Numerical observation of Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates

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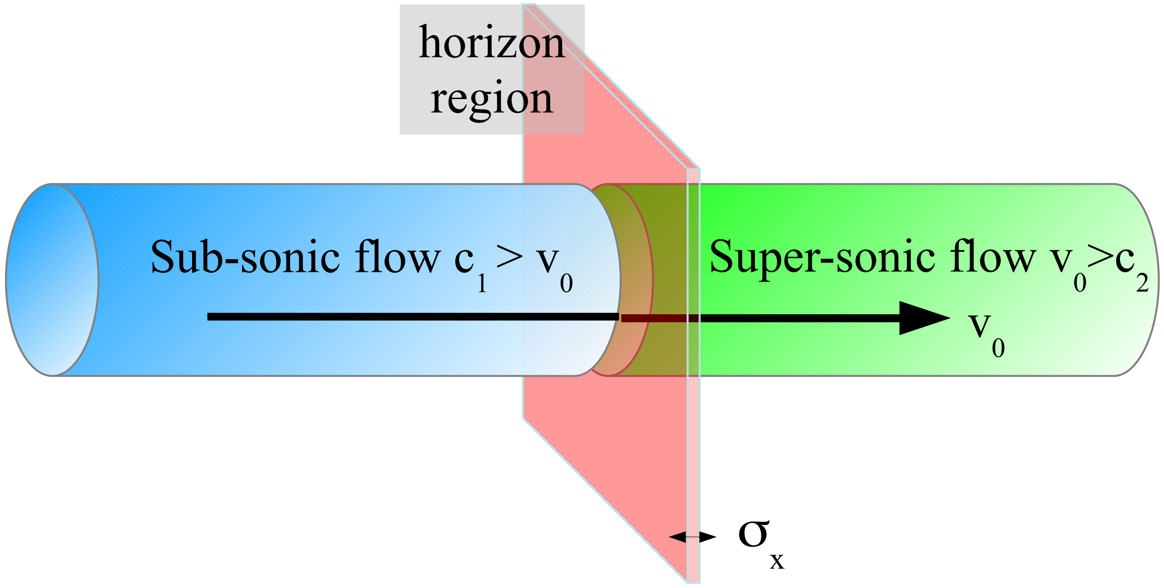
- Alessandro Fabbri (IFIC - Univ. de Valencia and CSIC, Spain)
- Nicolas Pavloff (Université Paris-Sud, Orsay, France)

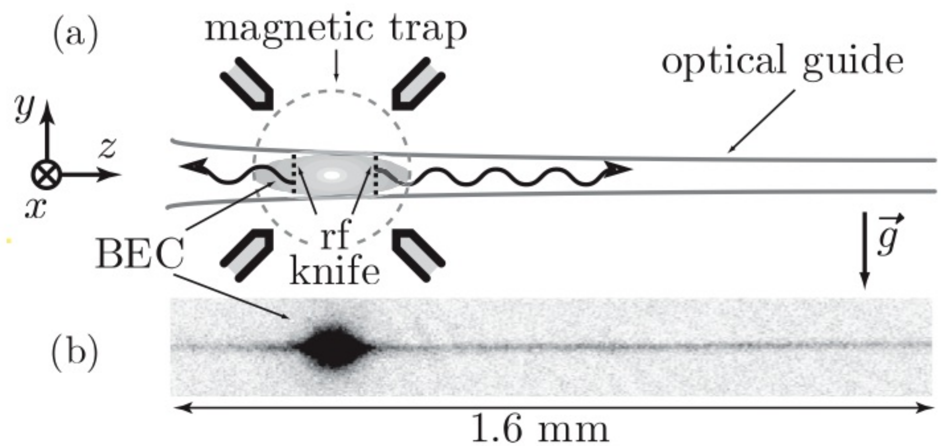
The main "experimental" challenges



Numerical simulation of BEC dynamics using Wigner-MC method

- start from uniform condensate in motion at v_0
- switch on horizon at a given time t=0 and go to black-hole regime $c_1 > v_0 > c_2$
  - minimize deterministic disturbances, e.g. Landau processes (in super-sonic region) and soliton shedding during and after switch-on
- concentrate on effect of quantum fluctuations
  - isolate (thermal) Hawking emission from background phonons (also thermal)

(i) How to create a clean black hole?



From: W. Guerin *et al*., PRL **97**, 200402 (2006)

- Out-coupled atom laser beam: uniform density and velocity v_0
- Atom-atom interaction constant initially uniform and equal to g_1
- Within σ_t around $t=0$: modulation $g_1 \to g_2$ and $V_1 \to V_2$ in $x>0$ region only
  via: Feshbach resonance (g depends on applied B) or modify transverse confinement
- Step in nonlinear coupling constant g $\Rightarrow$ step in sound speed.
- Black-hole formed if $c_1>v_0>c_2$, thickness σ_x of crossover region determines surface gravity
- Chemical potential jump to be compensated by external potential $V_1+ ng_1=V_2+ ng_2$
  allows to avoid Cerenkov-Landau phonon emission, soliton shedding

(ii) How to detect Hawking radiation?

Density-density correlation function:

$$G^{(2)}(x,x')=\frac{\langle :n(x)\,n(x'):\rangle}{\langle n(x)\rangle\,\langle n(x')\rangle}$$

Prediction of gravitational analogy:

→ entanglement in Hawking pairs gives peculiar Hawking signal in $G^{(2)}$

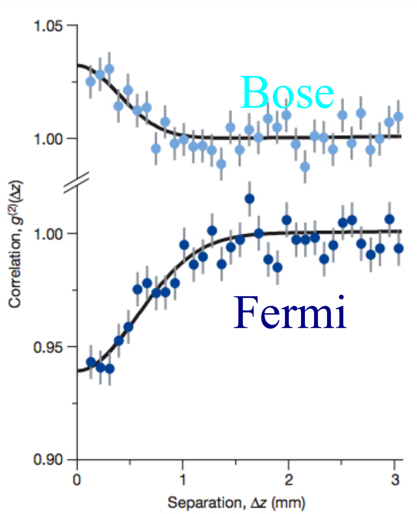
→ long-range in/out density correlations

$$G_2(x,x') = 1 - \frac{\xi_1\xi_2}{16\pi c_1 c_2}\,\frac{k^2}{\sqrt{n^2\xi_1\xi_2}}\,\frac{c_1c_2}{(c_1-v)(v-c_2)}\,\cosh^{-2}\left[\frac{k}{2}\left(\frac{x}{c_1-v}+\frac{x'}{v-c_2}\right)\right]$$

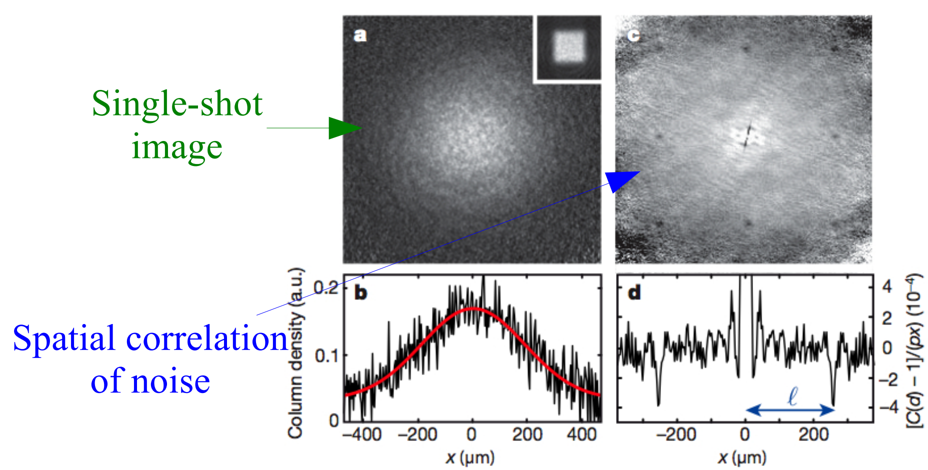
R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008).

Experimental measurements of $G^{(2)}(x,x')$

- Fully coherent BEC: $G^{(2)}(x,x') = 1$
- **Atomic HB-T**: positive correlation due to thermal Bose atoms (negative for fermions)
- Quantum correlations after collision of two BECs revealed
- Noise correlations in **TOF picture after expansion from lattice**
- Correlations in products of molecular dissociation from molecular BEC

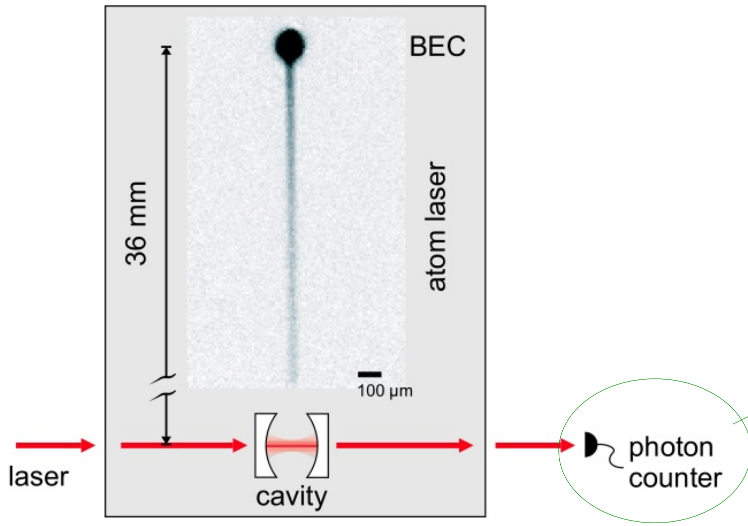


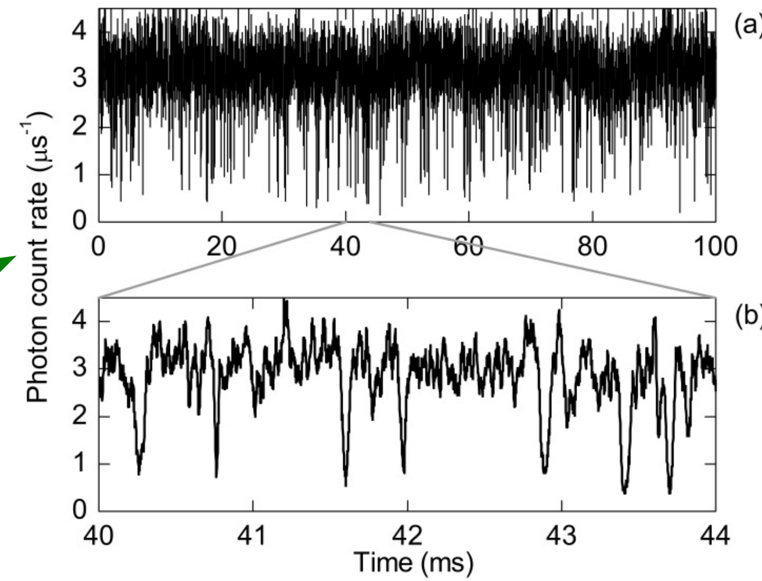
Jeltes *et al.*, Nature 445, 402 (2007)

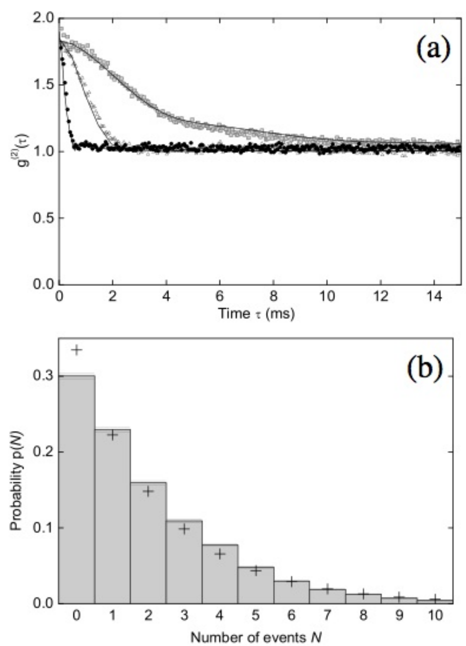


Rom *et al.*, Nature 444, 733 (2006)

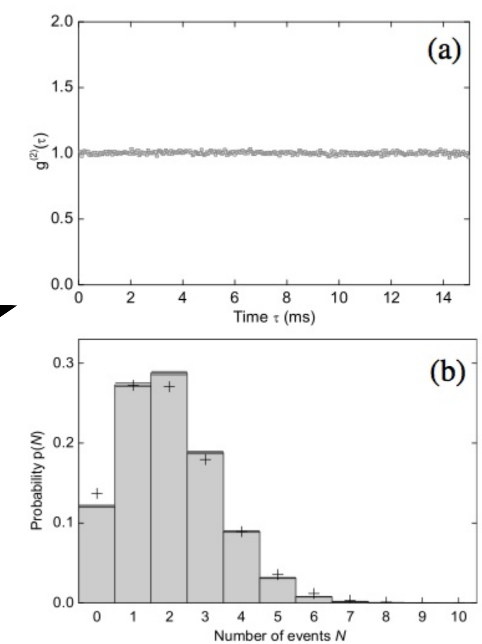
Atom counting at the single atom level







Thermal atom beam



Coherent atom laser beam
Poissonian statistics

A. Öttl, *et al.* Phys. Rev. Lett. **95**, 090404 (2005)

The numerical method: Wigner-Monte Carlo

At t=0, homogeneous system:

- Condensate wavefunction in plane-wave state
- Quantum + thermal fluctuations in plane wave Bogoliubov modes
- Gaussian α_k, variance $<|\alpha_k|^2> = [2 \tanh(E_k / 2k_B T)]^{-1} \rightarrow ½$ for $T \rightarrow 0$.

$$\psi(x, t=0) = e^{i k_0 x} \left[\sqrt{n_0} + \sum_k \left(u_k e^{i k x} \alpha_k + v_k e^{-i k x} \alpha_k^* \right) \right]$$

At later times: evolution under GPE

$$i\hbar \, \partial_t \psi(x) = -\frac{\hbar^2}{2\mathrm{m}} \partial_x^2 \psi(x) + V(x) \psi(x) + g(x) |\psi(x)|^2 \psi(x)$$

Expectation values of observables:

- Average over noise provides symmetrically-ordered observables

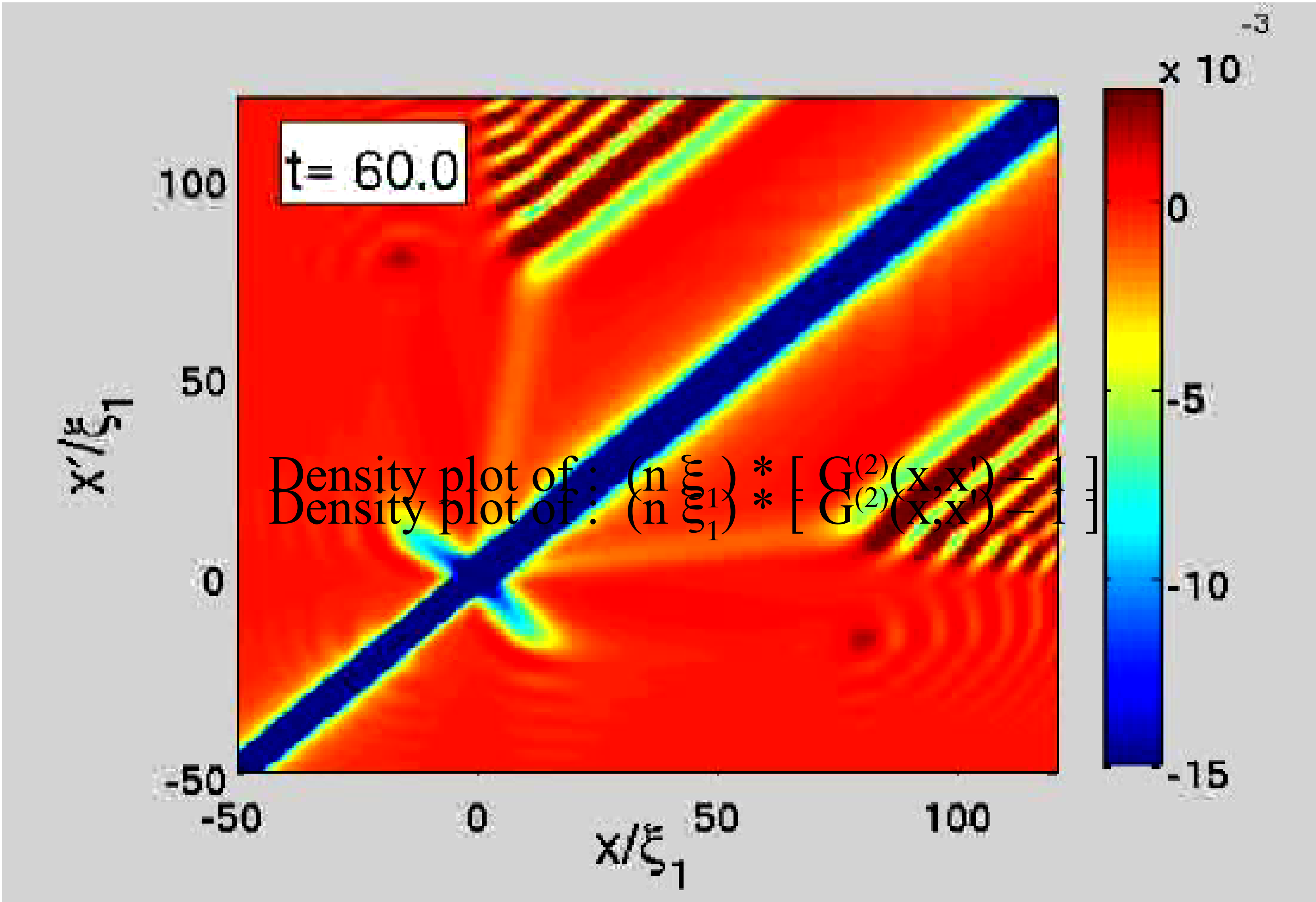
$$\langle \psi^*(x) \, \psi(x') \rangle_W = \frac{1}{2} \langle \hat{\psi}^\dagger(x) \, \hat{\psi}(x') + \hat{\psi}(x') \, \hat{\psi}^\dagger(x) \rangle_Q$$

Equivalent to Bogoliubov, but can explore longer-time dynamics

A. Sinatra, C. Lobo, Y. Castin, J. Phys. B 35, 3599 (2002)

The movie !!!

Density plot of : $(n\,\xi_1) * [\, G^{(2)}(x,x') - 1 \,]$



Density plot of : $(n\,\xi_1) * [\, G^{(2)}(x,x') - 1 \,]$
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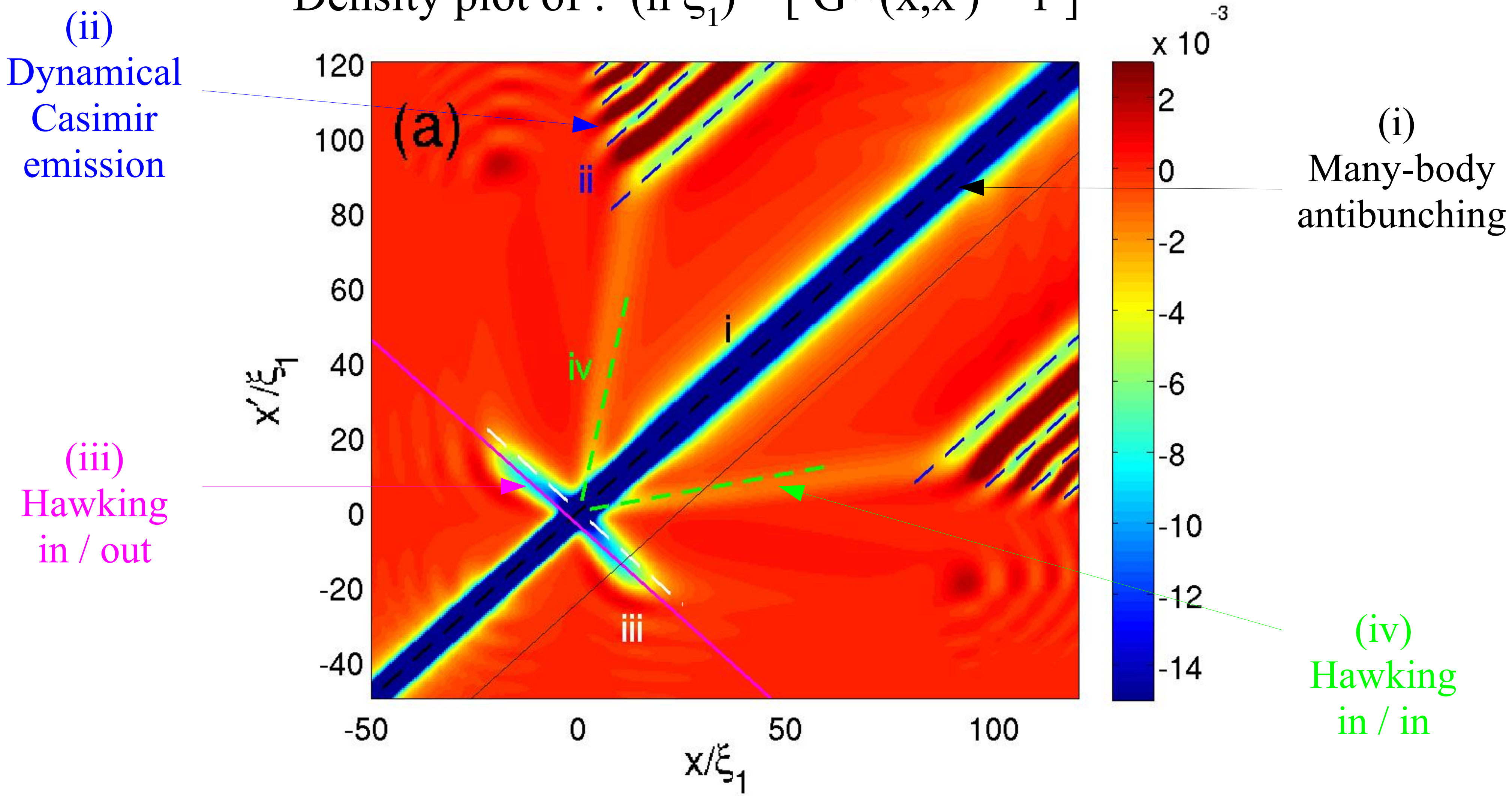
IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

The numerical observations

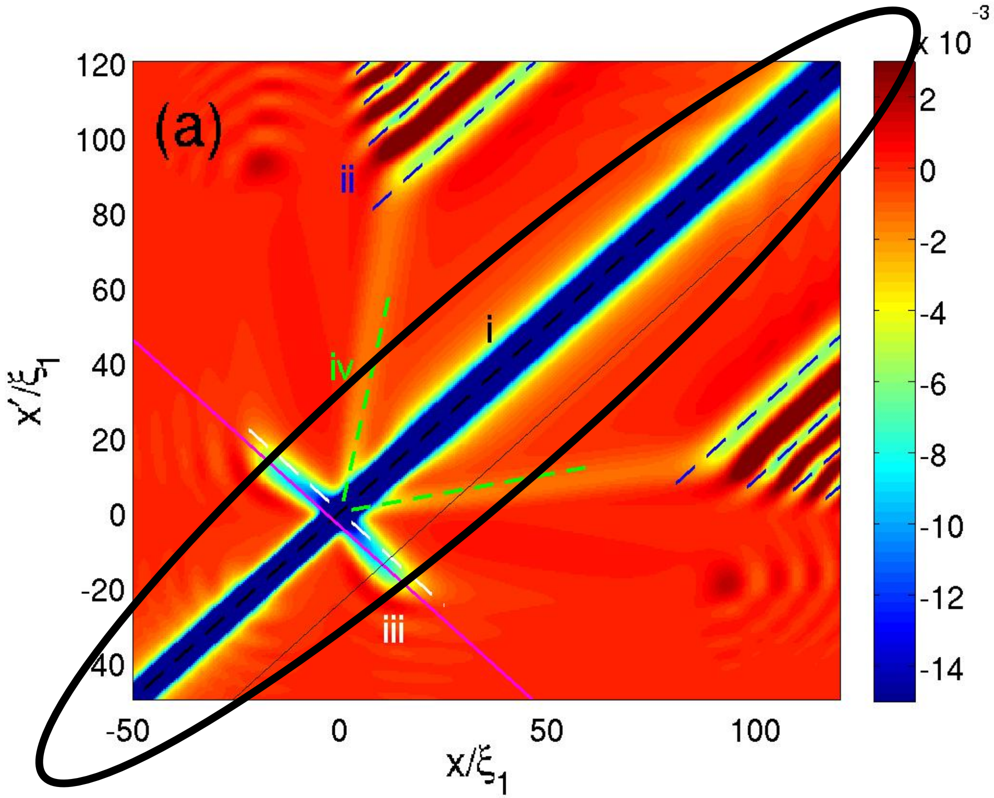
Density plot of : $(n\,\xi_1) * [\,G^{(2)}(x,x') - 1\,]$



IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

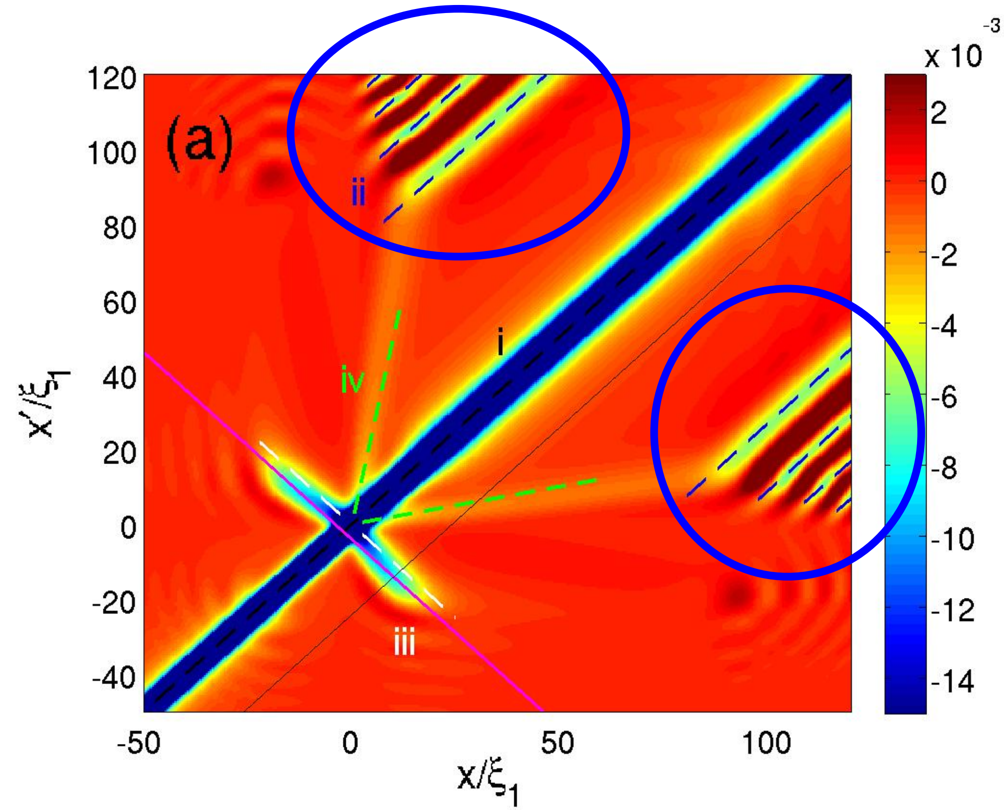
Feature (i) : Many-body antibunching

- present at all times
- due to repulsive interactions
- almost unaffected by flow



See e.g.: M. Naraschewski and R. J. Glauber, PRA 59, 4595 (1999)

Feature (ii): Dynamical Casimir emission of phonons



Fringes parallel to main diagonal

- intensity depends on speed of switch-on
- only in x>0 region, move away in time
- do not depend on flow pattern,

  also present in homogeneous system

Physical explanation:

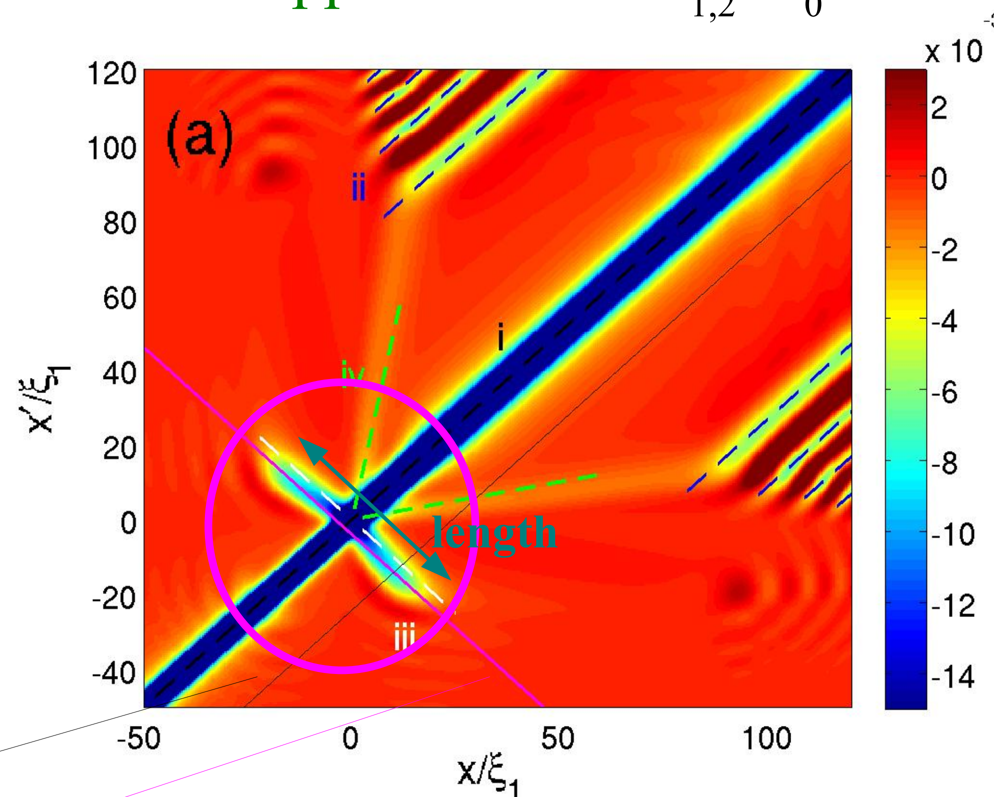
- in x>0 region $g_1 \rightarrow g_2$ within short time σ_t :
- non-adiabatic time modulation of Bogoliubov vacuum
- phonon pair emission at t=0, from all points x>0
- fringes depend on |x-x'|: quantum correlations in counter-propagating pairs
- correlations propagate away at speed ≥ $2c_s$

<u>See also:</u> M. Kramer *et al.* PRA **71**, 061602(R) (2005); K. Staliunas *el al.*, PRL **89**, 210406 (2002).

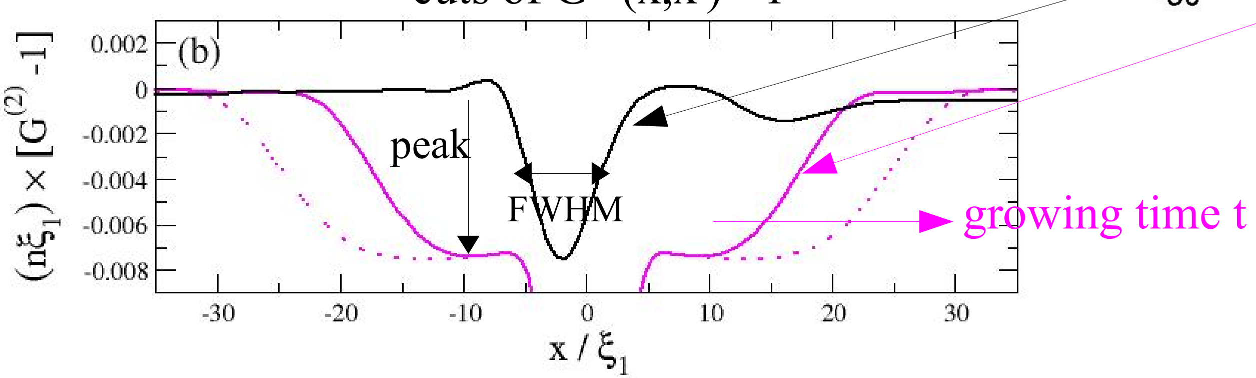
Feature (iii) : The Hawking signal

Negative correlation tongue extending from the horizon x=x'=0

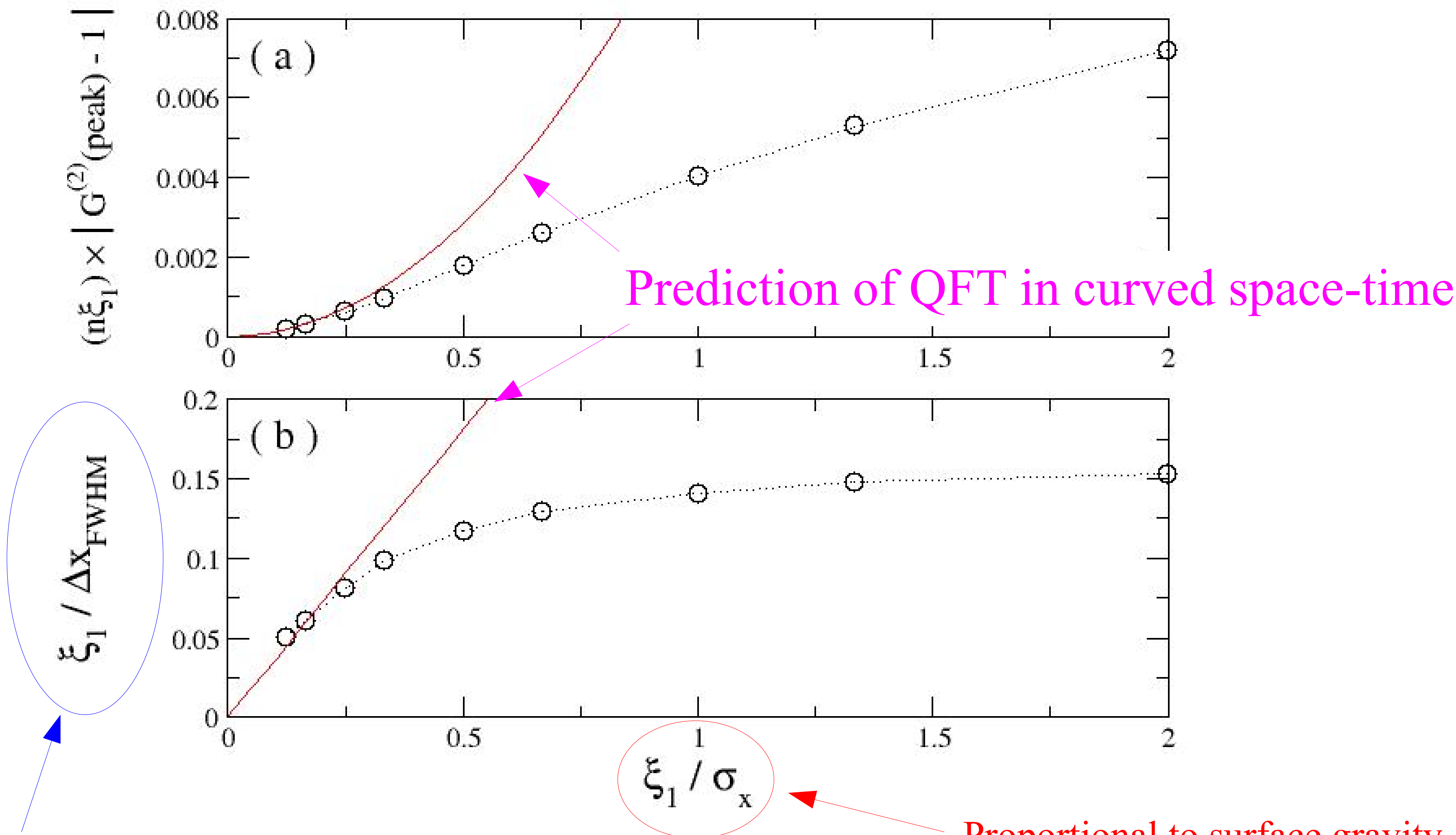
- long-range in/out density correlation which disappears if both $c_{1,2}<v_0$
- length grows linearly in t
- peak height, FWHM constant in t
- slope $\frac{v_0-c_2}{v_0-c_1}$ agrees with theory
  - pairs emitted at all t from horizon
  - propagate at sound speed



cuts of $G^{(2)}(x,x')$ - 1



Quantitative analysis

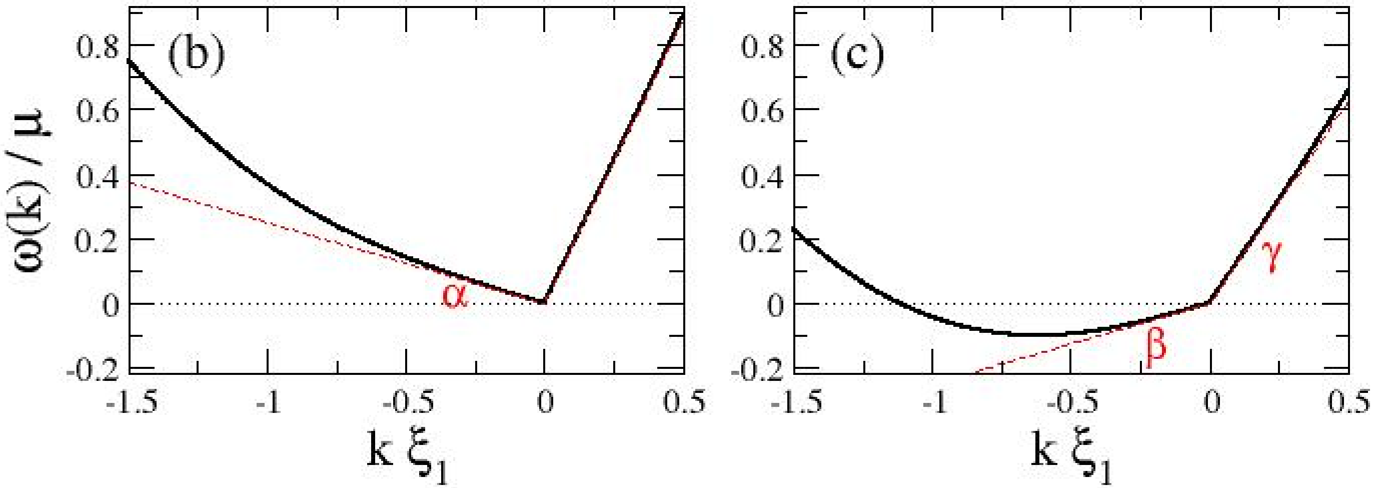


Proportional to surface gravity

Proportional to emission temperature

Analog model prediction quantitatively correct in hydrodynamic limit $\xi_1 / \sigma_x \ll 1$

Features (iii,iv) : More on the Hawking signal



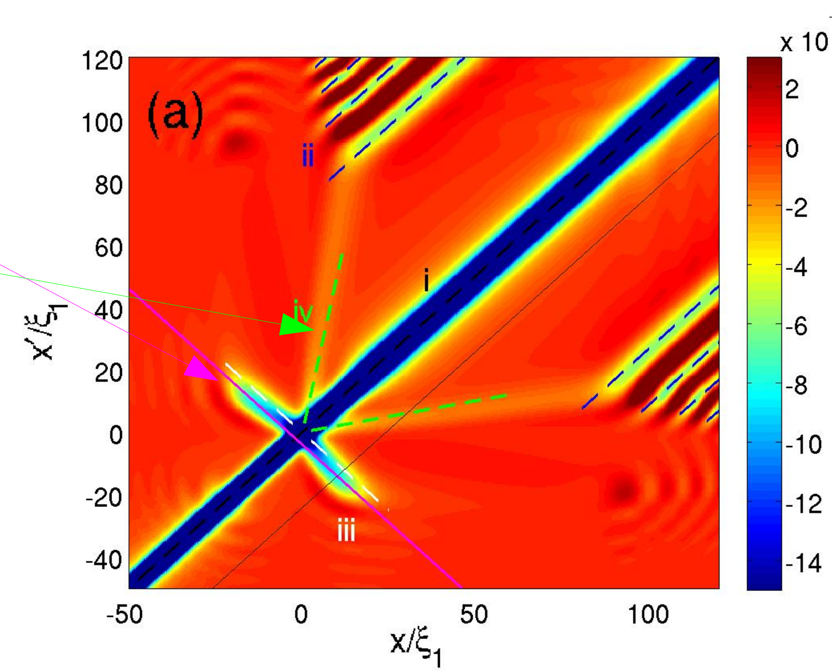
Two parametric "Hawking" processes:

- in/out: vacuum → α + β   (feature iii)
- in/in: vacuum → β + γ   (feature iv)

Energy conserved only if sub/super-sonic

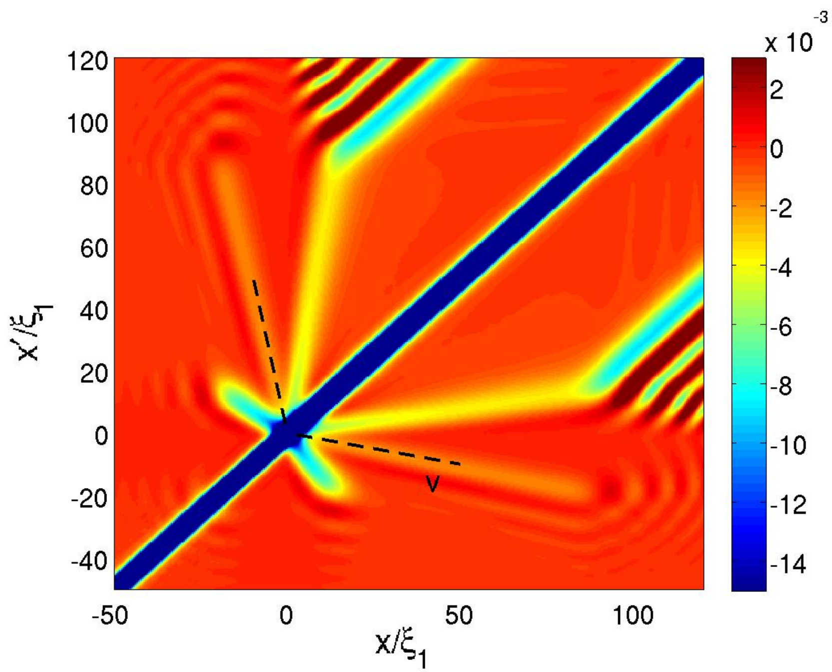
Momentum provided by horizon

Slope of tongues $\frac{v_0 - c_2}{v_0 - c_1} \simeq -1$, $\frac{v_0 - c_2}{v_0 + c_2} \simeq \frac{1}{5}$

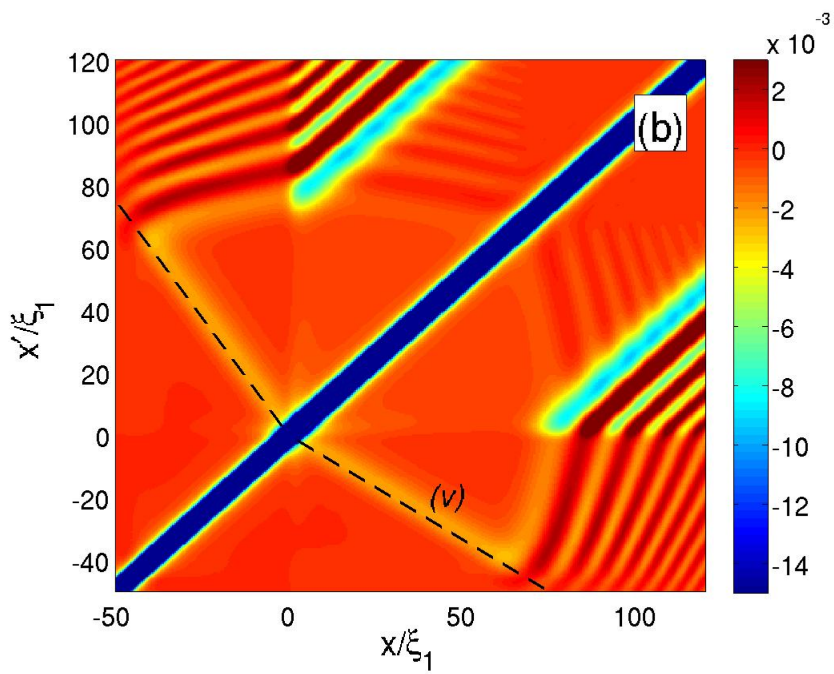


Effect of an initial T>0

- Hawking signal remains visible also for initial T comparable to T_H
- Stimulated Hawking emission
- Extra tongues (v) due to partial scattering of thermal phonons on horizon
- distinguishable from Hawking emission by different slope $\frac{(v_0 - c_1)}{(v_0 + c_2)}$



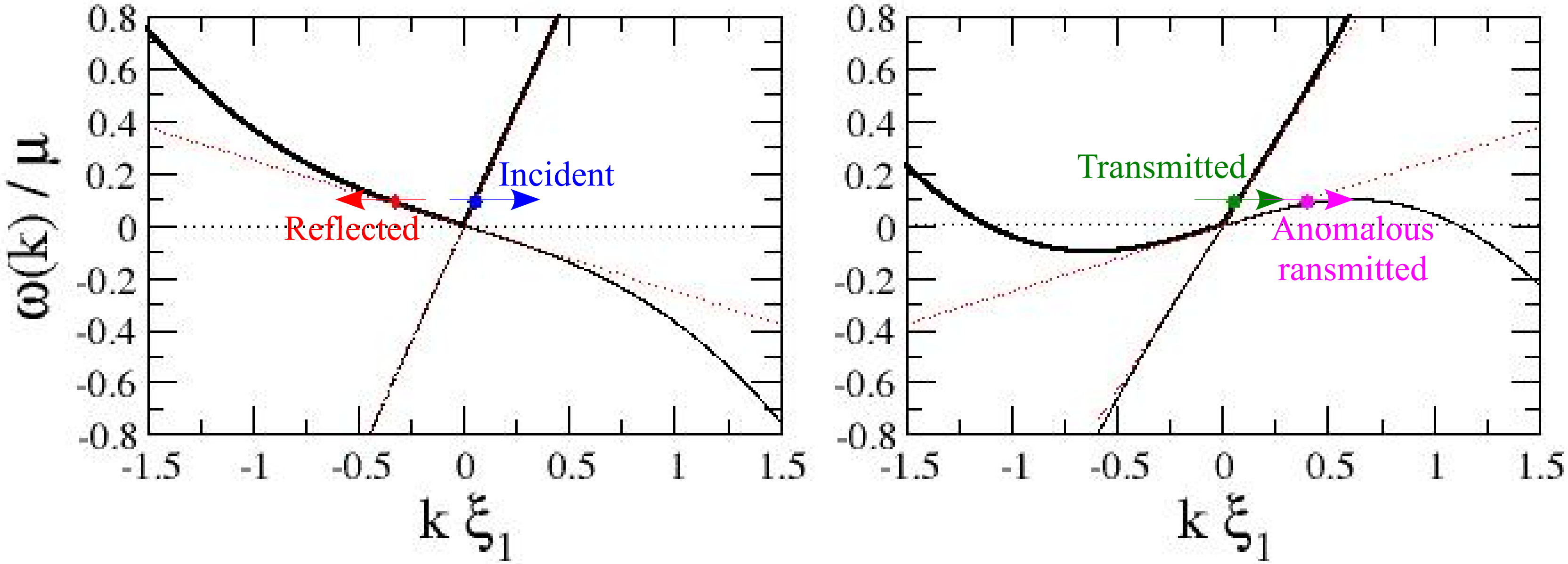
with Black Hole



without Black Hole

How I physically understand Hawking radiation

Bogoliubov dispersion on sub- and super-sonic sides

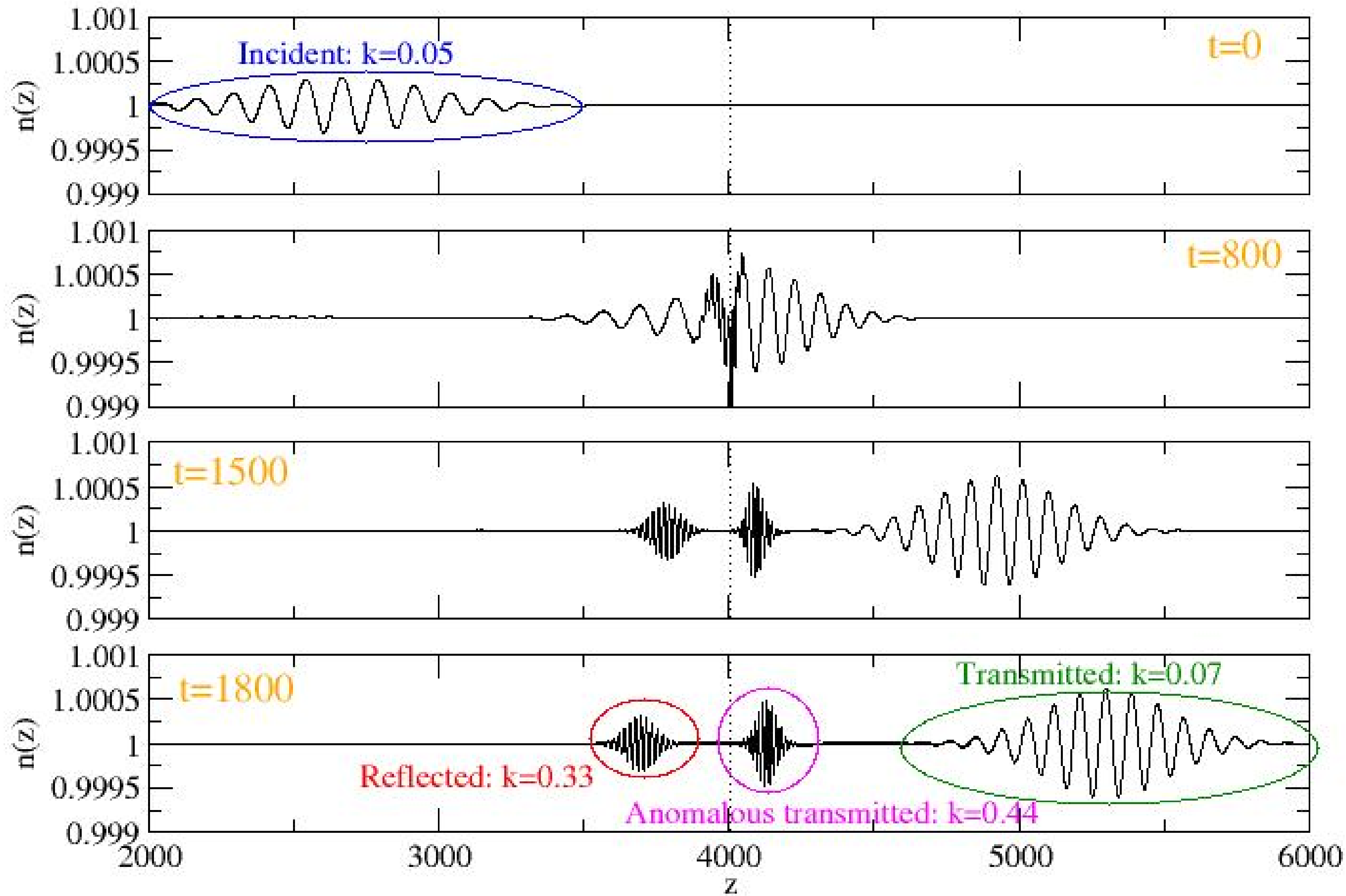


Incident plane wave → reflected, transmitted and anomalous transmitted

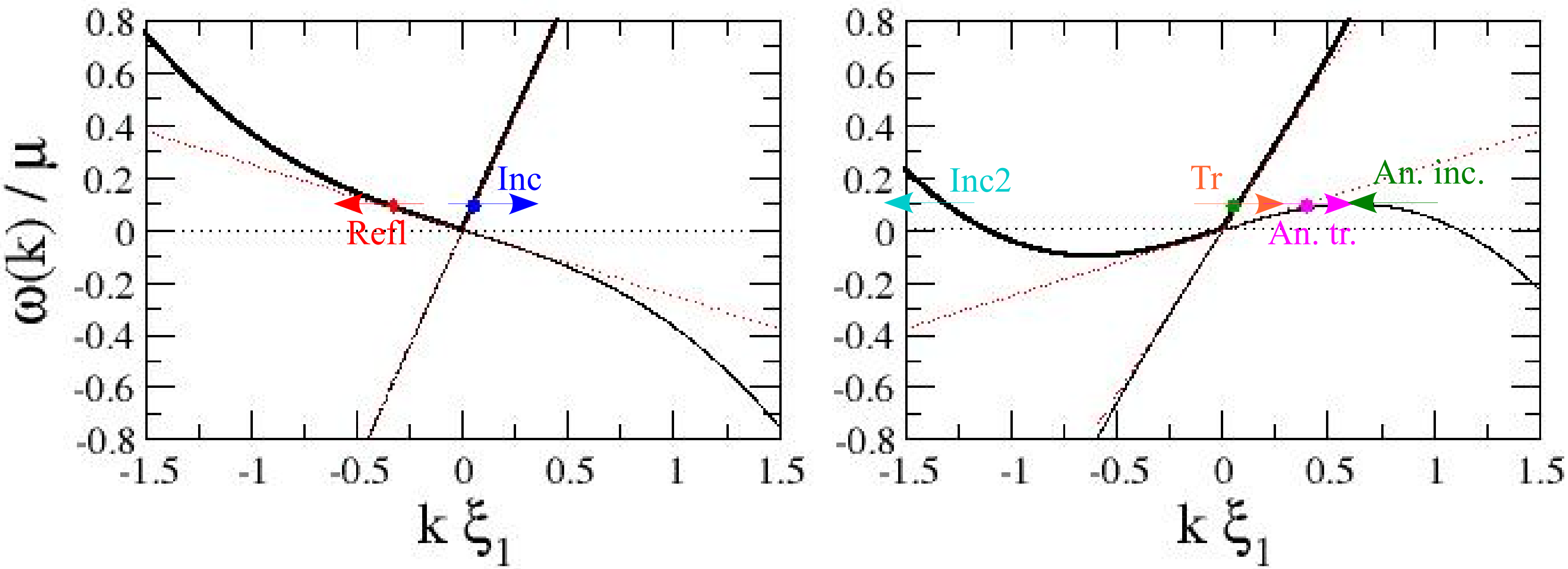
Anomalous transmitted wavepacket only exists if black hole $c_1 > v_0 > c_2$

Similar phenomenology to classical hydrodynamics experiments

(Classical) wavepacket dynamics



Hawking radiation : parametric emission of phonon pairs



Input-output formalism of quantum optics $\begin{pmatrix} a_{refl} \\ a_{tr} \\ a^{\dagger}_{an.tr} \end{pmatrix} = M \begin{pmatrix} a_{inc} \\ a_{inc2} \\ a^{\dagger}_{an.inc.} \end{pmatrix}$

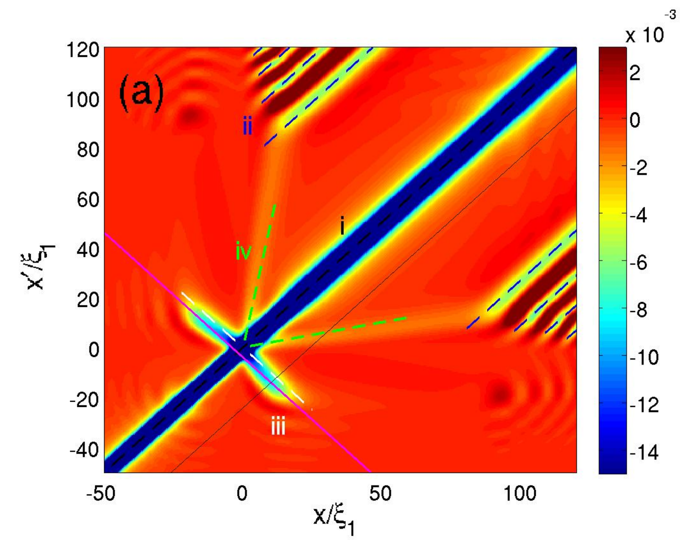
- $a^{\dagger}_{an.inc.}$, $a^{\dagger}_{an.tr.}$ creation operators for Bogoliubov "ghost" branch
- zero-point fluctuations in incident beam becomes real transmitted particles

$$\langle a^{\dagger}_{an.tr.} a_{an.tr.} \rangle = |M_{3,3}|^2 \langle a_{an.inc.} a^{\dagger}_{an.inc.} \rangle + \ldots$$

- energy conserved thanks to super-sonic flow; momentum provided by horizon

See also: Leonhardt & Philbin, cond-mat/arXiv:0803.0669 and Macher-Parentani's talks.

Why correlations?



Quantum correlations in emitted pairs:

- $\langle a_{refl.}\, a_{an.tr.}\rangle = M_{1,3}\ M^{*}_{3,3}\ \langle a_{an.inc.}\, a^{\dagger}_{an.inc.}\rangle$
- $\langle a_{tr.}\, a_{an.tr.}\rangle = M_{2,3}\ M^{*}_{3,3}\ \langle a_{an.inc.}\, a^{\dagger}_{an.inc.}\rangle$

- two-mode squeezing, thermal statistics when looking at one component
- simultaneous emission at all times t at horizon position
- propagate from the horizon with group velocity
- visible in density correlation function as signal peaked on lines $\frac{x}{v_{g1}} = \frac{x'}{v_{g2}}$
- slopes determined by c_1, v_0, c_2:
  - in-out: $v_{g1}=v_0-c_1$, $v_{g2}=v_0-c_2$ $\qquad \frac{v_0-c_2}{v_0-c_1} \simeq -1$
  - in-in: $v_{g2}=v_0+c_2$, $v_{g2}=v_0-c_2$ $\qquad \frac{v_0-c_2}{v_0+c_2} \simeq \frac{1}{5}$

Outlook

Analog Hawking radiation of phonons from acoustic black-hole numerically observed via density correlation function

- microscopic simulations of condensate dynamics
- parametric emission of entangled phonon pairs from the horizon
- propagating phonons responsible for correlated density fluctuations
- Hawking signal easily distinguished from other processes
  (e.g. Landau-Cerenkov, background thermal phonons)
- appreciable signal intensity for realistic parameters,
  worst enemy: atomic shot noise
- trans-Planckian, high-k modes in horizon region under control

R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008)
IC, S. Fagnocchi, A. Recati, R. Balbinot, A. Fabbri, New J. Phys. 10, 103001 (2008)