# Singular quasilinear elliptic problems

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We review some existence results [1, 2, 4, 6, 7, 8] for the quasilinear boundary value problem

$$\begin{aligned} -\Delta u + g(u)|\nabla u|^2 = f(x), \quad x \in \Omega, \\ u = 0, \quad x \in \partial\Omega, \end{aligned}$$

where $\Omega$ is a bounded open in $\mathbb{R}^N$, $g$ is a continuous function with a singularity at zero, and $f$ is a suitable integrable function.

Supplementing this result with the uniqueness result proved jointly with Sergio Segura in [5], we use by the first time the Bifurcation Theory to study [3] the case of nonlinear data, i.e. of the problem

$$\begin{aligned} -\Delta u + g(u)|\nabla u|^2 = f(x,u), \quad x \in \Omega, \\ u = 0, \quad x \in \partial\Omega, \end{aligned}$$

with $f$ a suitable Carathéodory function.

# References

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