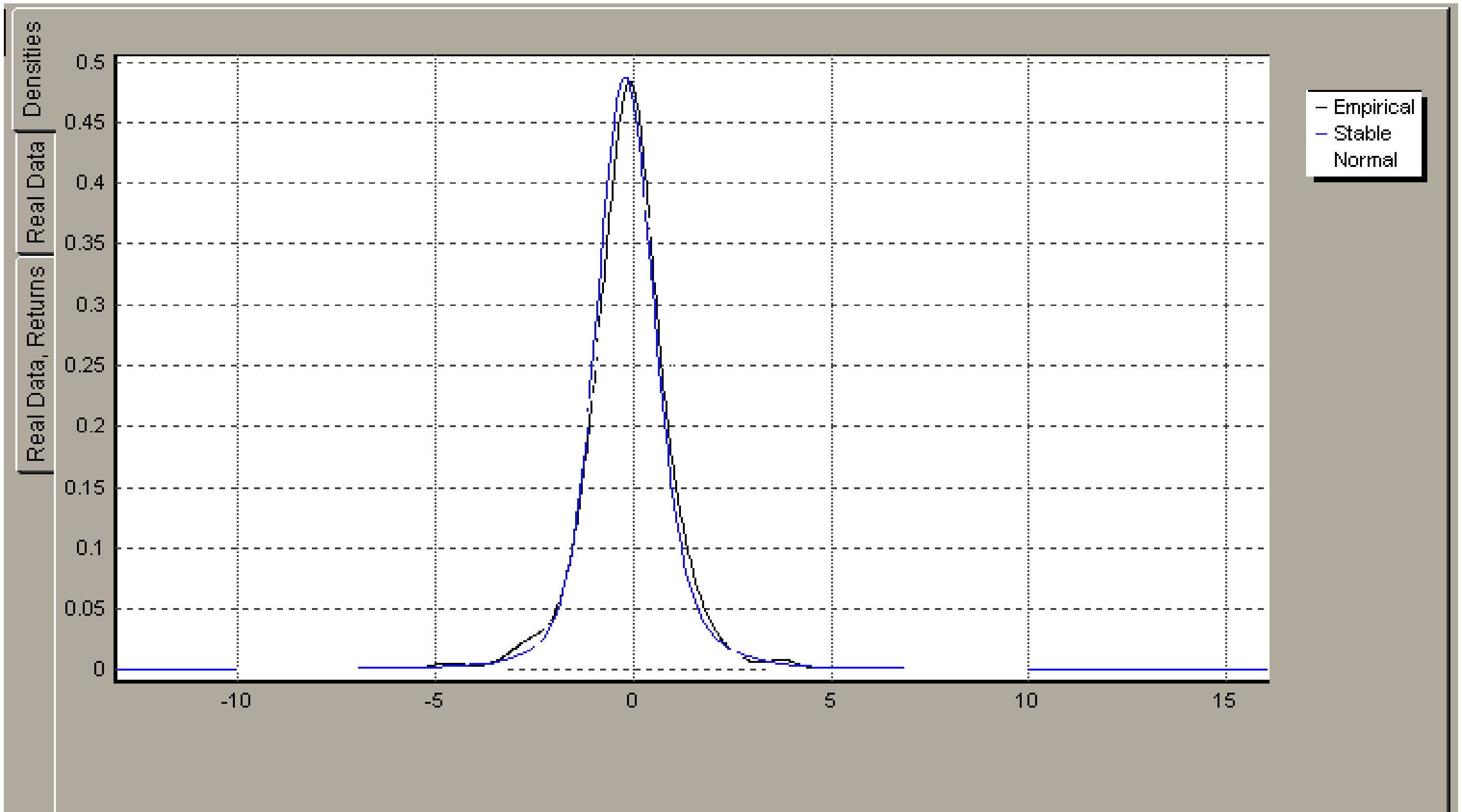
Gaussian distribution is not a realistic assumption for stock returns

- High empirical kurtosis $\Rightarrow$ heavy tailedness
- Asymmetric empirical distribution
- Slowly decaying correlation of squared returns $\Rightarrow$ long-range dependence
- Heteroskedasticity (volatility clustering)

Example 1: Empirical PDF of 10, 15, 30 and 60-minutes returns of Deutsche Bank stock prices; normal and stable fits

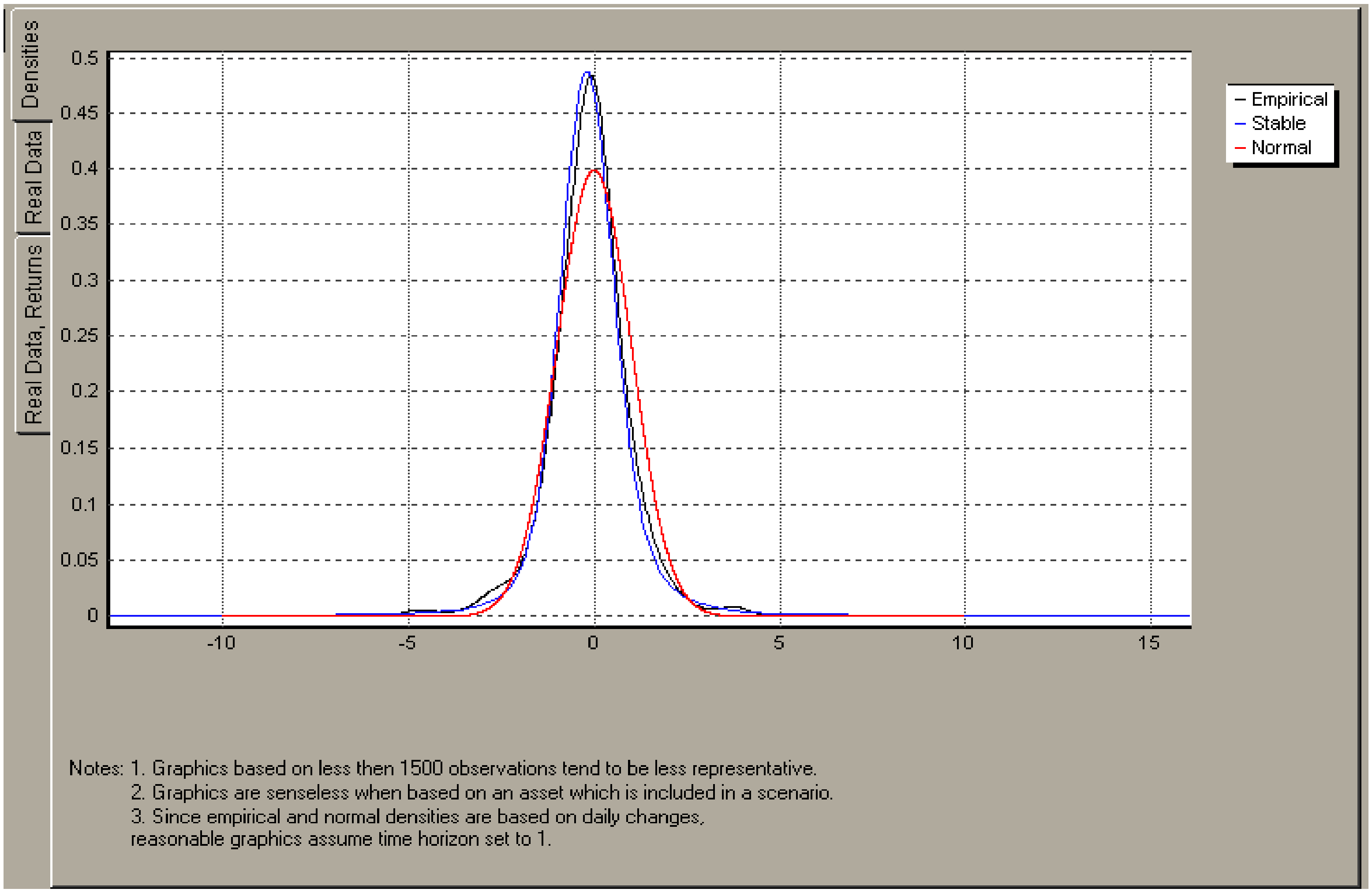


Example 2: Empirical and Stable Densities of DAX 30



Notes: 1. Graphics based on less then 1500 observations tend to be less representative.
2. Graphics are senseless when based on an asset which is included in a scenario.
3. Since empirical and normal densities are based on daily changes, reasonable graphics assume time horizon set to 1.

Example 2 (cont.): Comparison between Empirical and Stable Densities of DAX 30



Stable distributions and idea of stochastic subordination enable postulation of richer models for price behaviour

Subordinated model for stock prices (Bochner (1955), Mandelbrot & Taylor (1967), Clark (1973), Mandelbrot et al. (1998))

- Stochastically subordinated stock price

$$Z(t) = S \circ T(t) = S(T(t))$$

- α-stable processes clock effects (on all time scales)
- Assumptions:

  S(t) and T(t) independent,

  S(t): $R^{+}_{0} \to R$,

  T(t): $R^{+}_{0} \to R^{+}_{0}$, T(0) = 0 a.s., non-decreasing paths

- Where do heavy tails and long memory come from?
- What is the probability structure of S(t) and T(t)?

Features of the subordinated model for stock prices

Subordinated model expresses the original idea: (log) prices follow Brownian motion under a suitable transformation of the time scale.[2]

- T(*t*) = No. of transactions up to time t
- S(*t*) = tick-by-tick price

Features of the subordinated model

- Heavy tails, with the stability index depending on time
- Market clock effects (on all time scales)
- Long-range dependence

Heavy tailedness comes from S(t) and
LRD comes from T(t)

[2]) On days when no new information is available, trading is slow, and the price process evolves slowly. On days when new information violates old expectations, trading is brisk, and the price process evolves much faster. (Clark, 1973)

Application of the stable models extends the common applications in assessment of market and credit risk, portfolio optimization and forecasting

Extended stable GARCH (Phi-Alpha) models are developed for range of applications (Cognity Integrated Risk Management System, FinAnalytica, Inc.)

- Value at Risk Analysis
- Credit Risk Modeling
- Portfolio Optimization
- Forecasting / (Factor models)
- Asset Allocation

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Setting a key decision on the momentum strategy within risk-return framework, several issues can be addressed simultaneously

What is the aim of the risk-adjusted decision rule?

- Align conceptual risk-return framework of investment strategy with the momentum trading decision rule (e.g. capture risk-return profile of stocks)
- Allow treatment of the non-Gaussian data which was disregarded in previous and contemporary studies
- Apply various risk measures within risk-adjusted criterion that pay more attention to the left-tail of the return-distribution
- Obtain balanced risk-return performance
- Use risk-adjusted criteria in portfolio optimization problem and devise alternative optimized-weighted strategy

Risk-adjusted criterion is compared to cumulative return benchmark and takes form of a risk-return ratio

What we would like to examine?

- Whether the risk-adjusted criteria can generate more profitable strategies than those based on simple cumulative return criterion
- What is the appropriate risk measure embedded in a risk-return criterion that obtains the best results (e.g., Variance, ETL/CVaR)
- Evaluate and compare performance of ratios based on different distributional assumption and measures of risk
- Which criterion gives the most robust strategy regarding transaction costs ?
- What is the marginal benefit of the optimized-weighted strategy?

Expected Tail Loss (ETL) is a coherent risk measure and superior to VaR

ETL (Expected Tail Loss, or CVaR) is the average loss below VaR

- Intuitive downside risk measure that is coherent
- Remarkable portfolio optimization properties[4]

Stable GARCH models for ETL (Phi-Alpha ETL), FinAnalytica's Approach

- Fit multivariate stable distribution model
- With stable distribution volatility clustering model
- Generate scenarios and compute ETL=CVaR

[4] Rockafellar & Uryasev (2000), Journal of Risk

Momentum methodology is extended by applying stock selection criteria in risk-return framework

We analyze several risk-return ratios that differ in treatment of risk and distributional behavior of data

- The **Sharpe Ratio** is the ratio between the expected excess return and its standard deviation:

$$\rho(r) = \frac{E(r - r_f)}{STD_{(r - r_f)}}$$

- **STARR$_{(1-\alpha)100\%}$** (CVaR$_{(1-\alpha)100\%}$ Ratio) is the ratio between the expected excess return and its conditional value at risk:

$$\rho(r) = \frac{E(r - r_f)}{CVaR_{(1-\alpha)100\%}(r - r_f)}$$

Alternative risk-return ratios use ETL as a measure of risk and are able to capture heavy tail behaviour in the data

Alternative R-Ratio is the ratio of the expected tail return above the level, divided by the expected tail loss.

- **R-ratio** with parameters *α* and *β* in [0,1].

$$\rho(r) = \frac{ETL_{\alpha 100\%}(r_f - r)}{ETL_{\beta 100\%}(r - r_f)}$$

- Here, if *r* is a return on a portfolio or asset, and *ETLα(r)* is defined as

  $ETL_{\alpha 100\%}(r) = E(l|l > VaR_{(1-\alpha)100\%}(r))$, *where*

  $CVaR_{(1-\alpha)100\%}(r) = ETL_{\alpha 100\%}(r)$ *(we assume continuous return distr.)*

- The idea behind the R-ratio is to try to simultaneously capture the maximum level of return and get insurance for the maximum loss

- The choice of a specific tail probabilities selects a particular risk measure in the ETL class of measures and reflects the risk and return objectives of an investor

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Momentum portfolios are formed using risk-return criteria and daily data

Data and Methodology

- We use **daily data of 382 stocks included in the S&P Index** in the period January 1, 1992 to December 31, 2003. (stocks with equal and complete return history)

- **Four "J-month/K-month" strategies** based on the ranking and holding periods of 6 and 12 months and **10 criteria** are examined (i.e., **6/6, 6/12, 12/6 and 12/12 strategies, overall 40 strategies**)

- Zero-investment portfolio is constructed at the end of each ranking period by simultaneously selling winners and losers

- 10% of the stocks with the highest value of stock selection criterion in the ranking period constitute winner portfolio (e.g., highest decile) and 10% of the stocks with the lowest values the loser portfolio

- Winner and loser portfolio are equally weighted at formation and held for subsequent K-months of the holding period

Optimization of winner and loser portfolios based on risk-return criteria leads to an optimized-weighted strategy

Portfolio selection and optimization approach follows usual Markowitz (1962) approach with portfolio choice based on reward-risk criteria

- For every risk-return criterion $\rho(\cdot)$, we compute the optimal winner portfolio of the max optimization problem and optimal loser portfolio of the min optimization problem:

$$\max_x \rho(x'r) \quad s.t. \quad \sum_{i=1}^{n} x_i = 1;\ x_i \geq 0;\ \ i = 1,\dots,N$$

$$\min_y \rho(y'r) \quad s.t. \quad \sum_{i=1}^{n} y_i = 1;\ y_i \geq 0;\ \ i = 1,\dots,N$$

- where ρ is the ratio criterion, x_i and y_i are optimized weights in the winner and loser portfolios respectively, and N equals the number of stocks in winner or loser portfolio

Impact of transaction costs on momentum portfolios

Transaction Costs as a cost of implementing a trading strategy

- Korajczyk and Sadka (2004) find for long positions in winner-based strategy that proportional spread costs do not eliminate statistical significance of momentum profits.

Issues in consideration with measuring transaction costs impact

- What realistic model of transaction cost impact to apply?
- Risk and liquidity characteristics of extreme portfolios may have impact on the assumptions of the trading cost model
- Realistic assessment shall focus on the actual turnover of the portfolios – Tradeoff between profitability and turnover
- Adjustment applied at portfolio rebalancing periods.

Momentum profits are adjusted for transaction costs using realistic assumptions

Analysis of transaction costs is based on total trading cost and actual turnover

- We use as estimate of the one-way total trading costs that averages 0.78% of the value of the traded stock (assumption based on Chalmers, Edelen, and Kadlec, 2002)
- We apply an optimization model which estimates net adjusted return of momentum strategy using one-way transaction cost *c*, that is proportional to the actual value of portfolio's long or short position
- We analyze the final wealth of the portfolio over all holding periods
- By **tracking the actual turnover** within the winner and loser portfolio for each ranking and holding period, we obtain more precise estimation of the incurred transaction costs as compared to other methods.

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Performance evaluation of momentum strategies is conducted using different performance measures

Performance evaluation measures

- The performance of strategies is examined using **average returns**, **cumulative realized profits** (accumulated winner – loser spreads) and **independent risk-adjusted performance measure**
- Risk-adjusted performance is evaluated using risk-adjusted measures – Sharpe Ratio and an independent performance ratio
- **Risk-adjusted independent performance measure** is in the form of the $\mathbf{STARR_{99\%}}$, $E(X_t)/CVaR_{99\%}(X_t)$, where X_t is the sequence of the daily winner minus loser spreads in the holding period
- Independent performance measure can use different significance levels of the ETL measure reflecting different risk-return profile objectives and levels of risk-aversion of an investor
- The criterion that obtains the best risk-adjusted performance is the one with the highest value of the independent performance measure.

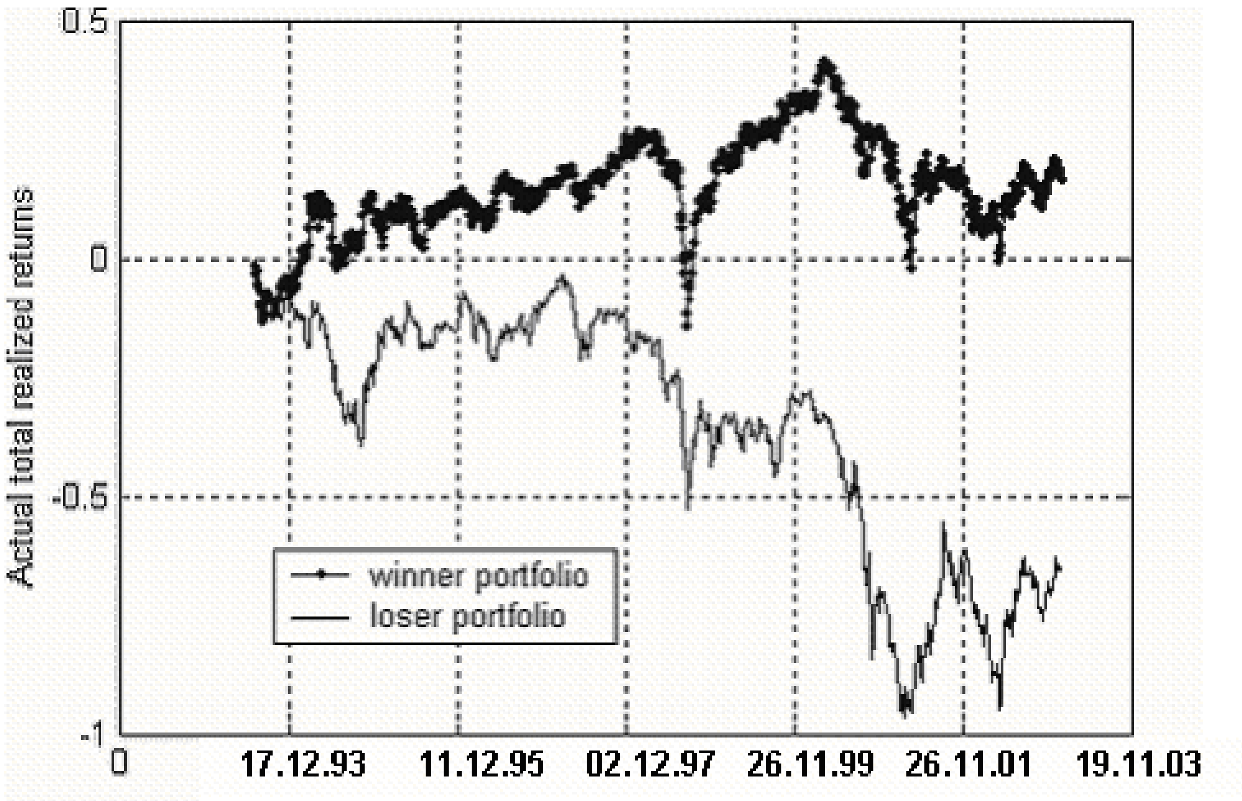
Comparison of equal-weighted and optimized-weighted 6/6 strategy after adjustment for transaction costs shows advantage of using alternative ratio

Analysis of the final wealth of momentum portfolio for equal-weighted and optimized-weighted 6/6 strategy using different risk-return criteria, January 92-December 2003

Risk-Return Ratio	Portfolio Final Wealth	Stock Ranking Criteria		
		Cumulative Return	Sharpe Ratio	R-ratio (0.05, 0.05)
Equal-weighted Strategy	No transaction cost	1.0774 (8.98%)	0.5185 (4.32%)	1.1147 (9.29%)
	Transaction cost 0.78%	0.7221 (6.02%)	0.0393 (0.33%)	0.6323 (5.27%)
	Transaction cost 0.485%	0.8905 (7.42%)	0.2206 (1.84%)	0.8148 (6.79%)
Optimized-weighted Strategy	No transaction cost	n.a.	0.7608 (6.34%)	1.8941 (15.78%)
	Transaction cost 0.78%	n.a.	0.28749 (2.40%)	1.4245 (11.87%)
	Transaction cost 0.485%	n.a.	0.4687 (3.90%)	1.6069 (13.39%)

We analyze the graph of a sample path of cumulative realized profits of winner and loser portfolios across holding periods for different criteria

Cumulative realized returns of winner and loser portfolios for a 6-month/6-month momentum strategy and STARR(50%) criterion



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Further optimization approaches based on a two-step and one-step procedure are evaluated and compared- FinAnalytica's Approach

Two-step procedure optimal portfolio problem:

1. **Selection of winners and losers** according to selection criterion (cumulative return, risk-adjusted criteria or alphas from specific factor model)
2. **Solving the zero-value optimal portfolio problem** with the winners and losers chosen in the step 1).

One-step procedure uses modified objective of the two-step procedure

- Search for the best optimal solution in one step zero-value optimization with objective function reflecting the tradeoff between the expected excess return at given level, and (i) tail risk at a given level and (ii) transaction costs.

We compare the two-step procedure (benchmark model of a large FI) with one-step procedure using different search methods

The results on one-step procedure provide better results than the two-step benchmark by using the genetic algorithm search

Empirical results and observations for the one-step strategy in the 10 year period 09/1995 – 08/2005

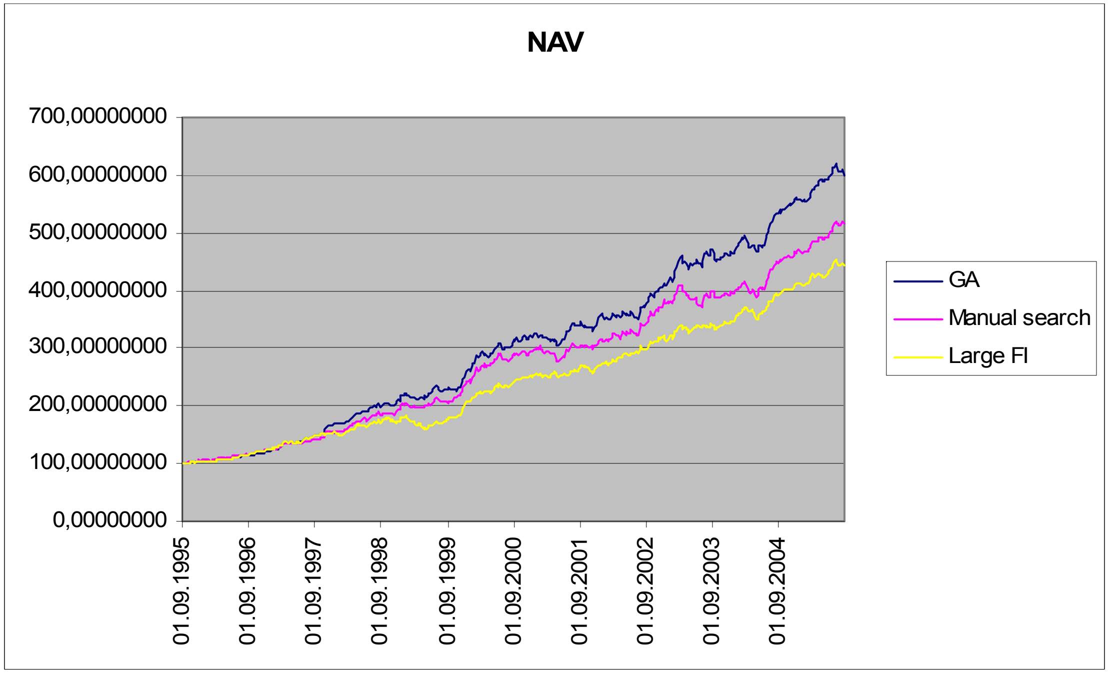
- **Characteristics of the best solution**:

  - Compounded return 19.54%,

  - Average return 18.19%

The results show clear advantage of using one-step procedure with direct GA method over the benchmark model based on two-step procedure

Comparison of Cumulative Return of the two-step benchmark model and the one-step optimization approach



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The results show that alternative risk-return criteria using expected tail loss produce better risk-adjusted results and less heavy tailed distributions

Conclusions

- Alternative risk-return ratios using the ETL can be conveniently applied at the individual stock level for distributions exhibiting heavy tails
- Additional advantage of alternative criteria is to model an investor's risk aversion for downside risk and desired risk-return profile
- Alternative ratios provide better risk-adjusted performance, outperforming cumulative return criterion will generally depend on the data sample properties
- Traditional Sharpe ratio criterion provides the worst performance indicating inappropriateness of the dispersion risk measure
- An investor pursuing alternative strategies using R-ratio will accept lower risk due to heavy tails than the cumulative return investor
- Formulation of the zero-value portfolio problem using one-step optimization outperforms two-step procedure

Research publications

- Rachev, S. , Jašić, T., Stoyanov, S. and Fabozzi, F. "Momentum strategies based on reward–risk stock selection criteria" *Journal of Banking and Finance.* **31/8,** 2325-2346, 2007
- Rachev, S., and Mittnik, S. *Stable Paretian Models in Finance*, Wiley –Financial Economics and Quantitative Analysis, 2000
- Rachev, S. , Menn C. and Fabozzi F. *Fat Tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection, and Option Pricing,* Wiley-Finance, 2005
- Rachev, S., Stoyanov, S, and Fabozzi F. *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization: The Ideal Risk, Uncertainty, and Performance Measures,* Wiley, Finance, 2007

Agenda

- Back –Up – Not to be used for printed materials

Next step – Advanced Portfolio Optimization within the Framework of the Phi-Alpha (Stable GARCH) Approach

Formulation of the objective function with regards to expected return and ETL of the portfolio

- Find $x_1, x_2, \ldots, x_N$ (instruments weights):

  max $E(R_P) - c*ETL(R_P)$
  subject to
  $x_1 + x_2 + \ldots + x_N = 1$
  $x_i > 0$, $i = 1 \ldots N$

  $E(R_P) = x_1 E(I_1) + x_2 E(I_2) + \ldots + x_N E(I_N)$
  $ETL(R_P) = E(-R_P | R_P < a)$
  ! $Prob(R_P < a)$ should be small (~ 1%)

- Input – simulations for possible changes in portfolio assets produced for VaR calculations,
- Properties : for the normal distribution case this problem is equivalent to the traditional Markovitz optimal portfolio problem (that is – they will produce one and the same results)

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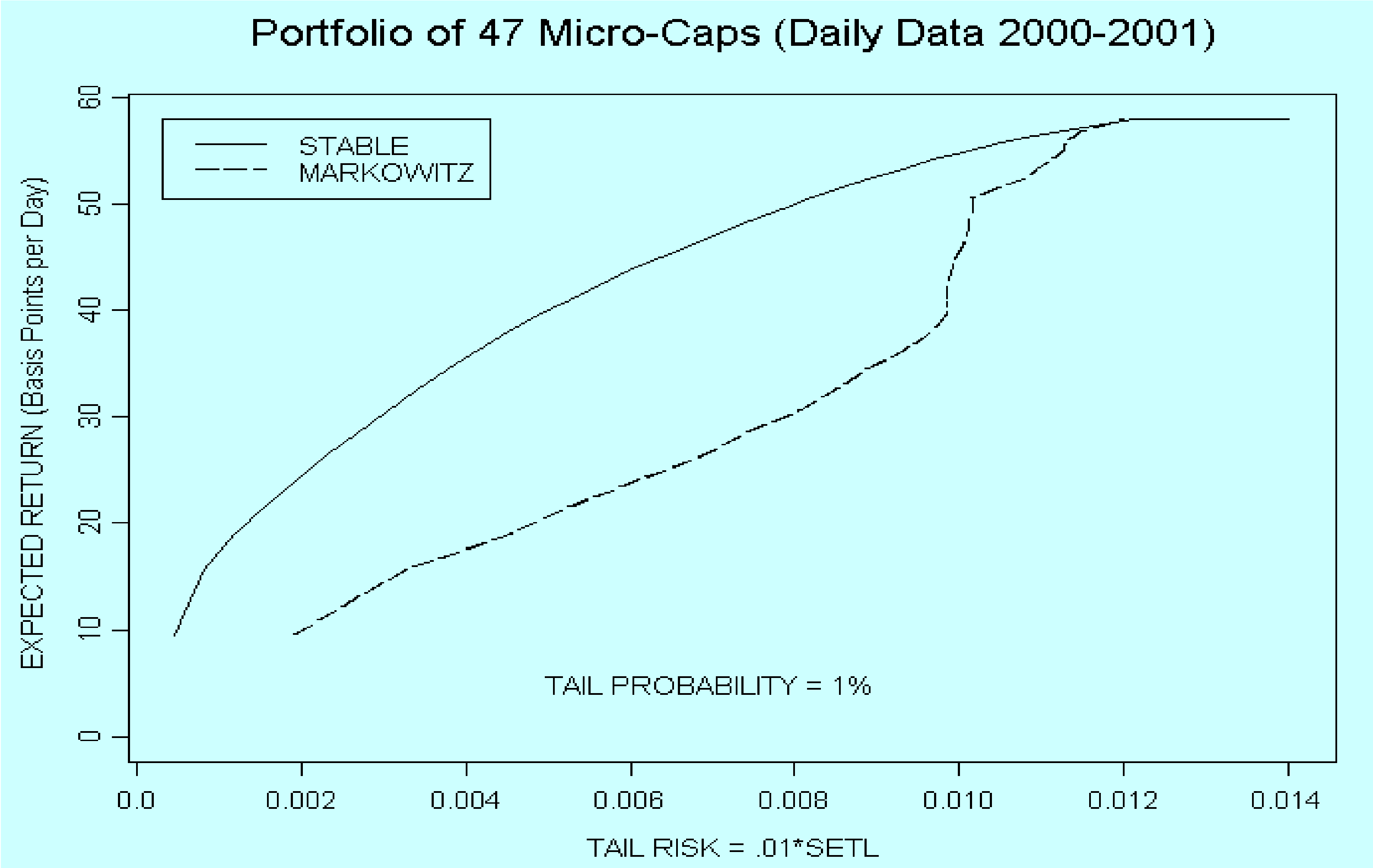
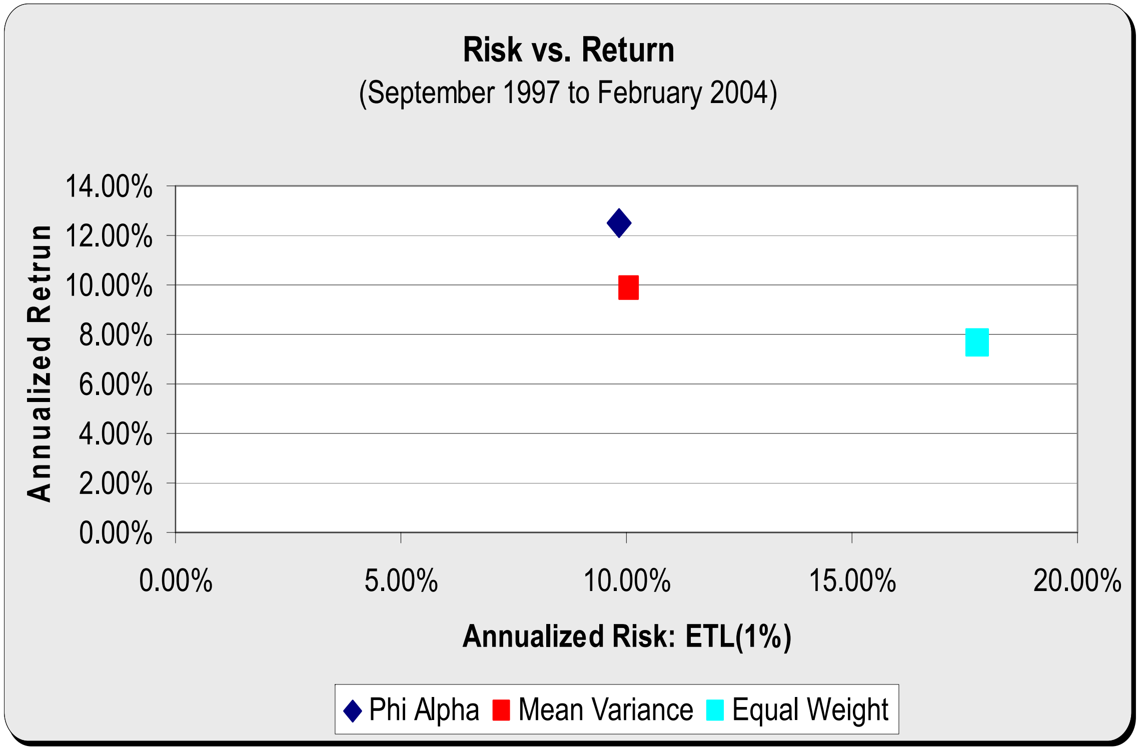
Example: Advantage of Optimizing Phi-Alpha Portfolios (47 Micro-Caps, Data from 2000 – 2001)



Example: Phi-Alpha is the best investment strategy for Vanguard Index Funds



Methodology	STARR (annual)
Phi Alpha	1.27
Mean Variance	0.98
Equal Weight	0.38

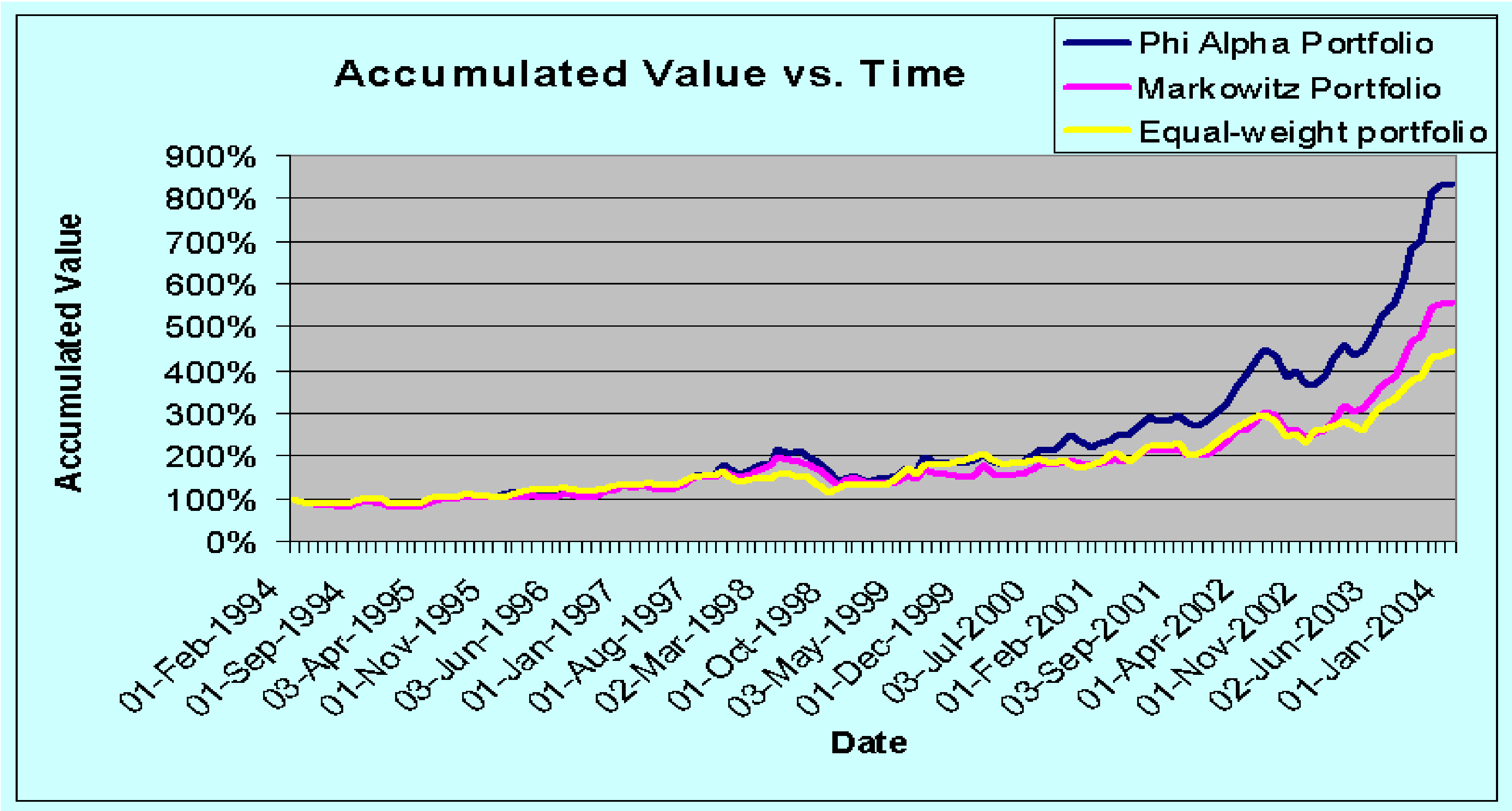
Example: International Equity Portfolio

- **Goal:**
  - Greater total return than MSCI – World index
  - High dividend yield
  - Manage risk

- **Portfolio constructed from dividend paying ADRs**
  - Predominantly large Cap
  - Methodology addresses inconsistent dividend payments

- **Selection of dividend paying ADRs**
  - For each year select top 60 (20) ADRs by risk adjusted yield
  - Use equal weights on 20 ADRs for each year

Example: International Equity Portfolio – Extension of Mergent Dividend Achievers



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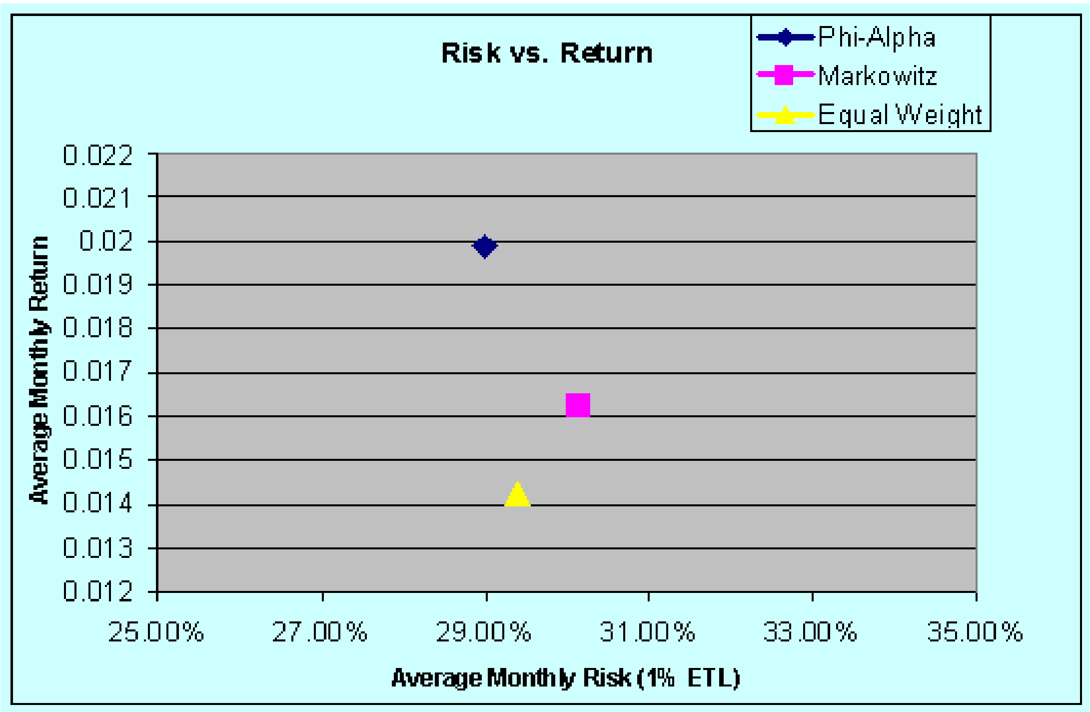
Example: International Equity Portfolio – Extension of Mergent Dividend Achievers



Methodology	STARR (monthly)
Phi Alpha	0.14
Markowitz	0.11
Equal Weight	0.10

The results show that risk-return ratios criteria compliant with coherent risk measure of expected shortfall produce better results than traditional criteria

Conclusion and future research

- *Conceptually*, risk-return ratios that are based on the coherent risk measure of the expected shortfall can be conveniently applied at the individual stock and portfolio level.
- *Methodologically*, we utilize daily data and capture the distributional properties of stock returns and their risk component at a different threshold level of the tail distribution.
- *Empirically,* results show our ratios drive balanced risk-return performance and for every examined strategy produce better results than a simple cumulative return and the traditional Sharpe ratio criterion.
- *Future research:* General analysis of winners and winners portfolios (total optimization)
- *Future Research:* Stable Factor models “explaining” the profits from winners and losers R-ratio optimization

The results on two-step procedure indicate sensitivity to high transaction costs and advantage of using stable scenarios

Empirical results and observations of the two-step strategy

- The best result was obtained with stable scenarios.
- High transaction costs reduce the overall performance and are difficult to control due to two-step process
- Using mixed integer programming (MIP) in which the sets of winners and losers are not necessary tp be known in advance is cumbersome due to computational burden
- Results shows that it is better to include transaction costs in the objective
- Use of stable scenarios outperform the historical method and the normal Monte Carlo

Incorporating the winners/losers selection in the zero-value optimal portfolio problem for one-step problem

Modification of the objective function of the two-step problem with introduction of the constant C_0 and additional constraints

- Find $w_1, w_2, \ldots, w_N$ (instruments weights):

$$(P_2)\begin{cases} \max_{w} C_0 \sum_i w_i Er_i - C_1 ETL_\alpha(w) - C_2 TC \\ s.t. \\ \sum_i w_i^+ = 1 \\ \sum_i w_i^- = 1 \\ |w_i| \le a,\ i = 1, n \\ w_i = w_i^+ - w_i^- \end{cases}$$

where C_0, C_1 and C_2 are positive constants.

- The constants C_1 and C_2 remain fixed for the entire back-testing and C_0 is calculated for each separate optimization starting from 1 and using multiplicative algorithm with coefficient M (constant used to increment C_0)