Flowers & Friends in Frankfurt

A workshop on geometric analysis

Abstracts

On the area of the graph of a discontinuous map from the plane into the plane

Giovanni Bellettini (Tor Vergata, Rome)

We shall discuss one possible way of determining the area of the graph of a discontinuous map from the plane into the plane. The problem consists in understanding the optimal way to approximate a two-codimensional nonsmooth graph with two-codimensional smooth graphs. In contrast to the codimension one case, it turns out that the area functional is non local. We shall focus attention on this nonlocality phenomenon, in particular for a triple junction type map, and for a map which is discontinuous along a smooth simple compact curve.

Classification of decomposable open 3-manifolds

Gérard Besson (Grenoble)

We will consider open 3-manifolds which are modelled on a graph, the vertices being taken in a finite family of closed and prime 3-manifolds and the edges corresponding to connected sums. We will address the question : when are two such manifolds homeomorphic? A criterion will be given for the case when the finite family of building blocks is fixed.

Multiparameter sweepouts and the existence of minimal hypersurfaces – II

Fernando Codá Marques (Princeton)

This is the second part of the talk. For the abstract, see André Neves (below).

Approaching Plateau's problem with minimizing sequences of sets

Camillo De Lellis (Uni Zurich)

In a joint paper with Francesco Ghiraldin and Francesco Maggi we provide a compactness principle which is applicable to different formulations of Plateau's problem in codimension one and which is exclusively based on the theory of Radon measures and elementary comparison arguments. Exploiting some additional techniques in geometric measure theory, we can use this principle to give a different proof of a theorem by Harrison and Pugh and to answer a question raised by Guy David.

The surgery construction for 2-dimensional mean curvature flow

Gerhard Huisken (MFO/Tübingen)

The lecture explains joint work with Simon Brendle on mean curvature flow with surgery for embedded mean-convex surfaces in 3-manifolds. In particular it is explained how the surgery construction in a neck can be adjusted to preserve the non-collapsing property with arbitrarily small errors.

Flow of curves by curvature in R^n

Tom Ilmanen (ETH)

(partly j/w J. Hättenschweiler) Let c be a closed curve immersed in R^n. Following Altschuler-Grayson & Perelman, we prove existence of an (essentially unique) weak evolution c(t) by curvature in R^n satisfying:

(1) Each c(t) is a piecewise C^1 curve that is smooth except for a finite set of singular points S(t).

(2) c(s) converges in C^0 to c(t) as s -> t from the past (indeed in C^1 except near the singular points).

(3) Possibly, some "doubled" segments (= hairs) of c(t) can instantaneously vanish for s > t. There are only finitely many hairs in space-time.

(4) The singular set $S = U_t S(t)\times\{t\}$ together with the union of the hairs is a closed set in spacetime with 1-dim parabolic Hausdorff measure bounded. In particular, c(t) is smooth except on a closed set of times of Hausdorff dimension 1/2.

(5) Each singularity is either a simple cusp, modelled on a grim reaper on a small scale, or a multicusp, modelled on several grim reapers with a common axis but opposite directions. There are only finitely many multicusps.

Universal behaviors in geometric heat flows

Dan Knopf (UT Austin)

Geometric heat flows like Ricci flow and mean curvature flow have proven to be remarkably successful tools to investigate the geometry and topology of manifolds. They are nonlinear PDE with a diffusion-reaction structure that makes them prone to finite-time singularities. But these singularities are aids, not obstacles, to these flows' applications — because regions of high curvature tend to be very special. In this talk, we will survey these phenomena and present evidence in favor of the conjectures that (1) solutions asymptotically acquire extra symmetries as they become singular, and (2) generic solutions may be constrained to a small catalog of universal asymptotic profiles.

Local solutions to a free boundary problem for the Willmore functional

Ernst Kuwert (Freiburg)

We consider a free boundary problem for the Willmore functional. Given a smooth domain Ω in R^3, we construct Willmore disks which are critical in the class of surfaces meeting Ω orthogonally along their boundary and having small prescribed area. Using rescaling we first obtain constrained solutions with prescribed two-dimensional barycenter, and then study the variation of the barycenter.

Rigidity and non-rigidity results for conformal immersions

Tobias Lamm (KIT)

In this talk we review some rigidity and non-rigidity results for conformal immersions from the sphere, which we obtained in joint works with Huy Nguyen resp. Reiner Schätzle.

Asymptotics of the hyperkähler metric on the Hitchin moduli space

Rafe Mazzeo (Stanford)

This is a report on some recent and ongoing work, with Swoboda, Weiss and Witt, on a new construction of large solutions in the moduli space of stable Higgs bundles on a Riemann surface. The ultimate goal is to obtain information about the L^2 metric on this moduli space which should hopefully correlate with some far-ranging predictions from physics. The broader context here is the study of degenerating solutions in gauge theory, and if time permits, there will also be a discussion of some related work concerning the Kapustin-Witten equations.

Einstein Manifolds and the Codimension Four Conjecture

Aaron Naber (Northwestern)

In this talk we discuss the recent solution of the codimension four conjecture. Roughly, this tells us that if we study a noncollapsing limit of Einstein manifolds $(M^n,g_i)\to(X,d)$, or more generally just manifolds with bounded Ricci curvature, then X is smooth away from a closed set of codimension four. Using the quantitative stratification one can use this to prove a priori L^p estimates for the curvature $|Rm|$ on Einstein manifolds for all $p<2$, and L^2 bounds in dimension four. Another application is the proof of Anderson's finite diffeomorphism conjecture. This is joint work with Jeff Cheeger.

Multiparameter sweepouts and the existence of minimal hypersurfaces – I

André Neves (Imperial)

It follows from the work of Almgren in the 1960s that the space of unoriented closed hypersurfaces, in a compact Riemannian manifold M, endowed with the flat topology, is weakly homotopically equivalent to the infinite dimensional real projective space. We have used this nontrivial structure, together with previous work of Gromov and Guth on the associated multiparameter sweepouts, to prove the existence of infinitely many smooth embedded closed minimal hypersurfaces in manifolds with positive Ricci curvature. This is motivated by a conjecture of Yau (1982). We will discuss the Almgren-Pitts min-max theory in general and if time permits the problem of the Morse index.

Qualitative properties of dynamical systems arising in biology

Alan Rendall (JGU Mainz)

This talk is concerned with the application of geometric singular perturbation theory and bifurcation theory to issues related to the qualitative behaviour of solutions of systems of differential equations describing biological processes. With these methods it is possible to prove the existence of multiple stable stationary solutions and periodic solutions of systems of equations of high dimension. The general theory will be illustrated by a prime example, the MAP kinase cascade. It is both a test case for learning about models for more general biochemical systems and a subject with potential medical applications.

A local regularity theorem for mean curvature flow with triple edges

Felix Schulze (UCL)

We consider the evolution by mean curvature flow of surface clusters, where along triple edges three surfaces are allowed to meet under an equal angle condition. We show that any such smooth flow, which is weakly close to the static flow consisting of three half-planes meeting along the common boundary, is smoothly close with estimates. Furthermore, we show how this can be used to prove a smooth short-time existence result. This is joint work with B. White.

Scattering for a critical nonlinear wave equation in 2 space dimensions

Michael Struwe (ETH)

In joint work with Martin Sack we show that the solutions to the Cauchy problem for a wave equation with critical exponential nonlinearity in 2 space dimensions scatter for arbitrary smooth, compactly supported initial data.

On Singular Liouville systems in the study of non-abelian Chern-Simons vortices

Gabriella Tarantello (Tor Vergata, Rome)

Motivated by the construction of non-abelian Chern-Simons vortex configurations of "non-topological" type as well as other type of soliton configurations, we discuss recent results concerning the solvability of planar (singular) Liouville systems in terms of some total integrals (fluxes).

Refined asymptotics of the Teichmueller harmonic map flow

Peter Topping (Warwick)

The field of geometric flows started when Eells and Sampson performed a gradient flow on the harmonic map energy in 1964, in order to find harmonic maps. Rupflin and I showed how a slightly different gradient flow on the same functional can be crafted to find minimal immersions instead. In this talk I will sketch the background to this flow, and then go on to describe some new work that will appear around the time of the workshop. Joint work with Melanie Rupflin and Tobias Huxol.

On isoperimetric profiles for the 1st nontrivial Neumann & Steklov eigenvalues in Riemannian manifolds

Tobias Weth (Frankfurt)

We consider the geometric variational problems of maximizing the first nontrivial Neumann and Steklov eigenvalues of the Laplace-Beltrami operator among subdomains of a Riemannian manifold under a fixed volume constraint. We will mainly be concerned with the corresponding local isoperimetric (or, more precisely, isochoric) profiles. As a corollary of our analysis, we deduce local isoperimetric comparison principles depending only on the scalar curvature.

Boundary behavior in mean curvature flow

Brian White (Stanford)

Consider a one-parameter family of hypersurfaces whose interiors move by mean curvature and whose boundaries either are fixed or, more generally, have prescribed motion. We discuss existence of solutions, a strong boundary regularity theorem, and also examples of boundary singularities.

Averaging holonomy & curvature on manifolds with lower sectional curvature bound

Burkhard Wilking (Münster)

We show that an almost nonnegatively curved spin manifold has a vanishing Â-genus. We will argue by contradiction and assume that the Dirac operator has kernel. This gives rise to an almost parallel spinor. The spinor is thus nearly parallel along most curves. We will explain how this affects the averaged curvature operator in suitable coordinates.