Enrique Llorens: maestro, amigo, compañero

❑ Why me?

- ❑ Why me?
- ❑ Thanks, Jesús!

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- Why me?
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    - His three mathematical passions (not including paella)

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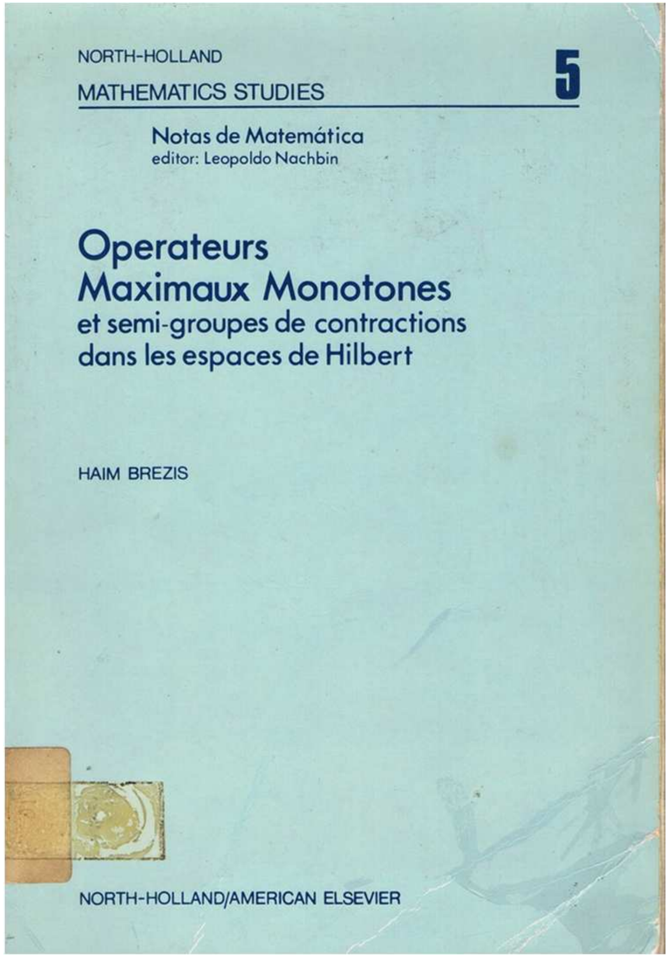
  - Popurrí of pictures

Málaga, 1983

Málaga, 1983

The fixed point property for reflexive spaces: an open problem in Brezis' book



REFERENCES BIBLIOGRAPHIQUES; COMPLEMENTS ET PROBLEMES OUVERTS

CHAP. I

§.I.1. Le théorème du min-max sous sa forme la plus élémentaire est dû à Von Neumann (1937). La démonstration du théorème 1.1 repose sur une idée de Shiffman (cf. KARLIN [1]). On trouvera dans la littérature de nombreuses généralisations du théorème du min-max ; citons entre autres celles de BROWDER [14], GHOUILA-HOURI [1], KY FAN [1] [2] [3], MOREAU [1] (qui établit la relation avec la théorie des fonctions convexes conjuguées) et SION [1].

ROCKAFELLAR [5] met en évidence le lien qui existe entre la recherche des points selle et celle des zéros d'un opérateur monotone ; notons simplement que si $K(x,y)$ est une fonction convexe-concave différentiable sur $E \times F$ et si $K_x(x,y)$ (resp. $K_y(x,y)$) désignent les différentielles de K par rapport à x (resp. y), alors $[x_o,y_o]$ est un point selle de K si et seulement si $K_x(x_o,y_o) = K_y(x_o,y_o) = 0$ i.e. $M(x_o,y_o) = 0$ où $M(x,y) = (K_x(x,y), -K_y(x,y))$ est un opérateur monotone de $E \times F$ dans $E' \times F'$

§.I.2. Le théorème 1.2 a été prouvé indépendamment par BROWDER [6], KIRK [1] et GÖHDE [1]. La démonstration que nous présentons n'est pas la plus simple, mais on en retiendra surtout la propriété remarquable de fermeture indiquée à la proposition 1.3. Cette proposition est due à F. BROWDER [17] . De nombreux travaux ont été consacrés à l'étude des points fixes d'une contraction (resp. communs à une famille de contractions ainsi qu'à la convergence de diverses méthodes itératives (cf. BELLUCE - KIRK [1], BROWDER - PETRYSHYN [1], KANIEL [2] ainsi que les monographies de de FIGUEIREDO [1] et OPIAL [1]). Le problème suivant semble être encore ouvert :

Pb.1. Soit E un espace de Banach réflexif et soit C un convexe fermé borné de E. Soit T une contraction de C dans C ; est-ce que T admet un point fixe ?

§.I.3. Le théorème 1.4 a été remarqué indépendamment par CRANDALL

- Integración en Espacios Localmente Compactos, Rosa Tejeda Salvatierra, 1981.

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- Introducción a la Optimización en Espacios de Banach, J.A. Pulido del Río, 1983.

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- Teoría del punto fijo para aplicaciones no expansivas en espacios de Banach, Catalina Fernández Escalona, 1984.
- Algunas propiedades geométricas de los espacios de Banach relacionadas con la teoría del punto fijo, Antonio Jiménez Melado, 1984.


Valencia

Perito agrónomo (1967)

Perito agrónomo (1967)
Licenciado en ciencias matemáticas (1973)


Gerona
Valencia

Profesor en C.U. Gerona

Doctor en Matemáticas (1977), Univ. Aut. Barcelona

Doctor en Matemáticas (1977), Univ. Aut. Barcelona
Director: Florencio del Castillo Abánades

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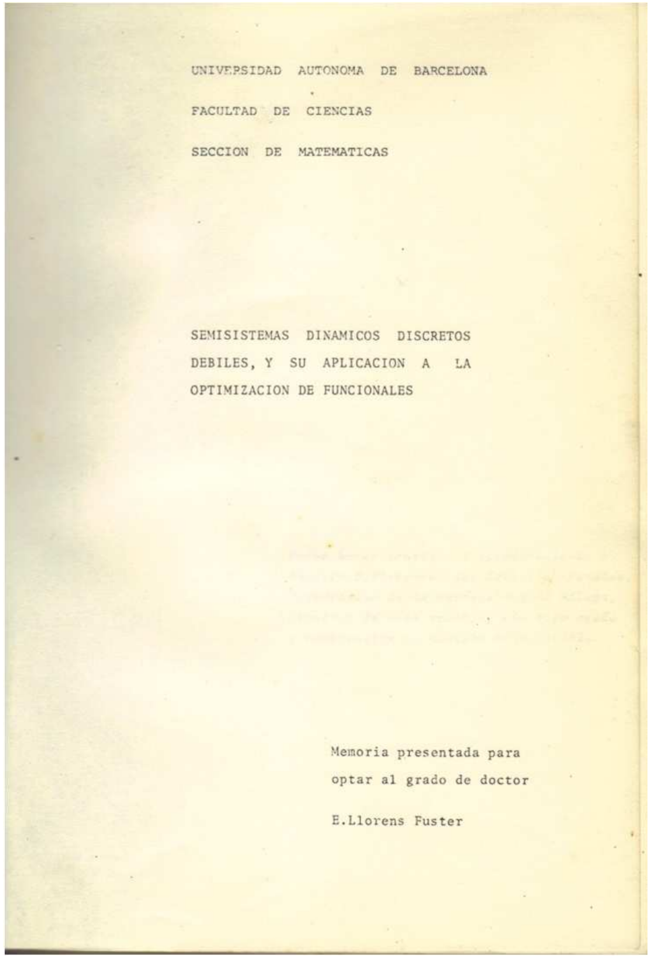


UNIVERSIDAD AUTONOMA DE BARCELONA

FACULTAD DE CIENCIAS

SECCION DE MATEMATICAS

SEMISISTEMAS DINAMICOS DISCRETOS
DEBILES, Y SU APLICACION A LA
OPTIMIZACION DE FUNCIONALES

Memoria presentada para
optar al grado de doctor

E.Llorens Fuster

Gerona, 1977

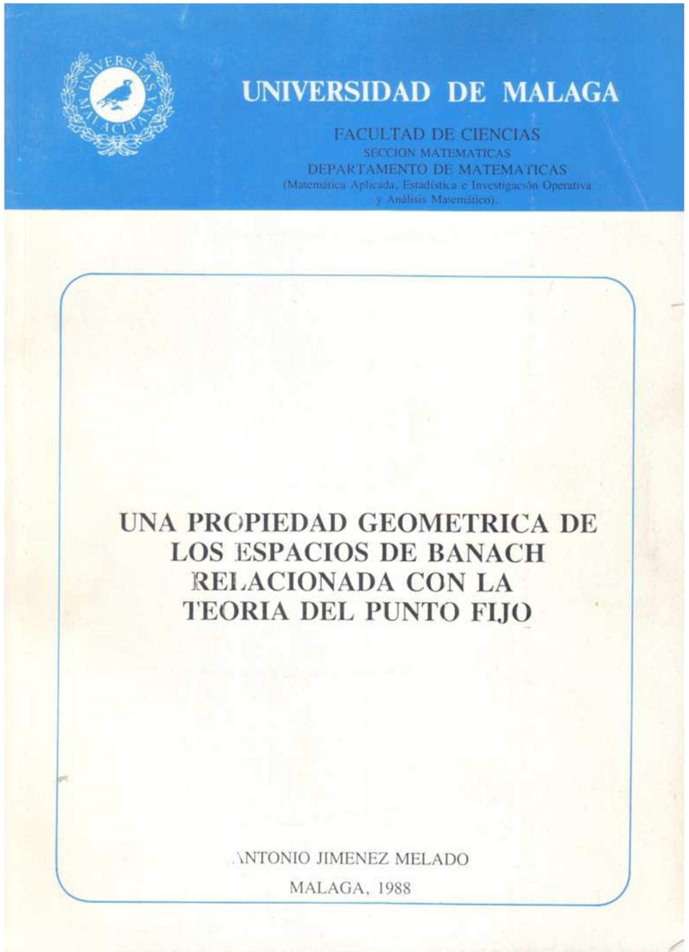
Málaga, 1977

With the family

Málaga, 1977

Valencia, 1984

Mi tesis: Málaga, 1988

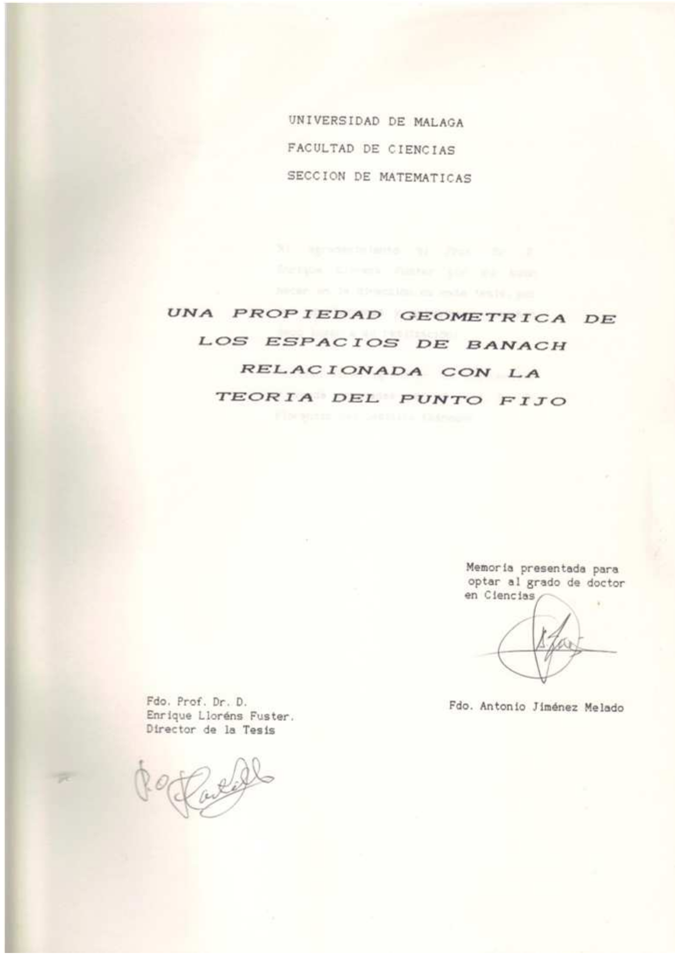


UNIVERSIDAD DE MALAGA

FACULTAD DE CIENCIAS
SECCION MATEMATICAS
DEPARTAMENTO DE MATEMATICAS
(Matemática Aplicada, Estadística e Investigación Operativa y Análisis Matemático).

UNA PROPIEDAD GEOMETRICA DE LOS ESPACIOS DE BANACH RELACIONADA CON LA TEORIA DEL PUNTO FIJO

ANTONIO JIMENEZ MELADO
MALAGA, 1988



UNIVERSIDAD DE MALAGA
FACULTAD DE CIENCIAS
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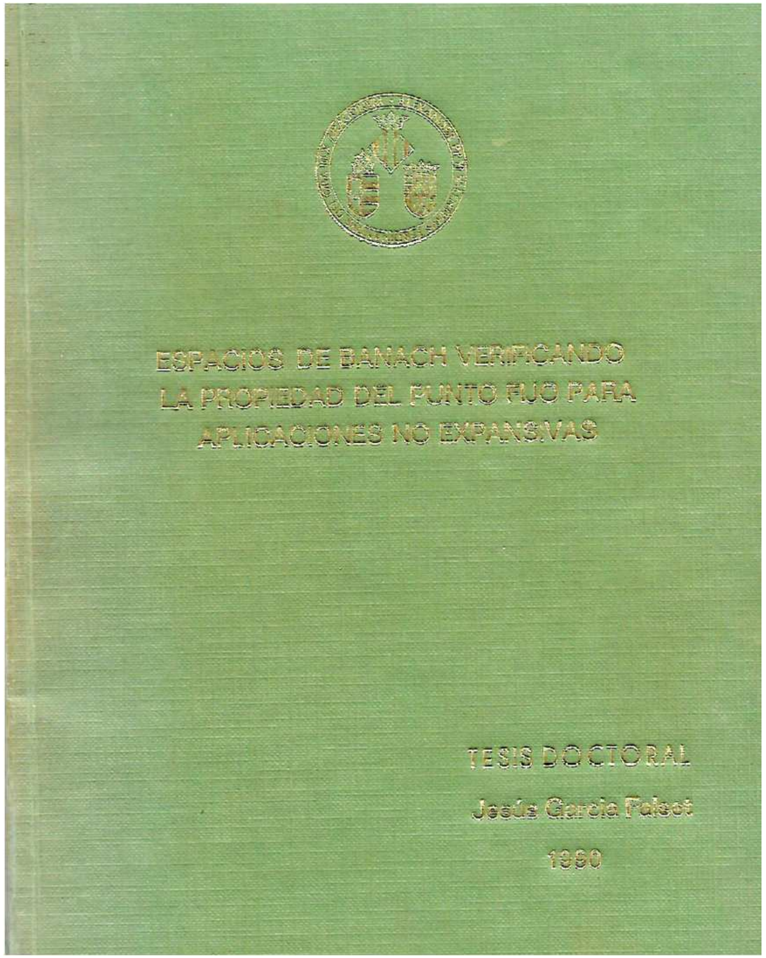
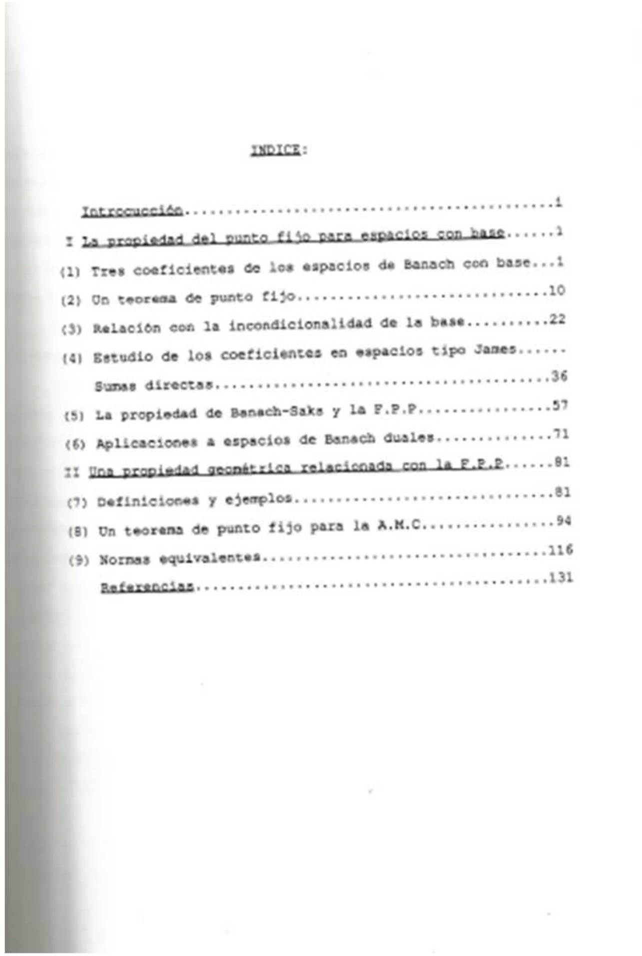
Memoria presentada para optar al grado de doctor en Ciencias

Fdo. Antonio Jiménez Melado

Fdo. Prof. Dr. D. Enrique Lloréns Fuster. Director de la Tesis

Tesis de Jesús: Valencia, 1990





INDICE:

Halifax, 1991

Tesis de Eva: Valencia, 2003

VNIVERSITAT DE VALÈNCIA — Facultat de Ciències Matemàtiques

DEPARTAMENTO DE ANÁLISIS MATEMÁTICO

Geometría de los Espacios de Banach en Teoría Métrica del Punto Fijo

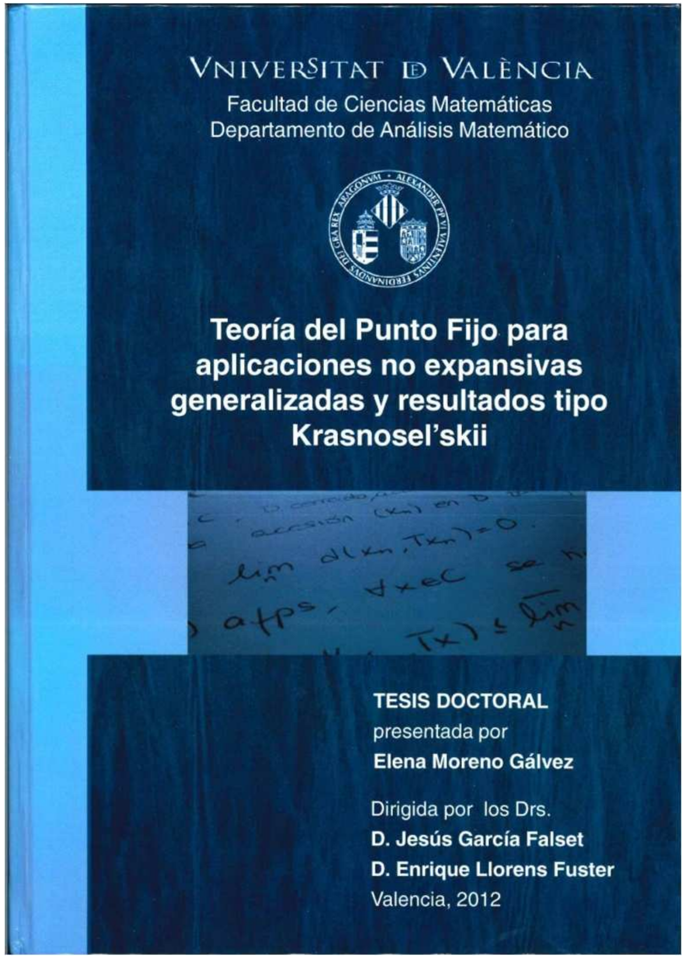
TESIS DOCTORAL
presentada por
Eva María Mazcuñán Navarro

dirigida por los Doctores
D. Jesús García Falset y
D. Enrique Llorens Fuster

Valencia, 2003

Índice general

Tesis de Elena: Valencia, 2012



Índice

Master and disciples

Three mathematical passions

The fixed point property (FPP)

Suppose that $(X, \|\cdot\|)$ is a Banach space. We say that

- $T : C \subset X \to X$ is nonexpansive if

$$\|T(x) - T(y)\| \leq \|x - y\|$$

  for all $x, y \in X$.

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- $(X, \|\cdot\|)$ has the fixed point property (FPP) if every nonexpansive map $T : C \to C$ defined on a nonempty, bounded, closed and convex subset C of X has a fixed point in C.
- $(X, \|\cdot\|)$ has the weak fixed point property (w-FPP) if the above property is satisfied replacing bounded, convex by weakly compact.

Browder-Göhde-Kirk, 1965

A Banach space $(X, \|\cdot\|)$ has the w-FPP if:

Browder-Göhde-Kirk, 1965

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Browder-Göhde-Kirk, 1965

A Banach space $(X, \|\cdot\|)$ has the w-FPP if:

- $(X, \|\cdot\|)$ is a Hilbert space;
- $(X, \|\cdot\|)$ is uniformly convex;
- $(X, \|\cdot\|)$ has weak normal structure: any nonempty, convex and weakly compact subset C of X, with $diam(C) > 0$ is contained in a ball centered at a point $x_0 \in C$ and radius $r < diam(C)$.

Big (still open) problems

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Big (still open) problems

- Does every reflexive Banach space have the w-FPP?
- Does every renorming of a uniformly convex space have the w-FPP?
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A (not so big) Problem : find geometric properties implying the w-FPP, but not normal structure.

Uniformly nonsquare Banach spaces

Uniformly nonsquare Banach spaces

The modulus of convexity and the characteristic of convexity:

$$\delta_X(\varepsilon) = \inf\left\{1 - \left\|\frac{x+y}{2}\right\| : x, y \in B_X, \|x-y\| \geq \varepsilon\right\}$$

$$\varepsilon_0(X) = \sup\{\varepsilon \in [0,2] : \delta_X(\varepsilon) = 0\}$$

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Then,

- X is uniformly convex iff $\varepsilon_0(X) = 0$;
- X is uniformly nonsquare iff $\varepsilon_0(X) < 2$.

Theorem (Goebel, 1970)

If $\varepsilon_0(X) < 1$, then $(X, \|\cdot\|)$ has weak normal structure (and then w-FPP).

K. Goebel, Convexity of balls and fixed point theorems for mappings with nonexpansive square. Compositio Mathematica 1970.

Some unsolved problems

The following questions seem to be interesting:

1°. Does exist the B-space with $\varepsilon_0 = 1$ and without normal structure?

2°. Is the condition (6) exact? It means, does exist the space B, the convex set $C \subset B$ and the involution F of C satisfying the condition (6) with k such that

$$\frac{k}{2}\left(1-\delta\left(\frac{2}{k}\right)\right) = 1$$

and without fixed points?

3°. What are the sufficient condition for existence of the fixed points for the involutions of higher order (i.e. such mappings F that $F^n = I$ for some integer n)?

4°. It is not known whether Kirk's theorem is true in arbitrary reflexive B-space. Is it true in the spaces with $\varepsilon_0 = 1$? If yes, is it true in the space with $\varepsilon_0 < 2$? What is the greatest lower bound of such numbers ε that Kirk's theorem is true for all spaces with $\varepsilon_0 < \varepsilon$?

Uniformly nonsquare Banach spaces

Question: Does $\varepsilon_0(X) < 2$ imply w-FPP?

Uniformly nonsquare Banach spaces

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Partial answers. $(X, \|\cdot\|)$ has the w-FPP if

- $\varepsilon_0(X) < 2$ and X has the WORTH Property (Jesús García 1993)

Uniformly nonsquare Banach spaces

Question: Does $\varepsilon_0(X) < 2$ imply w-FPP?

Partial answers. $(X, \|\cdot\|)$ has the w-FPP if

- $\varepsilon_0(X) < 2$ and X has the WORTH Property (Jesús García 1993)
- $\frac{1}{4}\varepsilon_0(X) + \frac{1}{2}\mu(X) < 1$ (A.Jiménez, E.Llorens, 1996)

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- $\varepsilon_0(X) < 2$ (J.García, E.Llorens, E.Mazcuñán, 2006)

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Stability of the FPP in ℓ_2

Stability of the FPP in ℓ_2

Theorem (Lifshitz, 1975)

Let C be a nonempty, convex and weakly compact subset of ℓ_2 and let $T : C \to C$ be a uniformly lipschitzian map such that

$$\|T^n(x) - T^n(y)\|_2 \leq k\|x - y\|_2 \qquad x, y \in X, \qquad n = 1, 2, \dots \; .$$

If $k < \sqrt{2}$, then T has a fixed point in C.

Stability of the FPP in ℓ_2

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A consequence: Suppose that

- $|\cdot|$ is a norm on ℓ_2 such that $a\|x\|_2 \leq |x| \leq b\|x\|_2$ for all $x \in X$.
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Then T is $\|\cdot\|_2$-k-uniformly lipschitzian with $k = \frac{b}{a}$.

Corollary

If $d(Y, \ell_2) < \sqrt{2}$, then Y has the FPP.

Three questions

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(Q1) Is $lip(\ell_2) = \sup\left\{k > 0 : k \text{ can replace } \sqrt{2} \text{ in the Theorem }\right\} < \infty$?

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- $eqv(\ell_2) \geq \sqrt{2+\sqrt{2}} \approx 1.85$ (A.Jiménez, E.Llorens, 2000)

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Three questions

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$\sqrt{2} \leq lip(\ell_2) \leq \frac{\pi}{2}$ (Baillon, 1975)

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(Q3) Is Baillon's example $|\cdot|$-nonexpansive for some norm $|\cdot|$ on ℓ_2 equivalent to $\|\cdot\|_2$?

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No (J.García, A.Jiménez, E.Llorens, 1997)

Publications on stability

- A.Jiménez, E.Llorens, Stability of the fixed point property for nonexpansive mappings. Houston J. Math. 1992.
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- A.Jiménez, J.García, E.Llorens, Stability of the fixed point property for nonexpansive maps, Handbook of Metric F.P.T. 2002
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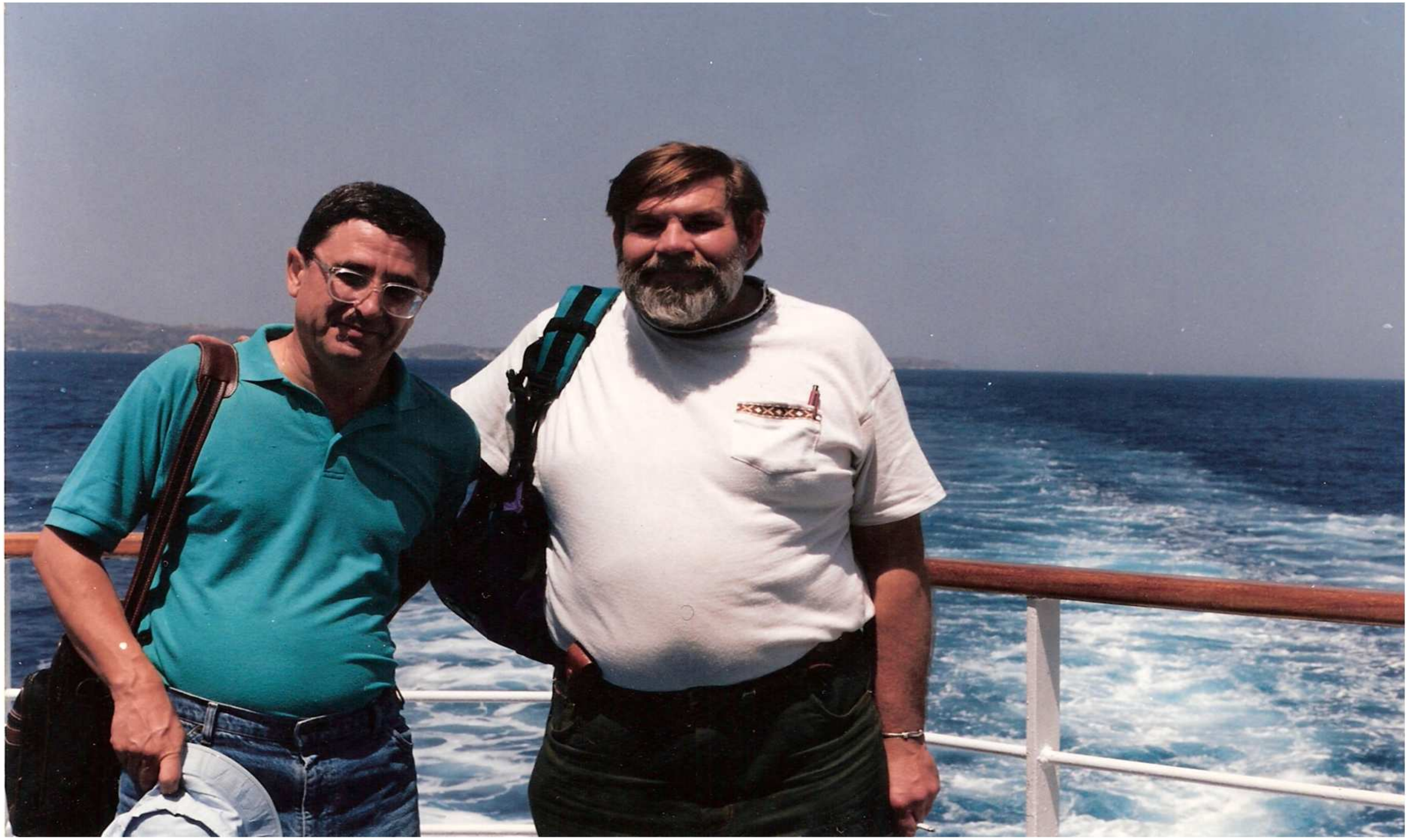
- C.A.Hernández, M.A.Japón, E.Llorens, On the structure of the set of equivalent norms on l_1 with the fixed point property, J.Math.Anal.Appl 2012.
- E. Llorens, The fixed point property for renormings of l_2, Arab.J.Math 2012.
- A.Jiménez, E.Llorens, A class of renormings of l_2 wih the fixed point property, JNCA 2013.

Popurrí of pictures

Maestro, amigo, compañero

Athens, 1996



Haifa 2001

Haifa 2001

Working in some place

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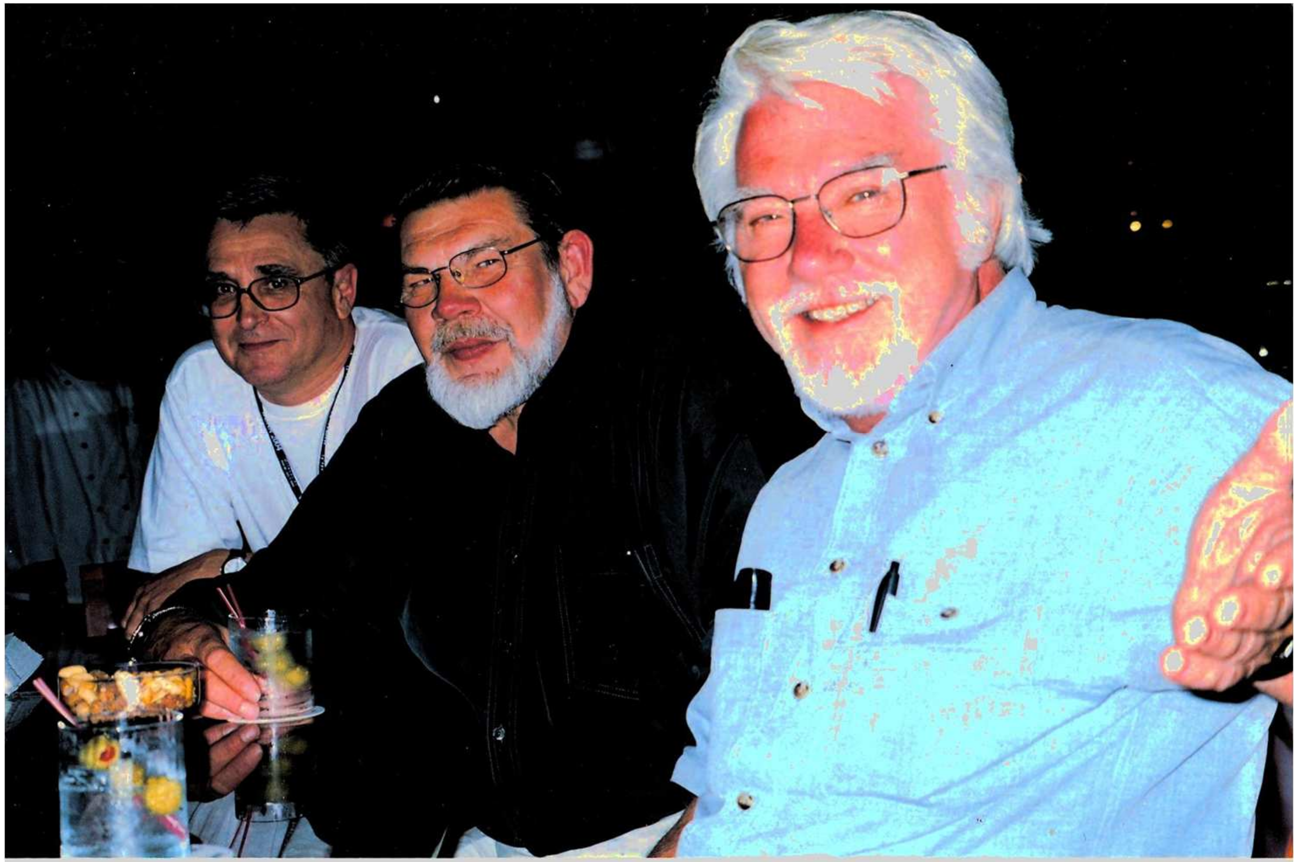
REGELE BERII ÎN ROMÂNIA

Working in Istanbul 2015

Working in Lublin

Orlando 2004



Orlando 2004

Guanajuato 2005

Cluj 2007

Aracena 2008

Aracena 2011

Málaga 2014

With Pino and family

Working again

With Adrian and family

Cluj 2007

David’s thesis

Antequera 2014

With Adrian and Jesús

Istanbul 2015

Lublin 2004

Valencia 2009

Pascua 2007

With Stan, Elisabetta and others

Rzsezow 2009

David's thesis

Istanbul 2015

Istanbul

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