# I. M. ISAACS

GABRIEL NAVARRO

## 1. Introduction

I. Martin Isaacs died on February 17, 2025, in Berkeley, California, where he had lived since his retirement in 2011 as a professor at the University of Wisconsin–Madison.

Isaacs is best known for his book *Character Theory of Finite Groups*, his masterpiece, from which mathematicians worldwide learned Character Theory. Of course, there were several somewhat earlier books on the subject—Curtis–Reiner, Dornhoff, Feit, etc.—but perhaps they did not have all the elegance, simplicity, immediacy, and even modernity of Marty's text. Stephen D. Smith writes in his Mathematical Review: "The style is clear and direct, and proofs are unusually easy to read." Various chapters of his book, such as those on character correspondences or character degrees, appear for the first time in a textbook and became research fields generating numerous articles, many of them written by himself. Marty approached the proofs of each lemma, each theorem, as a challenge he abandoned only when they were perfect. Writing mathematics well was so central to him that, not long ago, he founded the "Martin Isaacs Prize," awarded annually by the AMS for excellence in the writing of a research paper. When he said "I have cleaned up a proof" of something that interested him, it usually meant he had proved a result twice as strong in less than half the space. Throughout his career, Marty wrote other influential books, such as *Finite Group Theory* and *Algebra: A Graduate Course*, always with his unmistakable style: few preliminaries, the minimum necessary definitions, always getting to the "good stuff" (his words) as soon as possible.

Isaacs was born in the Bronx, New York, on April 14, 1940. Descendant of Hungarian Jewish immigrants, he studied at the Bronx High School of Science and then at the Polytechnic Institute of Brooklyn. The "Poly," of which he was so proud, was an institution that produced many top-level scientists, including several Nobel laureates. Isaacs, of prodigious intelligence, was part of the Poly team that won the Putnam competition, ahead of institutions like Harvard. Also on the team was his inseparable friend from adolescence, Donald Passman, the renowned specialist in Ring Theory.

2010 *Mathematics Subject Classification.* Primary 20C15.

This earned him admission and a scholarship to Harvard, where Marty pursued his graduate studies. It was there that he got closer to finite groups. His thesis advisor was none other than Richard Brauer, one of the greatest algebraists of the twentieth century. Brauer also supervised Passman's thesis at the same time, and there are many anecdotes from that time, especially involving Brauer's attempts to keep them apart since they always worked together. (Isaacs first published articles, from 1963 and before his dissertation, were co-authored with Passman on character degrees.) Both Isaacs and Passman completed their PhDs at Harvard in 1964.

It is impossible to ignore what happened in 1964. Marty had a terrible car accident that left him on the brink of death with severe burns and lifelong physical aftereffects. He was hospitalized first in Paris, then in a New York hospital for almost two years, during which time he survived against doctors predictions. In 1966 he joined the Mathematics Department at the University of Chicago, where he had been offered a position upon finishing his thesis. In 1969 he left Chicago and joined the Mathematics Department at Wisconsin–Madison, a department that boasted many great mathematicians, such as Walter and Mary Ellen Rudin in Analysis and Topology. In Algebra, Marty, Passman, and later Georgia Benkart were recruited. In 1971, at the age of 31, Isaacs was promoted to Full Professor. He became a central figure in a department of enormous prestige, and perhaps its best teacher. His office door, on the third floor of Van Vleck Hall, was always open, where he received students, colleagues, and visitors from around the world, almost all with questions or mathematical problems. Marty was particularly proud of directing the Mathematics Talent Search for many years, which involved middle and high school students from across Wisconsin, where he discovered talents such as Daniel Kane or the siblings Po-Ling, Po-Ru, and Po-Shen Loh, of whom he was so proud. On Mondays, his seminar took place at 3:30 pm, after Passman's, on the ninth floor of Van Vleck, with a view of Bascom Hill and Lake Mendota, where coffee was served before it started. After the seminar, he would ask "Who is interested in dinner?" The most frequent places were the Afghan restaurant, the Indian restaurant, or Smoky's, often at -20 degrees Celsius in the bitter cold of the American Midwest, where we drove in his car, always a Volvo.

Marty was a professor adored by his students. He made the most difficult things seem easy, bringing a kind of drama to each proof he wrote with perfect handwriting. He received numerous teaching awards. Passman, another exceptional teacher, recounts that once, when he had to substitute for Marty, "It was the first time I was booed by students *before* I started teaching!" Marty published around 200 papers. In the 42 years he was in Madison, he supervised 29 PhD students, until his retirement in 2011. Then he moved to Berkeley, where his longtime partner, Deborah Finch, lived.

## 2. The Mathematics of Isaacs

Marty was uncertain as to which he liked more: finite group theory or character theory. Although he also published some papers on Lie algebras (influenced by G. Benkart), the majority of his work focused on characters. Early on, Character Theory was used to glean more profound information about the group. Soon, guided by Isaacs, it became an object of study in its own right.

2.1. **Characters.** It is worth recalling that if $G$ is a finite group, a *complex representation* of $G$ is a multiplicative map

$$\mathcal{X} : G \to \mathrm{GL}_n(\mathbb{C}),$$

where $\mathrm{GL}_n(\mathbb{C})$ is the group of invertible $n \times n$ complex matrices. Each representation $\mathcal{X}$ of $G$ gives rise to a function $\chi : G \to \mathbb{C}$ defined by

$$\chi(g) = \mathrm{Trace}(\mathcal{X}(g)),$$

called the *character* of $G$ originating from $\mathcal{X}$. The number $n = \chi(1)$ is the *degree* of the representation. Since similar matrices have the same trace, we see that characters are class functions, i.e., functions $f : G \to \mathbb{C}$ satisfying

$$f(x^{-1}gx) = f(g)$$

for all $g, x \in G$. The set $\mathrm{cf}(G)$ of complex class functions forms a complex vector space whose dimension is the number $k$ of conjugacy classes in $G$. (The conjugacy class of $g \in G$ is the set $\{x^{-1}gx \mid x \in G\}$. The relation $g \sim h$ if and only if there exists $x \in G$ such that $g = x^{-1}hx$ partitions $G$ into its conjugacy classes.) Furthermore, $\mathrm{cf}(G)$ is a Hermitian space with the inner product

$$[f, h] = \frac{1}{|G|} \sum_{g \in G} f(g)\overline{h(g)}.$$

It is a nontrivial fact that two representations $\mathcal{X}$ and $\mathcal{Y}$, with characters $\chi$ and $\psi$ respectively, are *equivalent* (there is an invertible matrix $M$ such that $M^{-1}\mathcal{X}M = \mathcal{Y}$) if and only if $\chi = \psi$. Since the direct sum of representations is again a representation, we arrive at the fundamental definition: a character $\chi$ of $G$ is *irreducible* if it cannot be expressed as the sum of two characters of $G$. The fundamental theorem of character theory states that the set $\mathrm{Irr}(G)$ of irreducible characters of $G$ forms an orthonormal basis of $\mathrm{cf}(G)$. Other basic results are that the degrees of the irreducible characters divide the group order, and the famous equation

$$|G| = \sum_{\chi \in \mathrm{Irr}(G)} \chi(1)^2.$$

2.2. **The Glauberman–Isaacs correspondence.** Isaacs often regretted not having realized that the seed of the Glauberman correspondence was in his thesis. If $A$ is a finite group of automorphisms of a finite group $G$, then $A$ permutes all objects of $G$: elements, subgroups, characters, etc. The set of irreducible characters of $G$ fixed by $A$ is denoted by $\mathrm{Irr}_A(G)$; the set of elements of $G$ fixed by $A$ is the subgroup of fixed points $\mathbf{C}_G(A) = \{g \in G \mid g^a = g \text{ for all } a \in A\}$, etc. Marty liked to talk about the "$A$-glasses," which you put on and only see the $A$-invariant objects. For example, an $A$-Sylow theory would say that any $A$-invariant $p$-subgroup is contained in an $A$-invariant $p$-Sylow, and that all such $p$-Sylows are $\mathbf{C}_G(A)$-conjugate. In general, $A$-theories do not hold. But everything changes if $|A|$ and $|G|$ are coprime. George Glauberman proved that if $A$ is solvable, there is a natural bijection $^* : \mathrm{Irr}_A(G) \to \mathrm{Irr}(\mathbf{C}_G(A))$, called the *Glauberman correspondence* ([G68]). In the most important case where $A$ is a $p$-group, $\chi^*$ is the unique irreducible constituent of the restriction $\chi_C$ with multiplicity not divisible by $p$. Marty proved the latter in Proposition 2.2 of [Is66] if $|A| = p$, his article on his doctoral thesis, without recognizing the broader result. To remedy this, he wrote one of his best papers ([Is73]). If $A$ is not solvable, then by the Feit–Thompson theorem, $|A|$ is even, and thus $|G|$ is odd. Marty always said that to publish something about groups of odd order, the result had to be spectacular. In [Is73], Isaacs proved that if $|G|$ is odd, then there is also a natural bijection $\mathrm{Irr}_A(G) \to \mathrm{Irr}(\mathbf{C}_G(A))$, akin to Glauberman's. T. Wolf, in his 1977 doctoral thesis directed by Isaacs, showed that these two correspondences coincide whenever both exist [W78]. This correspondence is now known as the Glauberman–Isaacs correspondence, and it appears in the deepest results of character theory. But in [Is73] Isaacs proved another key result.

2.3. **The McKay conjecture.** In 1971 ([McK72]), in an article dedicated to Richard Brauer, J. McKay amazed group theory experts with a beautiful conjecture: If $S$ is a simple group, $P \in \mathrm{Syl}_2(S)$, $N = \mathbf{N}_S(P)$, and $\mathrm{Irr}_{2'}(S)$ is the set of irreducible characters of $S$ of degree not divisible by 2, then

$$|\mathrm{Irr}_{2'}(S)| = |\mathrm{Irr}_{2'}(\mathbf{N}_S(P))| \,.$$

It is simply unthinkable that the normalizer of a 2-Sylow (often just the 2-Sylow itself in simple groups) preserves such an important invariant of an often huge group. In [Is73], Isaacs proved that $|\mathrm{Irr}_{2'}(G)| = |\mathrm{Irr}_{2'}(\mathbf{N}_G(P))|$ if $G$ is solvable (the *opposite* of simple), and he also proved that

$$|\mathrm{Irr}_{p'}(G)| = |\mathrm{Irr}_{p'}(\mathbf{N}_G(P))|$$

for any prime $p$ and any group of odd order. That is, Isaacs provided more than enough evidence that McKay's equation might hold not only for simple groups and the prime 2, but for any group $G$ and any prime $p$. Marty told me on several occasions that he regretted not having formally conjectured in [Is73] that

$$|\mathrm{Irr}_{p'}(G)| = |\mathrm{Irr}_{p'}(\mathbf{N}_G(P))|.$$

This possible equality became known as the McKay conjecture for a long time, one of the main open problems in character theory.

But Marty had not yet said his last word on the subject. In [IN02], he conjectured the existence of a bijection

$$^* : \mathrm{Irr}_{p'}(G) \to \mathrm{Irr}_{p'}(\mathbf{N}_G(P))$$

such that $\chi(1) \equiv \pm\chi^*(1) \pmod p$. In [IMN], he reduced the McKay conjecture to a problem on simple groups: if all finite simple groups satisfy the so-called *inductive McKay condition*, then the McKay conjecture holds. Twenty years after the publication of [IMN], M. Cabanes and B. Späth ([CS24]) completed the proof that all finite simple groups satisfy this condition and, hence, the McKay conjecture is true. At the same time, they proved the character congruences predicted in [IN02]. The reduction in [IMN] became a model to reduce many other local/global conjectures.

2.4. **Characters and normal subgroups.** Marty liked to say his specialty was groups with many normal subgroups. He helped us understand how to handle them in character theory. If $N \trianglelefteq G$ and $\chi \in \mathrm{Irr}(G)$, Clifford's theorem (Thm 6.2 [Is76]) tells us that the restriction of $\chi$ to $N$ can be written as

$$\chi_N = e(\theta^{g_1} + \cdots + \theta^{g_t}),$$

where $e$ is a natural number, and $\{\theta^{g_1}, \ldots, \theta^{g_t}\}$ are all the $G$-conjugates of some $\theta \in \mathrm{Irr}(N)$. (If $g \in G$ and $n \in N$, then $\theta^g(n) = \theta(gng^{-1})$.) Restricting characters to a subgroup $H$ of $G$ has a dual called *induction*. If $\eta \in \mathrm{Irr}(H)$, we define

$$\eta^G(g) = \frac{1}{|H|} \sum_{\substack{x \in G \\ xgx^{-1} \in H}} \eta(xgx^{-1}).$$

It is straightforward to prove that

$$[\eta^G, \chi] = [\eta, \chi_H]$$

(this is *Frobenius reciprocity*, Lemma 5.2 [Is76]), and thus $\eta^G$ is a character of $G$. Note that $\eta^G(1) = |G : H|\eta(1)$, and usually $\eta^G$ is not irreducible.

Returning to $N \trianglelefteq G$ and $\theta \in \mathrm{Irr}(N)$, if we write $\mathrm{Irr}(G|\theta)$ for the set of irreducible characters of $G$ whose restriction to $N$ contains $\theta$, then the *Clifford correspondence* (Thm 6.11 [Is76]) tells us that induction defines a bijection

$$\mathrm{Irr}(G_\theta|\theta) \to \mathrm{Irr}(G|\theta),$$

where $G_\theta$ is the stabilizer of $\theta$ in $G$. This typically allows one to assume $G_\theta = G$ in inductive proofs on $|G|$. In that case, Marty calls $(G, N, \theta)$ a *character triple*, and by using Schur's work, Isaacs shows that $(G, N, \theta)$ can be "replaced" by another triple $(G^*, N^*, \theta^*)$ where $G/N \cong G^*/N^*$ and $N^*$ is central. For example, to prove that the integer $e$ in Clifford's theorem divides $|G|/|N|$, one may assume $\chi_N = e\theta$ and that $N \subseteq \mathbf{Z}(G)$ (a much easier case).

Suppose now that $\chi_N = e\theta$. By Frobenius reciprocity, we have $\theta^G = e\chi + \Delta$, where $\Delta$ is a character of $G$ (or zero) and hence $e^2 \leq |G : N|$. The extreme case $e^2 = |G : N|$ happens if and only if $\theta^G = e\chi$, namely, if the induced character $\theta^G$ contains exactly one irreducible constituent. In this case we say that $\chi$ (or $\theta$) is *fully ramified* over $G/N$. In 1964, N. Iwahori and M. Matsumoto conjectured that in this situation $G/N$ must be solvable. In 1982, Isaacs and R. Howlett proved this conjecture using the Classification of Finite Simple Groups (CFSG) [HI]. Marty did not particularly like using the classification, but when needed, he did so.

A simpler but crucial theorem of Marty's, used repeatedly, is the *Going Down Theorem* (Theorem 6.18 of [Is76]): if $K, L \trianglelefteq G$, $K/L$ is abelian, and $\theta \in \mathrm{Irr}(K)$ is $G$-invariant, then $\theta_L$ is irreducible, $\theta$ is induced from $L$, or $\theta$ is fully ramified with respect to $K/L$, i.e., $\theta_L = e\varphi$ with $e^2 = |K : L|$. In the latter case, under typical hypotheses, Marty, generalizing results of E. C. Dade, showed in [Is73] and [Is82] that there is a complement $U$ of $K/L$ in $G$ and a bijection

$$\mathrm{Irr}(G|\theta) \to \mathrm{Irr}(U|\varphi).$$

This deep result has been used in dozens of articles.

2.5. **$\pi$-theory.** The Department of Mathematics at the University of Chicago, together with Cambridge, was for a long time the best department in the world for group theory. One of the reasons was the presence of J. G. Thompson, a Fields medalist (1970) and Abel Prize laureate (2008). (They also had Glauberman and Jon Alperin.) Marty often recounted that Thompson, whom he greatly admired, told him: "As Brauer's student, how have you not studied modular theory and Brauer's papers?" So he went to do so. "When Thompson tells you to do something, you do it."

Modular character theory examines the relationship between characters and representations over fields of characteristic $p$, a prime. If ordinary (complex) character theory is already a powerful tool for understanding the group (for instance, there is no known proof of the Feit–Thompson theorem that avoids characters), modular theory is even more powerful. (For example, in the proof of Glauberman's Z* theorem, a key part of CFSG.) One of the most famous equations in modular theory is that if $\chi \in \mathrm{Irr}(G)$ and $\chi^0$ is the restriction of $\chi$ to the set of elements of $G$ whose orders are not divisible by $p$, then

$$\chi^0 = \sum_{\varphi \in \mathrm{IBr}(G)} d_{\chi\varphi}\varphi,$$

where $\mathrm{IBr}(G)$ is the set of Brauer characters (the complex lifts of irreducible characters in characteristic $p$), and the non-negative integers $d_{\chi\varphi}$ are the *decomposition numbers.*

In the works in which he discovered and developed $\pi$-theory, Marty produced some of his dearest results, especially [Is84]. What Isaacs did was to develop Brauer's

$p$-modular theory for a set of primes $\pi$, though only for solvable groups (his favorite ones). When $\pi$ is the set of all primes different from a certain prime $p$, Isaacs $\pi$-theory is precisely Brauer theory for $p$-solvable groups.

If $\pi$ is a set of primes, denote by $G_\pi$ the set of $\pi$-elements of $G$ (an element $g$ is a *$\pi$-element* if its order is a $\pi$-number, i.e., the set of primes dividing $o(g)$ is contained in $\pi$). Let $\mathrm{cf}(G_\pi)$ be the set of class functions $G_\pi \to \mathbb{C}$, and let ${}^0 : \mathrm{cf}(G) \to \mathrm{cf}(G_\pi)$ be the restriction of class functions. Isaacs proved that if $G$ is $\pi$-separable (a generalization of solvable), there exists a unique basis $I_\pi(G)$ of $\mathrm{cf}(G_\pi)$ such that if $\chi \in \mathrm{Irr}(G)$, then

$$\chi^0 = \sum_{\varphi \in I_\pi(G)} d_{\chi\varphi}\varphi,$$

for certain non-negative integers $d_{\chi\varphi}$. Moreover, if $\varphi \in I_\pi(G)$, then there exists $\psi \in \mathrm{Irr}(G)$ such that $\psi^0 = \varphi$. (The latter is Marty's version of the Fong–Swan theorem.) But even more: Isaacs discovered a canonical subset $B_\pi(G) \subseteq \mathrm{Irr}(G)$ such that the map $\chi \mapsto \chi^0$ defines a canonical bijection $B_\pi(G) \to I_\pi(G)$.

Isaacs construction of $B_\pi(G)$, always in $\pi$-separable groups, is extremely sophisticated and uses one of the objects Marty loved most and helped publicize: the $\pi$-special characters of D. Gajendragadkar [Ga79]. Gajendragadkar was a student of E. C. Dade at Urbana, Illinois, who visited Isaacs in his last year before completing his thesis in 1978. Marty helped him "clean up" his results. If $\chi \in \mathrm{Irr}(G)$, we say $\chi$ is *$\pi$-special* if $\chi(1)$ is a $\pi$-number and the order of the determinant of every $\theta \in \mathrm{Irr}(N)$, $N \trianglelefteq G$, covered by $\chi$ is also a $\pi$-number. Isaacs proved that any irreducible character of a $\pi$-separable group not induced from any proper subgroup (that is, a *primitive* character) can be written uniquely as a product of a $\pi$-special and a $\pi'$-special character. He was very proud of this result ([Is81]). Even in the classical case where $\pi$ is the set of primes different from $p$, Marty had proved a new statement: every Brauer character has a canonical lift to $G$, improving on his result in [Is74]. Using $\pi$-theory, Isaacs proved in [IN95] a $\pi$-version of the famous Alperin Weight Conjecture [Alp87], which included the first published proof of this conjecture for $p$-solvable groups.

2.6. **Character degrees.** Another recurring theme in Isaacs research, and one of his favorites, is the study of character degrees. As in so many other instances, the following notation is his: we let $\mathrm{cd}(G) = \{\chi(1) \mid \chi \in \mathrm{Irr}(G)\}$. N. Itô had proved that if $\chi \in \mathrm{Irr}(G)$ and $A \trianglelefteq G$ is abelian, then $\chi(1)$ divides $|G : A|$. It follows that if $G$ has a normal abelian $p$-Sylow $P$, then $p$ does not divide $\chi(1)$ for all $\chi \in \mathrm{Irr}(G)$ (Theorem 6.15 of [Is76]). Isaacs asked his thesis advisor, Brauer, about the converse, and Brauer's reply, which Marty often quoted, was: "The world is not ready for that." Indeed, about twenty years and CFSG were needed to prove the converse ([Mi86]).

Marty, together with Passman, began investigating and characterizing groups based on their character degrees (their first paper, written when they were still students, is [IP]). Isaacs showed that if $|\mathrm{cd}(G)| \leq 2$, then $G$ is metabelian (i.e., the commutator subgroup $G'$ is abelian). He also proved that if $|\mathrm{cd}(G)| \leq 3$, then $G''' = 1$. This gave rise to the Isaacs–Seitz conjecture: if $G$ is solvable, then $\mathrm{dl}(G) \leq |\mathrm{cd}(G)|$. This conjecture remains open.

2.7. **Various.** Isaacs had a deep passion for $M$-groups (or *monomial groups*). A group $G$ is an $M$-group if for every $\chi \in \mathrm{Irr}(G)$, there exists a subgroup $H \leq G$ and a linear character $\lambda \in \mathrm{Irr}(H)$ such that $\lambda^G = \chi$. $M$-groups are solvable but can be very intricate. The smallest solvable group that is not an $M$-group is $\mathrm{SL}_2(3)$. Dade showed that every solvable group is a subgroup of some $M$-group. However, for a while it was conjectured that normal subgroups of $M$-groups were also $M$-groups, but Dade and Van der Waall simultaneously found a counterexample. Both Isaacs and Dade attempted to prove that this does not occur in groups of odd order. M. Loukaki, a student of Dade, proved it for groups of order $p^a q^b$ in her PhD thesis. (A shorter proof is given in [L]). Though partial results exist, the difficulty of this topic makes further breakthroughs elusive.

Another well cited paper of Marty is [INW] in which he studies for the first time the so called *non-vanishing* elements: that is the elements $g \in G$ such that $\chi(g) \neq 0$ for all $\chi \in \mathrm{Irr}(G)$. If $G$ is solvable, it is suggested that $g \in \mathbf{F}(G)$, the Fitting subgroup of $G$, a problem that remains open.

A very significant aspect in Marty's last 20 years was his collaboration with Persi Diaconis. Initially interested in the symmetric group, Persi had a great passion for finite groups, and he found a friend and collaborator in Isaacs. Their joint article on supercharacters [DI] is Marty's most-cited paper. In it, they develop a supercharacter theory for "algebra groups," such as groups of unitriangular matrices over a field with $q$ elements. There are difficult conjectures concerning these groups, for example, a G. Higman conjecture stating that the number of conjugacy classes of these groups is a polynomial in $q$ with integer coefficients. Marty proved the conjecture that the degrees of the irreducible characters of these groups are powers of $q$ in [Is95]. With the help of MAGMA, in [IK98] he found a counterexample to a conjecture of Kirilov claiming that the characters of these $U_n(\mathbb{F}_2)$ were real. Marty loved using computers, and we sometimes had the "MAGMA vs. GAP" debates. Another conjecture we both spent a long time on, and eventually showed to be false, claimed that 2-Sylows of rational groups were rational [IN12]. This had been conjectured in the 1950s. After days of searching and a stroke of insight, we were lucky to find a counterexample.

## 3. Isaacs and Valencia

In my final undergraduate year (1987), I dared to write to Prof. Isaacs about some ideas I had, and we started corresponding by mail. At some point, we wondered whether the Glauberman–Isaacs correspondence commuted with restriction and induction of characters. Marty sent me his proof for the case $|G|$ odd (i.e., Isaacs correspondence). In 1988 I was fortunate to prove the remaining case (when $|A|$ is odd). In February 1989 I traveled to Berkeley, where Marty was a visiting professor in UC Berkeley, and we wrote our first joint article [IN91]. Among the 18 papers we co-authored by ourselves, the latest one [IN24] was the only one I wrote from start to finish. (Isaacs coauthors know that Marty always wrote the "final draft" of every paper.)

Isaacs came to Valencia for the first time in 1991, and he immediately fell in love with the city and its restaurants (especially so). He then returned with Deborah in 1992, 1996, 2003, 2004, and so on. (In 2003 he also visited Bilbao, invited by A. Moretó and J. Sangróniz.) Marty and Deborah liked to rent a car and drive around Spain. Needless to say, until he told me he was back safe and sound in the U.S., I was always somewhat anxious. In June 2009, we organized a conference in Valencia in his honor. Many legends of group and character theory came, for most of them their first time in Spain: E. C. Dade, G. Glauberman, R. Guralnick, B. Huppert, J. G. Thompson, etc., and his friend Persi Diaconis.

Several professors of the department in Valencia spent time in Madison, such as L. Sanus, J. Tent, and notably Alex Moretó, with whom Marty co-authored five articles (the first one when Alex was still in Bilbao; their most cited one is [ILM13]).

The last time I saw Marty was in April 2018, at the MSRI in Berkeley, where we spent a semester together, sharing an office. (We had spent two other semesters there: in 1990 and in 2008.) Then the pandemic arrived, and his health gradually deteriorated. We continued corresponding regularly until June 2024. From then on, I only wrote to Deborah for updates on his condition or to communicate with him through her. On February 18, 2025, she wrote to tell me that Marty had passed away.

## References


[Alp87] J. L. Alperin, Weights for finite groups. *Proc. Sympos. Pure Math.* **47-1** (1987), 369–379.

[CS24] M. Cabanes, B. Späth, The McKay Conjecture on character degrees. ArXiv: 2410.20392.

[DI] P. Diaconis, I. M. Isaacs, Supercharacters and superclasses for algebra groups. *Trans. Amer. Math. Soc.* **360** (2008), no. 5, 2359–2392.

[Ga79] D. Gajendragadkar, A characteristic class of characters of finite $\pi$-separable groups. *J. Algebra* **59** (1979), no. 2, 237–259.

[G68] G. Glauberman, Correspondences of characters for relatively prime operator groups. *Canadian J. Math.* **20** (1968), 1465–1488.

[HI] R. B. Howlett, I. M. Isaacs, On groups of central type. *Math. Z.* **179** (1982), no. 4, 555–569.

[Is66] I. M. Isaacs, Extensions of certain linear groups. *J. Algebra* **4** (1966), 3–12.

[Is73] I. M. Isaacs, Characters of solvable and symplectic groups. *Amer. J. Math.* **95** (1973), 594–635.

[Is74] I. M. Isaacs, Lifting Brauer characters of $p$-solvable groups. *Pacific J. Math.* **53** (1974), 171–188.

[Is81] I. M. Isaacs, Primitive characters, normal subgroups, and M-groups. *Math. Z.* **177** (1981), no. 2, 267–284.

[Is82] I. M. Isaacs, Character correspondences in solvable groups. *Adv. in Math.* **43** (1982), no. 3, 284–306.

[Is84] I. M. Isaacs, Characters of $\pi$-separable groups. *J. Algebra* **86** (1984), 98–128.

[Is76] I. M. Isaacs, *Character Theory of Finite Groups.* Academic Press, 1976.

[Is95] I. M. Isaacs, Characters of groups associated with finite algebras. *J. Algebra*, **177** (1995), 708–730.

[Is09$_1$] I. M. Isaacs, *Finite Group Theory*. Graduate Studies in Mathematics, **92**. American Mathematical Society, Providence, RI, 2008.

[Is09$_2$] I. M. Isaacs, *Algebra: a Graduate Course*. Reprint of the 1994 original. Graduate Studies in Mathematics, **100**. American Mathematical Society, Providence, RI, 2009.

[Is18] I. M. Isaacs, *Characters of Solvable Groups*. Graduate Studies in Mathematics, **189**. American Mathematical Society, Providence, RI, 2018

[IK98] I. M. Isaacs, D. Karagueuzian, Conjugacy in groups of upper triangular matrices. *J. Algebra* **202** (1998), no. 2, 704–711.

[ILM13] I. M. Isaacs, M. Loukaki, A. Moretó, The average degree of an irreducible character of a finite group. *Israel J. Math.* (2013) **197**, 55–67.

[IMN] I. M. Isaacs, G. Malle, G. Navarro, A reduction theorem for the McKay conjecture. *Invent. Math.* **170** (2007), 33–101.

[IN91] I. M. Isaacs, G. Navarro, Character correspondences and irreducible induction and restriction. *J. Algebra 140* (1991), no. 1, 131–140.

[IN95] I. M. Isaacs, G. Navarro, Weights and vertices for characters of $\pi$-separable groups. *J. Algebra* **177** (1995), 339–366.

[IN02] I. M. Isaacs, G. Navarro, New refinements of the McKay conjecture for arbitrary finite groups. *Ann. of Math. (2)* **156** (2002), 333–344.

[IN12] I. M. Isaacs, G. Navarro, Sylow 2-subgroups of rational solvable groups. *Math. Z.* **272** (2012), no. 3-4, 937–945.

[IN24] I. M. Isaacs, G. Navarro, Primes and conductors of characters. *J. Algebra* **653** (2024), 42–53.

[INW] I. M. Isaacs, G. Navarro, Thomas R. Wolf, Finite group elements where no irreducible character vanishes. *J. Algebra* **222** (1999), no. 2, 413–423.

[IP] I. M. Isaacs, D. Passman, Groups whose irreducible representations have degrees dividing $p^e$. *Illinois J. Math.* **8** (1964), 446–457.

[L] M. L. Lewis, M-groups of order $p^a q^b$: A theorem of Loukaki. Journal of Algebra and Its Applications **05**, No. 04, (2006), 465–503.

[McK72] J. McKay, Irreducible representations of odd degree. *J. Algebra* **20** (1972), 416–418.

[Mi86] G. O. Michler, A finite simple group of Lie type has $p$-blocks with different defects, $p \neq 2$. *J. Algebra* **104** (1986), no. 2, 220–230.

[W78] T. R. Wolf, Character correspondences in solvable groups. *Illinois J. Math.* **22** (1978), no. 2, 327–340.

Departament de Matemàtiques, Universitat de València, 46100 Burjassot, València, Spain

*E-mail address*: gabriel@uv.es